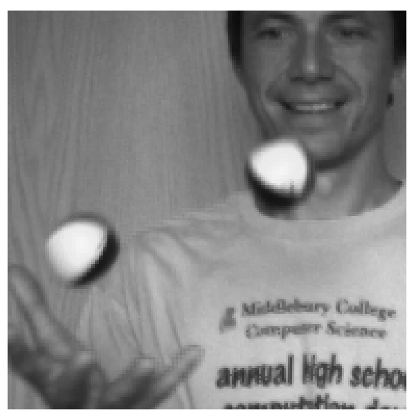
GESELLSCHAFT für
ANGEWANDTE MATHEMATIK und MECHANIK e.V.
INTERNATIONAL ASSOCIATION of APPLIED MATHEMATICS and MECHANICS

Spatio-temporal regularization and applications

Motivation

Variational methods for image processing heavily rely on appropriate regularization functionals. Our goal is to suitably extend spatial regularization approaches to the spatio-temporal setting. When considering spatio-temporal TV regularization for instance, the weighting of space with respect to time has to be fixed, a degree of freedom which balances between spatial and temporal regularization. Choosing $0 < \epsilon \ll 1$ and defining for example



Frame of original video

Frame of degraded video

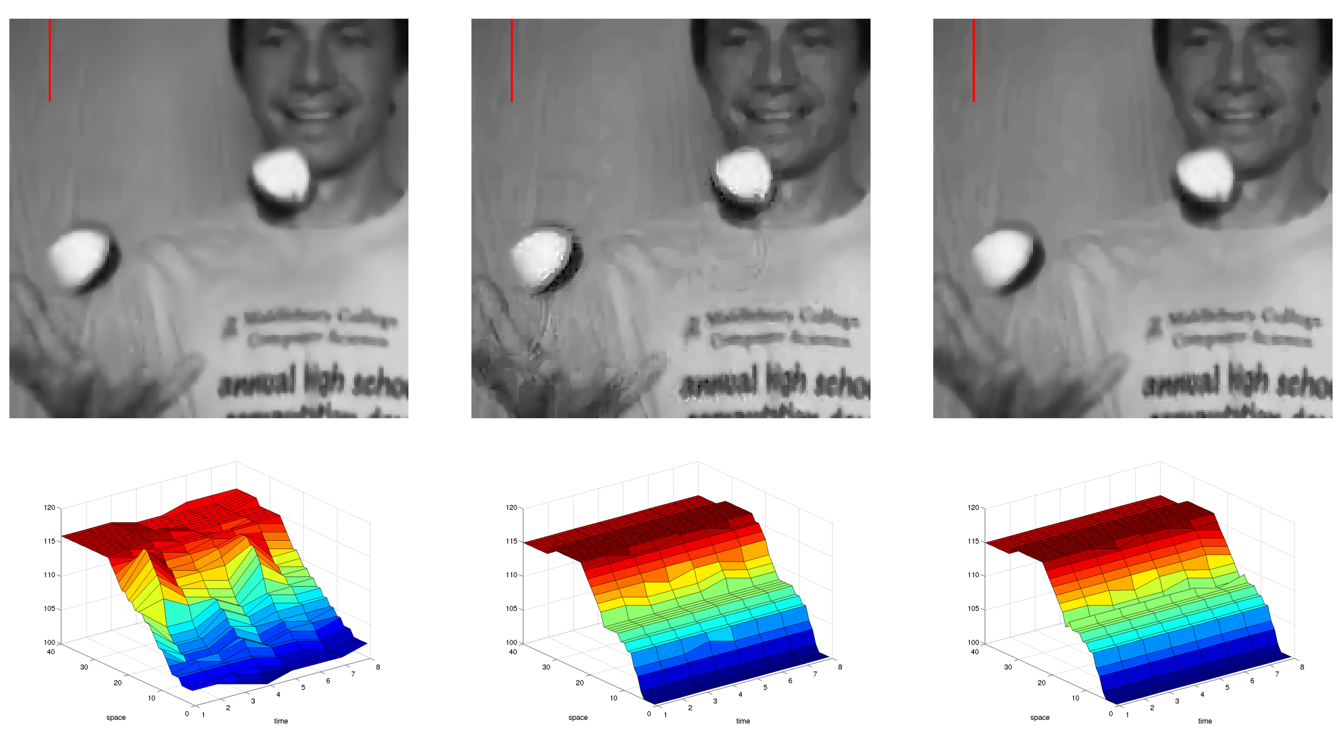
$$\mathrm{TV}_{\epsilon_t}(u) = \int \sqrt{(\partial_{x_1}u)^2 + (\partial_{x_2}u)^2 + \epsilon(\partial_t u)^2}$$

and

$$\mathrm{TV}_{\epsilon_x}(u) = \int \sqrt{\epsilon(\partial_{x_1}u)^2 + \epsilon(\partial_{x_2}u)^2 + (\partial_t u)^2},$$

it can be observed in the figure below, that TV_{ϵ_t} gives a good frame wise reconstruction but suffers from background flickering while TV_{ϵ_x} gives a stable background but suffers from motion artifacts. Our approach to overcome this is to use the infimal convolution of two total variation functionals with different norms as regularization:

$$\mathrm{ICTV}_{\epsilon}(u) = \min_v \Big(\mathrm{TV}_{\epsilon_t}(u - v) + \mathrm{TV}_{\epsilon_x}(v) \Big).$$



