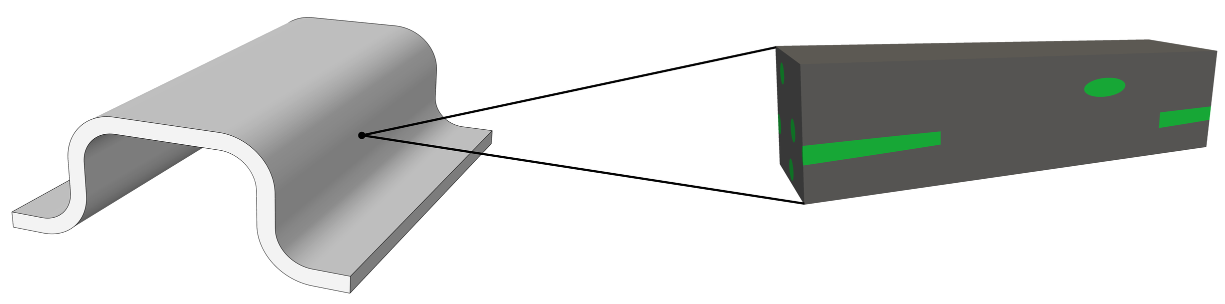
GESELLSCHAFT für
ANGEWANDTE MATHEMATIK und MECHANIK e.V.
INTERNATIONAL ASSOCIATION of APPLIED MATHEMATICS and MECHANICS

Efficient Nonlinear Homogenization

potential-based Reduced Basis Model Order Reduction (pRBMOR)

Introduction

Microscopic material heterogeneities are considered through *representative volume elements* (RVEs) in two-scale mechanical analyses. Nonlinearities generated by dissipative effects (e.g. viscoplasticity) or a separation at the phase boundaries necessitate iterative procedures to compute the RVE response. Due to the enormous numbers of RVE problems to be solved in realistic simulations, order reduction techniques are needed to reduce the computational cost to manageable amounts.



Microscopic material models

- bulk material is assumed to belong to the class of *generalized standard materials* (GSM) [1]
- interfacial behavior is characterized within the framework of *standard dissipative cohesive zones* (SD-CZ) [3]

	GSM (bulk)	SD-CZ (interface)
free energy density	$\psi_\Omega \equiv \psi_\Omega^{\mathrm{e}}(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}_{\mathrm{p}}) + \psi_\Omega^{\mathrm{h}}(\hat{q})$	$\psi_\mathscr{S} \equiv \psi_\mathscr{S}(\boldsymbol{\delta}, \hat{y})$
kinematic variable	infinitesimal strain $\boldsymbol{\varepsilon}$	separation $\boldsymbol{\delta}$
internal variables	plastic strain $\boldsymbol{\varepsilon}_{\mathrm{p}}$ hardening variables $\hat{q}$	$\hat{y}$
static variable	Cauchy stress $\boldsymbol{\sigma} = \frac{\partial \psi_\Omega}{\partial \boldsymbol{\varepsilon}}$	traction $\boldsymbol{t} = \frac{\partial \psi_\mathscr{S}}{\partial \boldsymbol{\delta}}$
driving forces	$(\boldsymbol{\sigma}_{\mathrm{p}}, \hat{r}) = -\frac{\partial \psi_\Omega}{\partial(\boldsymbol{\varepsilon}_{\mathrm{p}}, \hat{q})}$	$\hat{z} = -\frac{\partial \psi_\mathscr{S}}{\partial \hat{y}}$
dual dissipation	$\phi_\Omega^* \equiv \phi_\Omega^*(\boldsymbol{\sigma}_{\mathrm{p}}, \hat{r})$	$\phi_\mathscr{S}^* \equiv \phi_\mathscr{S}^*(\hat{z})$
evolution law	$\left(\dot{\boldsymbol{\varepsilon}}_{\mathrm{p}}, \dot{\hat{q}}\right) = \frac{\partial \phi_\Omega^*}{\partial(\boldsymbol{\sigma}_{\mathrm{p}}, \hat{r})}$	$\dot{\hat{y}} = \frac{\partial \phi_\mathscr{S}^*}{\partial \hat{z}}$

