GESELLSCHAFT für
ANGEWANDTE MATHEMATIK und MECHANIK e.V.
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A phase field model for FSI

Full Eulerian Fluid-Structure Interaction

Full Eulerian FSI model

Use a phase field ϕ to distinguish between fluid ($\phi = 1$) and an incompressible solid ($\phi = 0$). Now, global field variables can be defined from the variables in both phases (denoted by subscribts $\cdot_f, \cdot_s$) as:

$$
\begin{aligned}
\mathbf{v} &= \phi \mathbf{v}_f + (1-\phi)\mathbf{v}_s && \text{(velocity)}\\
p &= \phi p_f + (1-\phi) p_s && \text{(pressure)}\\
\rho &= \phi \rho_f + (1-\phi) \rho_s && \text{(density)}\\
\nu &= \phi \nu_f + (1-\phi) \nu_s && \text{(viscosity)}\\
\mathbf{T} &= \qquad (1-\phi)\mathbf{T}_s && \text{(elastic stress)}
\end{aligned}
$$

Hence we obtain the following governing equations valid in the whole domain $\Omega = \Omega_f \cup \Omega_s$:

1. Navier Stokes equation for fluid and solid phase:

$$
\begin{aligned}
\rho(\phi)(\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v}) + \nabla p - \nabla \cdot (\nu(\phi)\mathbf{D}) + \nabla \cdot \mathbf{T} &= \mathbf{f}\\
\nabla \cdot \mathbf{v} &= 0
\end{aligned}
$$

   where $\mathbf{D} = \nabla \mathbf{v} + \nabla \mathbf{v}^T$.

2. Evolution of the elastic stress:

$$
\partial_t \mathbf{T} + \mathbf{v} \cdot \nabla \mathbf{T} = \nabla \mathbf{v} \cdot \mathbf{T} + \mathbf{T} \cdot \nabla \mathbf{v}^T + \frac{E}{6}\mathbf{D} - k\phi \mathbf{T}
$$

   with Youngs modulus E and a large constant k to enforce $\mathbf{T} = 0$ in the fluid region.

3. Finally the fluid and solid phase are advected by a Cahn-Hilliard equation:

$$
\begin{aligned}
\partial_t \phi + \mathbf{v} \cdot \nabla \phi &= \nabla \cdot (M \nabla \mu)\\
\mu &= \epsilon^{-1}(4\phi^3 - 6\phi^2 + 2\phi) - \epsilon \Delta c.
\end{aligned}
$$

The interfacial stress boundary condition $[[p\mathbf{I} - \nu \mathbf{D}]]_s^f \cdot \mathbf{n} = \mathbf{T}_s \cdot \mathbf{n}$ is implicitly included.

Advantages

- single grid model
  - no mesh generation of the solid structure
  - no re-meshing
  - no extra treatment of coupling forces necessary
- simple to implement in standard PDE toolboxes
- implicit coupling of fluid and structure allows large time steps
- viscoelasticity can be included by adding $\lambda \mathbf{T}$ to the $\mathbf{T}$-equation
- easy to include additional surface equations (e.g. surface viscosity, surface tension)

Application: Cells in Flow

Biological cells behave similar to elastic solids with Young's moduli in the range of 1kPa. A flow scenario can be used to measure a cell's elastic response. Thereby single cells immersed in a viscous liquid are send through a narrow channel and their deformation is measured by camera image analysis.

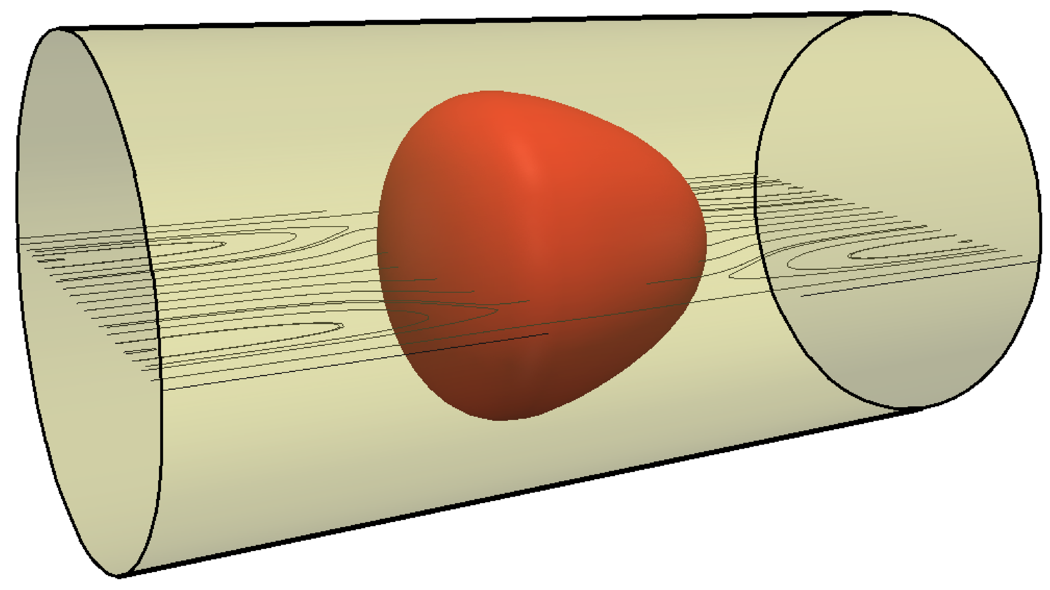

Figure 1: Simulation of a biological cell immersed in water during its flow through a circular channel. Using E=6kPa at a flow rate of 0.15m/s leads to significant deformation from the relaxed spherical shape.

