Lecce Conference in Calculus of Variations and Partial Differential Equations

October 4–7, 2016

E. ACERBI: Relaxation of second order one-dimensional energies

As a preliminary step to the case of surfaces, we study the relaxation of plastic and elastic energies depending on curvature and defined on regular curves in high codimension. Through an extension to suitable currents, the notions of generalized length and generalized curvature (in the plastic case) or p-curvature (in the elastic one) are given, through which an easily readable representation formula is provided.

L. AMBROSIO: New estimates on the matching problem

The matching problem consists in finding the optimal coupling between a random distribution of N points in a d-dimensional domain and another (possibly random) distribution. There is a large literature on the asymptotic behavior as N tends to infinity of the expectation of the minimum cost, and the results depend on the dimension d and the choice of cost, in this random optimal transport problem. In a recent work, Caracciolo, Lucibello, Parisi and Sicuro proposed an *ansatz* for the expansion in N of this expectation. I will ilustrate how a combination of semigroup smoothing techniques and Dacorogna-Moser interpolation provides first rigorous results for this *ansatz*.

Joint work, in progress, with Federico Stra and Dario Trevisan.

J. BALL: Remarks on incompatible and compatible sets of matrices

The talk will discuss various results (mostly drawn from joint work with R.D. James) concerning compact sets of matrices that are compatible or incompatible for gradient Young measures.

F. BETHUEL: Concentration sets for multiple-well problems in the $2D$ elliptic case

Whereas the formation of sharp interfaces in multiple well problems is quite well understood in the scalar case (e.g Modica-Mortola or Allen-Cahn problems), the vectorial case presents new difficulties and remains therefore widely open. I will present some progresses in the two-dimensional elliptic case.

L. BOCCARDO: No Fusco, no BV

We assume that Ω is a bounded, open subset of $\mathbb{R}^N$, $p > 1$, $f \in L^m$, $m \geq 1$, and we consider the <u>model case</u> of a nonlinear boundary value problem

$$\begin{cases} -\mathrm{div}\big(a(x)|\nabla u|^{p-2}\nabla u\big) = f(x), & \text{in } \Omega; \\ u = 0, & \text{on } \partial\Omega. \end{cases} \tag{1}$$

If $f \in L^m$, $\quad m \geq (p^*)'$, the summability of u (depending on m) was proved by G. Stampacchia and B-Giachetti (see also [B-Croce, book]). However, the summability of ∇u is not a strictly increasing function of m even in the linear case (see [Boccardo 2014]). In the case $f \in L^m$, $1 \leq m < (p^*)'$, is proved in [B-Gallouet, JFA 1989], [B-Gallouet, CPDE 1992], the existence of a distributional solution $u \in W_0^{1,[(p-1)m]^*}(\Omega)$ if $\sup\{1, \frac{N}{N(p-1)+1}\} < m < (p^*)'$. If $1 \leq m < \frac{N}{N(p-1)+1}$, the definition of entropy solution is needed ([paper-6]). If we assume $f \in L^m$, $m = \frac{N}{N(p-1)+1}$, $1 < p < 2 - \frac{1}{N}$, then there exists a distributional solution $\boxed{u \in W_0^{1,1}(\Omega)}$ [B-Gallouet 2012]. The presence of lower order terms depending on u [Cirmi 1995] or depending on ∇u [B-Gallouet 1993] produces finite energy solutions even if $m < (p^*)'$. In a joint paper [preprint] with R. Cirmi we study some borderline cases of m which give solutions in $\boxed{W_0^{1,1}(\Omega)}$.

A. BURCHARD: On the extremals of the Polya-Szego inequality

The Polya-Szego inequality states that the p-norms of the gradient generally decrease under symmetric decreasing rearrangement. Brothers and Ziemer [1988] discovered that there are non-trivial cases of equality even when $p > 1$. I will discuss recent work with A. Ferone, where, taking up from results of Cianchi and Fusco [2006], we use polar factorization to describe these equality cases.

G. BUTTAZZO: Symmetry breaking for a problem in optimal insulation

In the present talk we consider the problem of optimally insulating a given domain Ω of $\mathbb{R}^d$; this amounts to solve a nonlinear variational problem, where the optimal thickness of the insulator is obtained as the boundary trace of the solution. We deal with two different criteria of optimization: the first one consists in the minimization of the total energy of the system, while the second one involves the first eigenvalue of the related differential operator. Surprisingly, the second optimization problem presents a symmetry breaking in the sense that for a ball the optimal thickness is nonsymmetric when the total amount of insulator is small enough. In the last section we discuss the shape optimization problem in which Ω is allowed to vary, too.

References:

D. Bucur, G. Buttazzo, C. Nitsch: *Symmetry breaking for a problem in optimal insulation.* J. Math. Pures Appl., (to appear), available at `http://cvgmt.sns.it/`

S.J. Cox, B. Kawohl, B.P.X. Uhlig: *On the optimal insulation of conductors.* J. Optim. Theory Appl., **100** (2) (1999), 253–263.

A. Friedman: *Reinforcement of the principal eigenvalue of an elliptic operator.* Arch. Rational Mech. Anal., **73** (1) (1980), 1-17.

I. FONSECA: Homogenization of integral energies under periodically oscillating differential constraints

A homogenization result for a family of integral energies is presented, where the fields are subjected to periodic first order oscillating differential constraints in divergence form. We will give an example that illustrates that, in general, when the differential operators have non constant coefficients then the homogenized functional may be nonlocal, even when the energy density is convex. This is joint work with Elisa Davoli, and is based on the theory of A-quasiconvexity with variable coefficients and on two-scale convergence techniques.

