GESELLSCHAFT für
ANGEWANDTE MATHEMATIK und MECHANIK e.V.
INTERNATIONAL ASSOCIATION of APPLIED MATHEMATICS and MECHANICS

Optimal Control for Problems with Servo Constraints

joint work with Jan Heiland (MPI Magdeburg)

Introduction

The movement of a crane where the load must follow a prescribed trajectory is typically modeled by a multibody system with servo constraints. These models often are differential-algebraic equations (DAEs) of index 5,

$$\begin{aligned} M\ddot{x} &= A\dot{x} + Bu + f, \\ Cx &= y_{\text{opt}}. \end{aligned}$$

We search for the input u in order to fulfill the constraint $Cx = y_{\text{opt}}$. Because of the high index structure,

- the system is very sensible to perturbations,
- numerical simulations require an index reduction, e.g. by
  - $\to$ projection approach [BlaK04]
  - $\to$ minimal extension [AltBY14]

We replace the servo constraint by a minimization approach, i.e., we minimize the cost functional

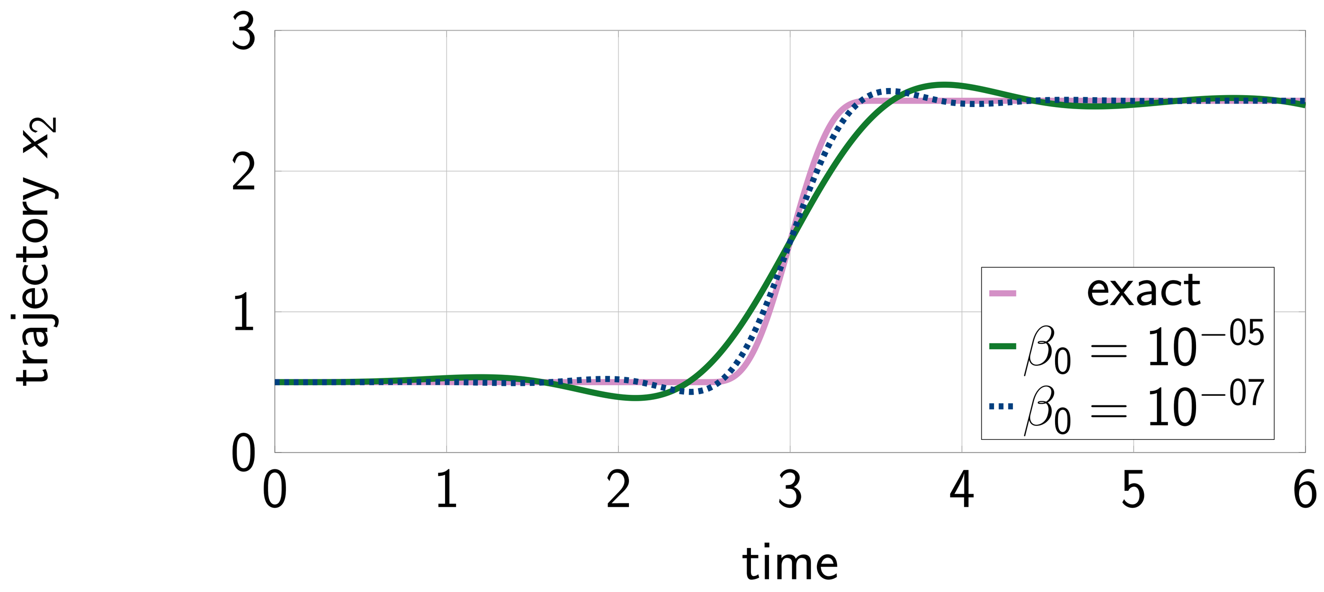
$$\mathcal{J}(x,u) := \mathcal{S}(x(T)) + \frac{1}{2}\int_0^T \|Cx - y_{\text{opt}}\|^2 + \sum_{i=0}^{\nu} \beta_i \|u^{(i)}\|^2 \,\mathrm{d}t,$$