Comparison of experimental data with numerical simulation results permits conclusions on cell elasticity. This can be used to identify cell phenotypes or morbidness.

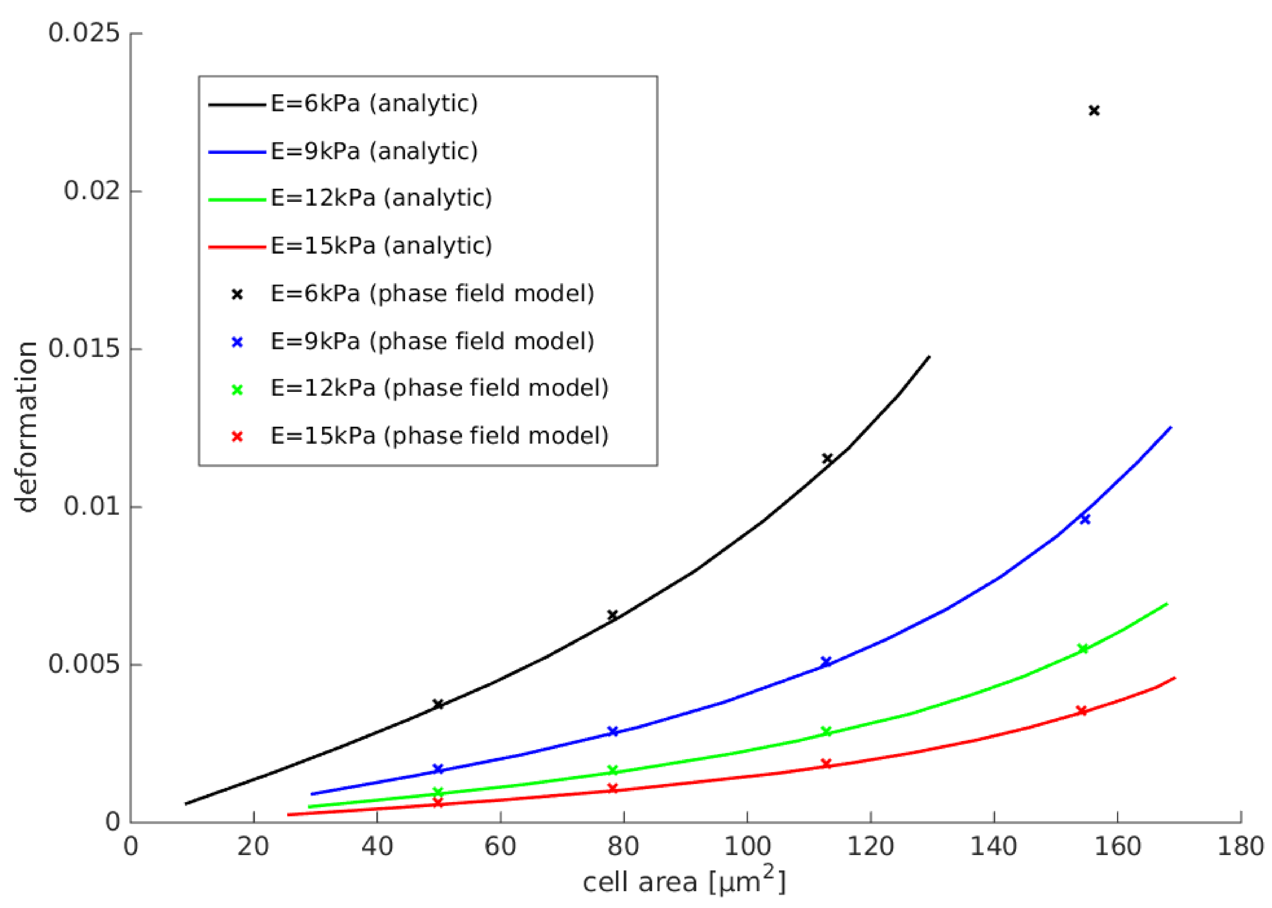

Figure 2: Comparison of numerical simulations with analytical calculations (valid in the small deformation limit). The cell deformation (1-circularity) is very well predicted by the numerical model.

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- Background
  - Ph.D. in mathematics, TU Dresden 2012
  - PostDoc at UC Irvine, USA
  - currently PI in DFG-SPP1506 at TU Dresden
- Research interests
  - numerical simulation of PDEs
  - modelling of complex two-phase flows
  - biological applications