N. FUSCO: A stability result for the first eigenvalue of the p-Laplacian

The Faber-Krahn inequality states that balls are the unique minimizers of the first eigenvalue of the

p-Laplacian among all sets with fixed volume. We shall present a sharp quantitative form of this inequality. This extends to the case $p > 1$ a recent result proved by Brasco, De Philippis and Velichkov for the Laplacian.

S. HENCL: Approximation of $W^{1,p}$ Sobolev homeomorphism by diffeomorphisms and the signs of the Jacobian

Let $\Omega \subset \mathbb{R}^n$, $n \geq 4$, be a domain and $1 \leq p < [n/2]$, where $[a]$ stands for the integer part of a. We construct a homeomorphism $f \in W^{1,p}((-1,1)^n, \mathbb{R}^n)$ such that $J_f = \det Df > 0$ on a set of positive measure and $J_f < 0$ on a set of positive measure. It follows that there are no diffeomorphisms (or piecewise affine homeomorphisms) f_k such that $f_k \to f$ in $W^{1,p}$. This is a joint work with D. Campbell and V. Tengvall.

V. JULIN: Stability of the Gaussian isoperimetric problem

P. KOSKELA: A density problem for Sobolev spaces

The density of smooth functions in a Sobolev space is often taken as the basis for the definition. However, functions smooth up to the boundary or global smooth functions need not necessarily be dense. Classical results give this density under a segment condition. In 1987, J.L. Lewis showed that global smooth functions are dense in $W^{1,p}(\Omega)$ for $1 < p < \infty$ when Ω is a planar Jordan domain. In 2007, A. Giacomini and P. Trebeschi showed that $W^{1,2}(\Omega)$ is dense in $W^{1,p}(\Omega)$ if Ω is a bounded simply connected planar domain. We describe our joint results with Tapio Rajala and Yi Ru-Ya Zhang that generalize the above results by Lewis, Giacomini and Trebeschi.

J. KRISTENSEN: Morse-Sard type results for Sobolev mappings

The Morse-Sard theorem, and the generalizations by Dubovitskii and Federer, have numerous applications and belong to the core results of multivariate calculus for smooth mappings. In this talk we discuss extensions of these results to suitable classes of Sobolev mappings. The quest for optimal versions of the results leads one to consider possibly nondifferentiable mappings that in turn warrant new interpretations. A key point of the proofs is to show that the considered Sobolev mappings enjoy Luzin N-type properties with respect to lower dimensional Hausdorff contents. The talk is based on joint work with Jean Bourgain, Piotr Hajlasz and Mikhail Korobkov.

J. MALY: A version of the Stokes theorem

Consider a parametrized surface $\mathcal{F}$ with values in a manifold Ω. How to recognize whether $\mathcal{F}$ induces a boundary (in the sense of currents) of an integer valued BV function on Ω? We give a characterization of this situation in terms of modulus of a path family. This is a joint work with Olli Martio and Vendula Honzlová Exnerová.

P. MARCELLINI: Some remarks in the Calculus of Variations

I will discuss about some new and old problems of the Calculus of the Variations.

R. MINGIONE: Recent Progresses in Nonlinear Potential Theory

Nonlinear Potential Theory aims at reproducing, in the nonlinear setting, the classical results of potential theory concerning the fine and regularity properties of solutions to linear elliptic and parabolic equations. Potential estimates, integrability, differentiability and continuity properties of solutions are at the heart of the matter. Here we give a brief survey of a few recent results.

M. MORINI: Nonlinear stability results for the Ohta-Kawasaki energy and for the nonlocal Mullins-Sekerka flow

It has been recently shown that strictly stable critical configurations for the Ohta-Kawasaki energy are in fact isolated local minimizers with respect to small L^1-perturbations. After reviewing such results and some of their applications, we consider the associated evolution problem. More precisely, we show that such strictly stable configurations are exponentially stable for the $H^{-1/2}$-gradient flow of the Ohta-Kawasaki energy, also known as the nonlocal (or modified) Mullins-Sekerka flow.

C. SBORDONE: Recent studies on Sobolev mappings

A homeomorphism U of the unit disk $\mathbb{D} \subset \mathbb{R}^2$, $U = (u_1, u_2) : \mathbb{D} \xrightarrow{\text{onto}} \mathbb{D}$ is a *quasiharmonic map*, if $u_i \in W^{1,1}_{loc}$, $i = 1, 2$ are finite energy solutions to the system

$$\begin{cases} \operatorname{div}\big(B(y)\nabla u_1\big) = 0 \quad \text{a.e. in } \mathbb{D} \\ \operatorname{div}\big(B(y)\nabla u_2\big) = 0 \quad \text{a.e. in } \mathbb{D} \end{cases} \tag{2}$$

for a symmetric degenerate elliptic *conductivity* $B = B(y)$ i.e.

$$\frac{|\xi|^2}{H(y)} \leq \langle B(y)\xi, \xi\rangle \leq H(y)|\xi|^2 \quad \text{a.e. in } y \in \mathbb{D} \quad \forall \xi \in \mathbb{R}^2 \tag{3}$$

where $H : \mathbb{D} \to [1, \infty[$ is measurable. A sufficient condition that the Sobolev homeomorphism $U \in W^{1,1}_{loc}$ is a quasiharmonic map is that $U^{-1} \in W^{1,1}_{loc}$. This condition is not necessary because we construct a quasiharmonic map U such that $U^{-1} \in \mathrm{BV} \setminus \mathrm{W}^{1,1}_{\mathrm{loc}}$. These results are obtained in a joint paper with Luigi D'Onofrio and Roberta Schiattarella.