TV_{ϵ_t} reconstruction | TV_{ϵ_x} reconstruction | ICTV_{ϵ} reconstruction

Second order ICTGV

A generalization of the above concept to higher order regularization in space time can be defined as

$$\mathrm{ICTGV}^2_{\beta}(u) = \sup \Big\{ \int_\Omega u\phi \,|\, \phi = \mathrm{div}^2 q_i, \text{ with } q_i \in \mathcal{C}^2_c(\Omega, S^{d\times d}),$$
$$\| |\mathrm{div}^l q_i|_{\beta^*_{l,i}} \|_\infty \le 1,\ l = 0, 1,\ i = 1, 2 \Big\},$$

where $|\cdot|_{\beta^*_{l,i}}$ denotes the dual norm of an arbitrary norm on $\mathbb{R}^d$. This optimally balances between two TGV functionals employing different norms and can equivalently be written as

$$\mathrm{ICTGV}^2_{\beta}(u) = \min_v \mathrm{TGV}^2_{\beta,1}(u - v) + \mathrm{TGV}^2_{\beta,2}(v).$$

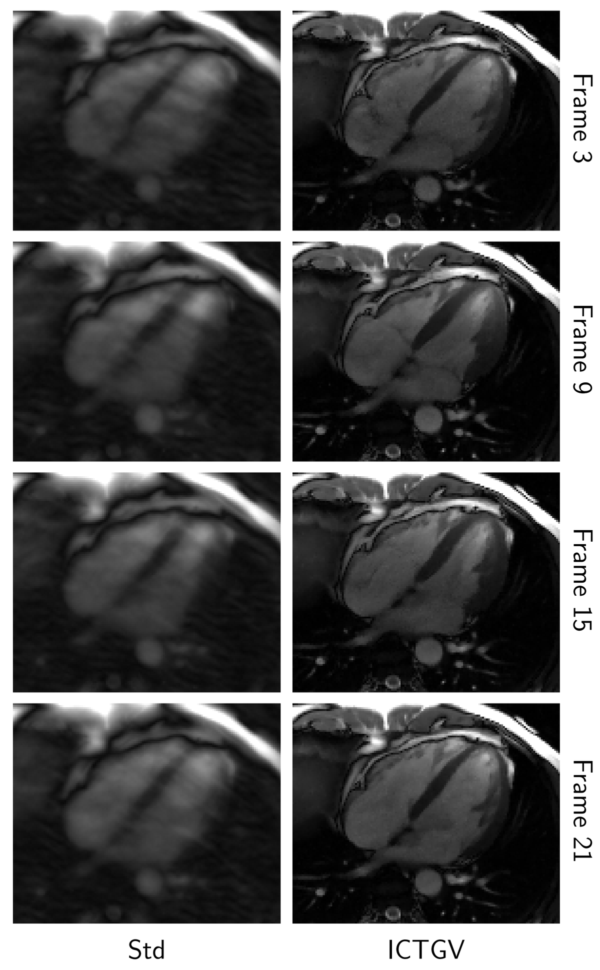
MRI Reconstruction

For the reconstruction of dynamic MRI data[1], we solve

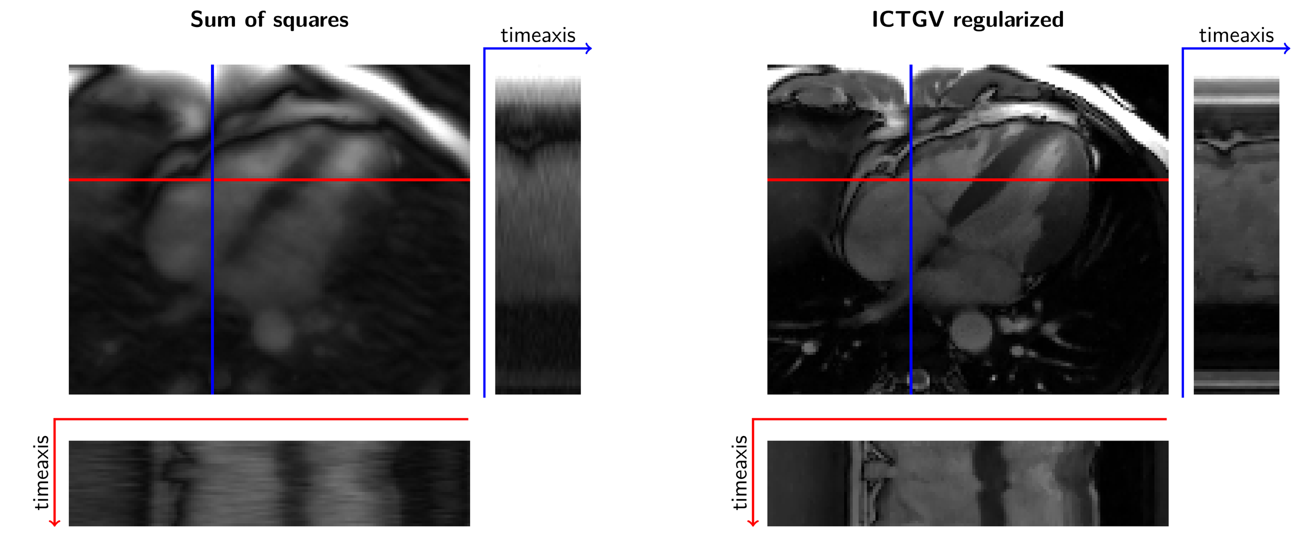
$$\min_u \mathrm{ICTGV}^2_{\beta}(u) + \lambda \sum_{t,c} \|\mathcal{F}_t(b_c u_t) - d_{t,c}\|_2^2,$$