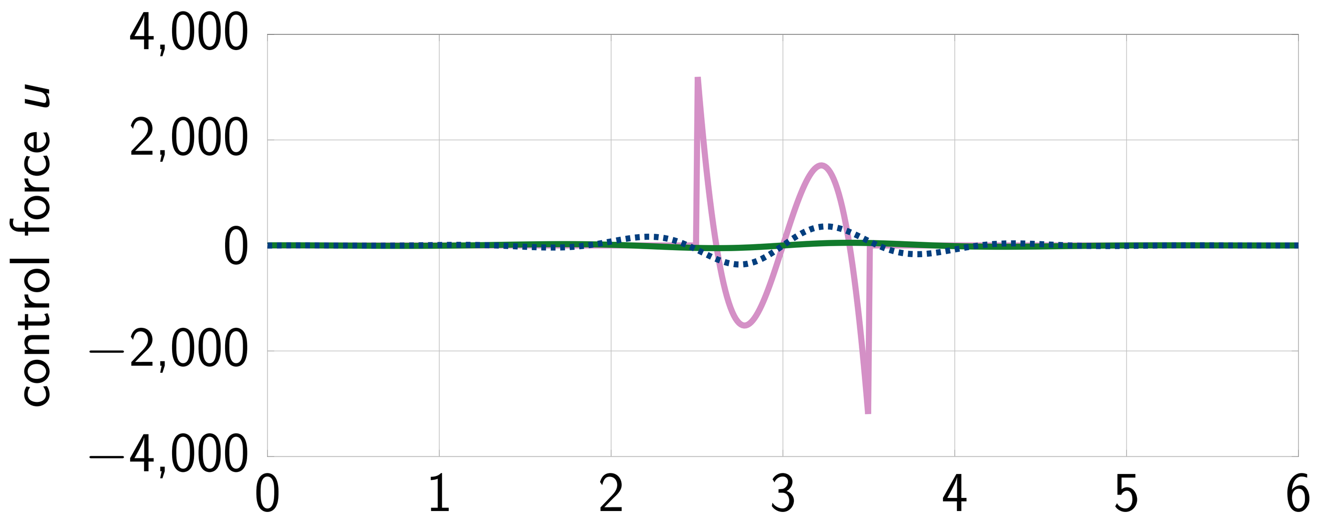
where x satisfies $M\ddot{x} = Ax + Bu + f$ and $\mathcal{S}(x(T)) := \gamma\frac{1}{2}\|Cx(T) - y_{\text{opt}}(T)\|^2$. Thus, we try to make the difference $Cx - y_{\text{opt}}$ small but also penalize the input variable u.

Numerical Example

Numerical simulation of the 2-car example for $\nu = 0$, different values of β_0, and parameters

- $m_1 = 1$, $m_2 = 2$, $k = 10$, $d = 0.5$



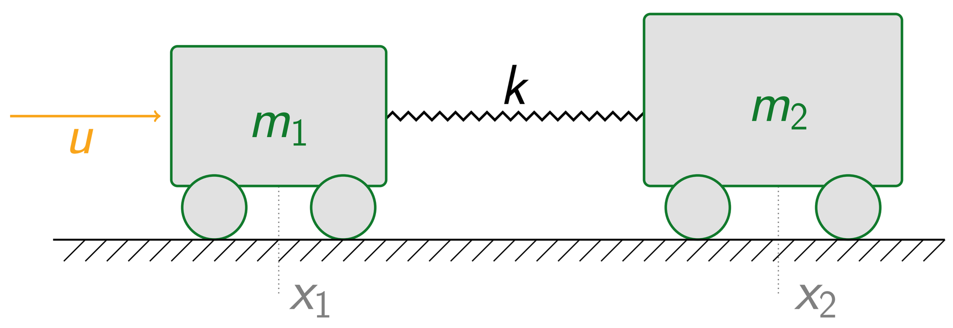


The figure shows that a good approximation of y_{opt} can already be achieved with a much smaller input force u than given by the exact DAE solution.

2-Car Example

Aim: control position of second car x_2 by the input force u.

- variables: positions x_1, x_2 and input force u
- spring constant k, resting distance d
- desired trajectory of x_2 given by y_{opt}



The dynamics are given by the index-5 DAE

$$\begin{aligned} m_1\ddot{x}_1 &= k(x_2 - x_1 - d) + u, \\ m_2\ddot{x}_2 &= -k(x_2 - x_1 - d), \\ x_2 &= y_{\text{opt}}. \end{aligned}$$

Note that a continuous solution u requires $y_{\text{opt}} \in C^4(0,T)$.

Optimality System

The formal optimality system is given as

$$\begin{aligned} M\ddot{x} &= Ax + Bu + f, \\ M^{\mathsf{T}}\ddot{\lambda} &= A^{\mathsf{T}}\lambda - C^{\mathsf{T}}Cx + C^{\mathsf{T}}y_{\text{opt}}, \\ 0 &= \sum_{i=0}^{\nu}(-1)^i \beta_i u^{(2i)} - B^{\mathsf{T}}\lambda \end{aligned}$$

Depending on ν, the last equation reads, e.g.,

- case $\nu = 0$: $\quad 0 = \beta_0 u - B^{\mathsf{T}}\lambda$
  - $\to$ DAE of index 1
- case $\nu = 1$: $\quad 0 = \beta_0 u - \beta_1 \ddot{u} - B^{\mathsf{T}}\lambda$
  - $\to$ ODE

The resulting boundary-value problems can be solved via

- finite differences (full discretization)
- Riccati ansatz
- shooting methods

Remark: With additional holonomic constraints, the formal optimality system contains consistency conditions. Thus, the system may not have a solution because of inconsistent boundary conditions.

Robert Altmann

- Education
  - Diploma in mathematics, HU Berlin
  - Research stays in Seoul and Paris
  - Scientific assistant, TU Berlin
- Research Interests
  - Constrained PDEs (operator DAEs)
  - Adaptive simulation
  - Convergence of time-discretization methods

Ref: **[BlaK04]** W. Blajer and K. Kołodziejczyk. A geometric approach to solving problems of control constraints: theory and a DAE framework. *Multibody Syst. Dyn.*, 11(4):343–364, 2004.
[AltBY14] R. Altmann, P. Betsch, and Y. Yang. Minimal extension for mechanical systems with control contraints. *Proceedings of IMSD 2014, Multibody System Dynamics*, 2014.

Creation date 08.03.2015. The author's work was supported by the ERC Advanced Grant MODSIMCONMP.