Reduced basis ansatz

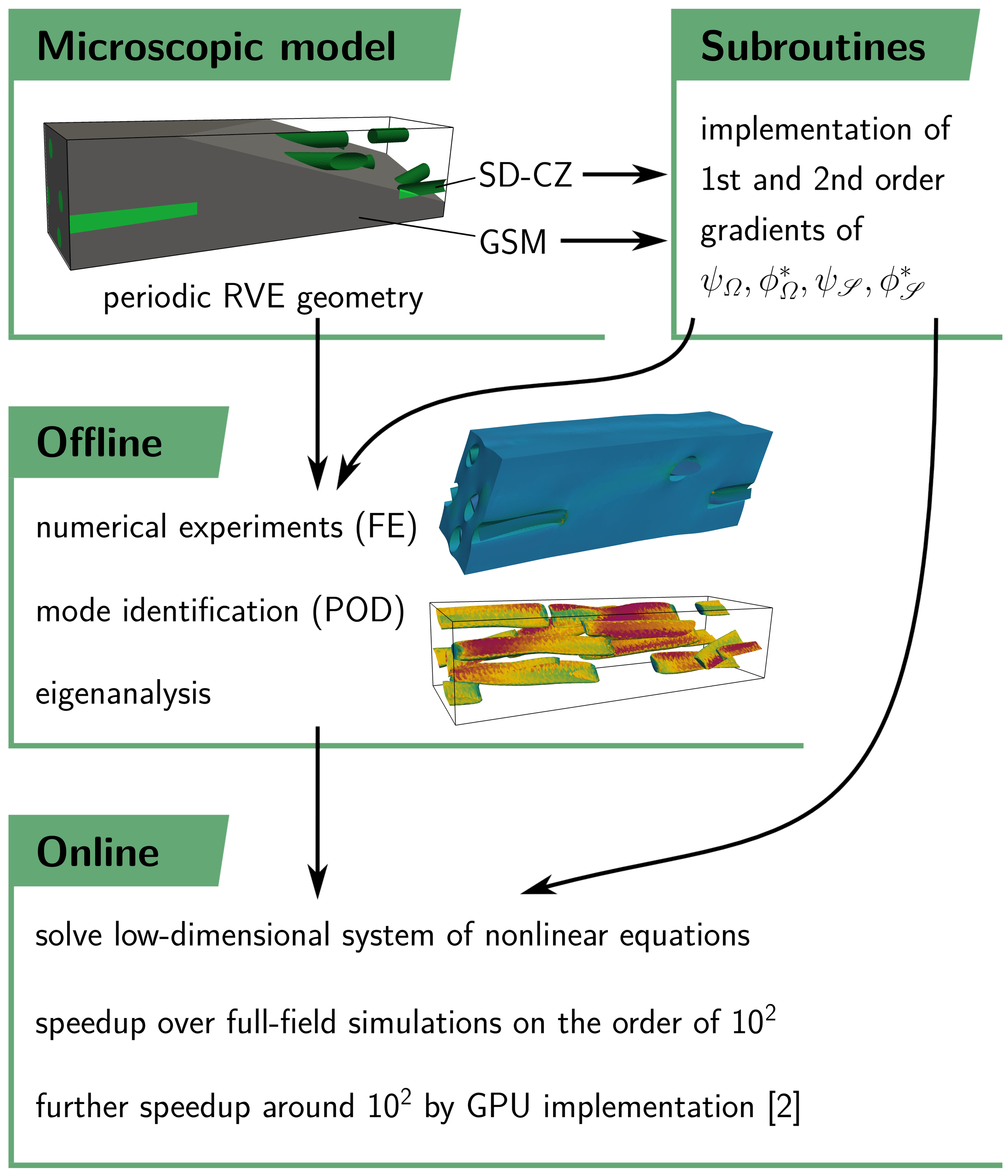
$$\boldsymbol{\delta}(\boldsymbol{x}) = \hat{\Delta}(\boldsymbol{x})\hat{\zeta}, \quad \hat{y}(\boldsymbol{x}) = \hat{Y}(\boldsymbol{x})\hat{\nu}, \quad \boldsymbol{\varepsilon}_{\mathrm{p}}(\boldsymbol{x}) = \hat{P}(\boldsymbol{x})\hat{\xi}, \quad \hat{q}(\boldsymbol{x}) = \hat{Q}(\boldsymbol{x})\hat{\lambda}$$

- express separation and internal variables via global bases
- define relocalization operators $\hat{\Delta}, \hat{Y}, \hat{P}, \hat{Q}$ column-wise by the global basis vectors (modes)
- derive modes from pre-computed solutions of the RVE problem through a *proper orthogonal decomposition* (POD)
- solve auxiliary problems to find self-equilibrated stress fields that ensure static admissibility for any $\hat{\zeta}, \hat{\xi}$ (eigenanalysis)

Effective material model

- RVE state is characterized by the mode coefficient vectors $\hat{\zeta}, \hat{\nu}, \hat{\xi}, \hat{\lambda}$ and the prescribed effective strain $\bar{\varepsilon}$
- effective material law is expressed as a GSM using averages of the microscopic potentials $\psi_\Omega, \phi_\Omega^*, \psi_\mathscr{S}, \phi_\mathscr{S}^*$
- variational calculus leads to nonlinear equations which define the evolution of the mode coefficients

pRBMOR scheme



References

[1] B. Halphen, Q.S. Nguyen, *Journal de Mécanique* **14**, 1975.

[2] F. Fritzen, M. Hodapp, M. Leuschner, *Computer Methods in Applied Mechanics and Engineering* **278**, 2014.

[3] M. Leuschner, F. Fritzen, J.A.W. van Dommelen, J.P.M. Hoefnagels, *Composites Part B: Engineering* **68**, 2015.

Matthias Leuschner

- Education
  - Diploma in Technomathematics, KIT (Karlsruhe)
  - Ph.D. student at KIT (April 2013 – March 2015)
  - Ph.D. student at University of Stuttgart (from April 2015)
- Research interests
  - Reduced order methods for multiscale material models
  - Potential-based material modeling

Young Investigator Group “Computer Aided Material Modeling”, Institute of Engineering Mechanics, Karlsruhe Institute of Technology, Dipl. Math.-techn. Matthias Leuschner, March 2015