where

- t, c index different timeframes and coils, respectively,
- $\mathcal{F}_t$ is a Fourier transform with time dependent masking,
- b_c are the complex valued coil sensitivities,
- $d_{t,c}$ is the given, sub-sampled data.



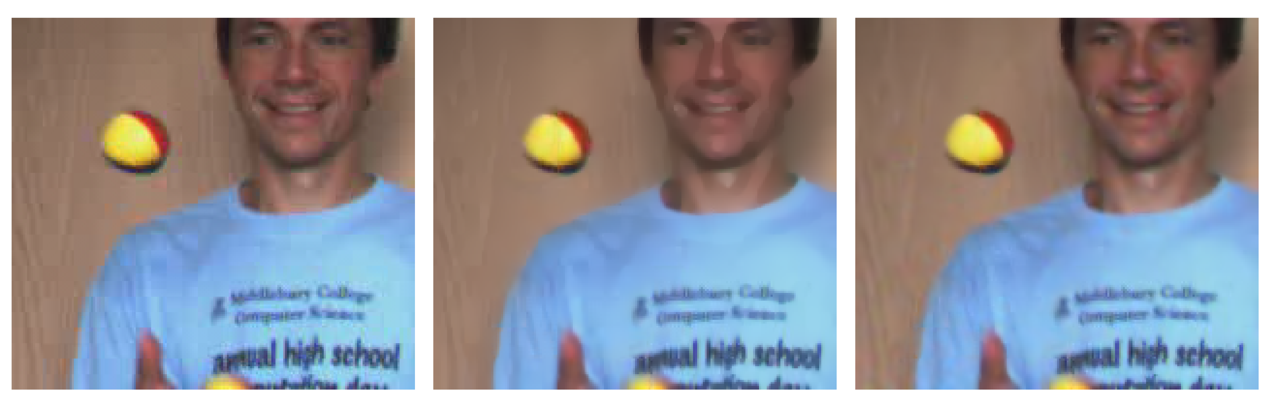
[1]Data from the ISMRM, subsampling factor ≈ 11



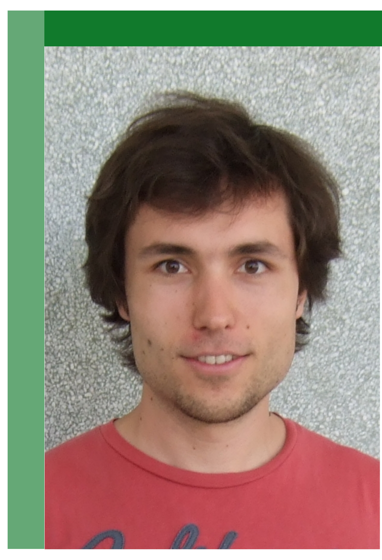
MPEG Decompression

MPEG operates blockwise and performs block-matching and a quantization of block-cosine coefficients for compression. Consequently, an MPEG compressed file provides information about a linear *block-cosine data* to *movie* operator A and interval bounds for block-cosine data $(J_i)_i$. Denoting by S a subsampling operator to account for additional color subsampling and defining $\widehat{S}$ such that $S\widehat{S} = \mathrm{Id}$, variational MPEG decompression using the spatio-temporal regularization functional $\mathcal{R}$ can be written as

$$\min_{d,s} \mathcal{R}(s + \widehat{S}Ad) + \mathcal{I}_{\ker(S)}(s) + \mathcal{I}_{\{d_i \in J_i\}}(d)$$



Standard reconstruction | Variational, $\mathcal{R} = \mathrm{TGV}^2_{\alpha}$ | Variational, $\mathcal{R} = \mathrm{ICTGV}^2_{\beta}$



Martin Holler

- Education
  - 2005 - 2010: Diploma in mathematics
  - 2013: Ph.D. from University of Graz
- Research interests
  - Variational methods in imaging
  - Higher order regularization
  - Image/video decompression

University of Graz, 2015. Cooperation with K. Bredies, K. Kunisch and M. Schlögl