GESELLSCHAFT für
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Nonlinear Energy-Harvester under Random Excitation

Energy Harvesting

The term energy harvesting adresses the conversion of otherwise unused ambient energy into usable electric energy. A well suited energy source which can be found in the vast majority of build environments and machines is kinetic energy in the form of structural vibrations.
A widely used method to harvest energy from such vibrations is to connect a beam-type structure (i.e. the energy harvester) which incorporates a piezoactive material to the vibrating host structure.

Nonlinear Energy Harvester

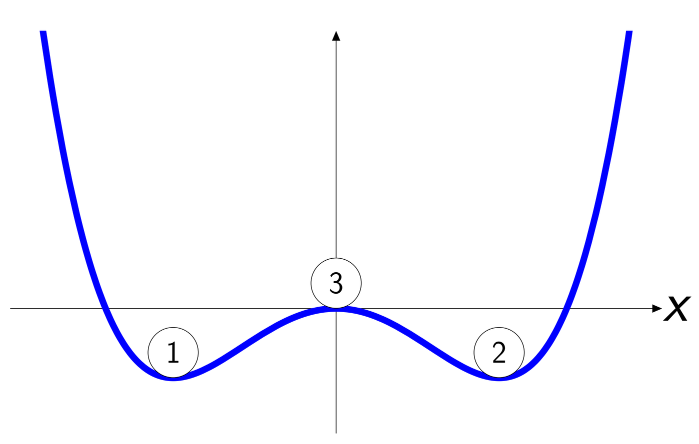
Since most ambient vibrations are frequency-varying or even totally random, it is not convinient to design an energy harvester as a linear oscillator which achieves the maximum performance only under resonance conditions. One method to improve the performance is to modify the properties of the system to obtain a nonlinear potential energy function $U(x) = \frac{1}{2}\alpha x^2 + \frac{1}{4}\beta x^4$. The dynamics of such an energy harvester are then described by the following system of coupled differential equations(1)

$$\ddot{x} + 2\delta\dot{x} + x(\alpha + \beta x^2) - \chi v = q(t) \qquad (1)$$

$$\dot{v} + \lambda v + \kappa \dot{x} = 0 \qquad (2)$$

where x represents the dimensionless displacement of the oscillator, v is the dimensionless voltage, δ is the viscious mechanical damping ratio, λ is the reziprocal of the dimensionless time constant, χ and κ are the dimensionless piezoelectric coupling coefficients in the mechanical and electrical circuit equations and q is the external excitation.
For $\alpha < 0$ and $\beta > 0$ the system becomes bistable with two potential wells 1 and 2 seperated by an unstable saddle 3 as can be seen in the figure below.

Potential Energy



If the supplied energy is large enough to overcome the potential barrier, the dynamic trajectories start to move between the potential wells activating the so called inter-well motions which can provide large displacement amplitudes even under non-resonance conditions.

(1): A. Erturk, J. Hoffmann, and D. J. Inman. A piezomagnetoelastic structure for broadband vibration energy harvesting. Applied Physics Letters, 94:11-14, 2009.

(2): W. Martens, U. von Wagner, and G. Litak. Stationary response of nonlinear magneto-piezoelectric energy harvester systems under stochastic excitation. The European Physical Journal Special Topics, 222:1665-1673, 2013.

Random Excitation

Modelling the random external excitation q as white GAUSSian noise, the set of differential equations (1) and (2) can be written as a stochastic differential equation

$$\mathrm{d}X_t = f(X_t, t)\mathrm{d}t + G\mathrm{d}W_t \qquad (3)$$

where X_t represents the vector of random variables corresponding to the state-space variables, f is the drift term resulting from the deterministic properties of the system, G is a meassurement for the intensity of the excitation and $\mathrm{d}W_t$ is the increment of a WIENER process. The probability density function p can be obtained as the solution of the corresponding FOKKER-PLANCK equation

$$\sum_{i=1}^{3} \frac{\partial}{\partial x_i}(f_i(x)p(x)) - \sum_{i,j=1}^{3} \frac{\partial^2}{\partial x_i x_j}(b_{ij}(x)p(x)) = 0 \qquad (4)$$

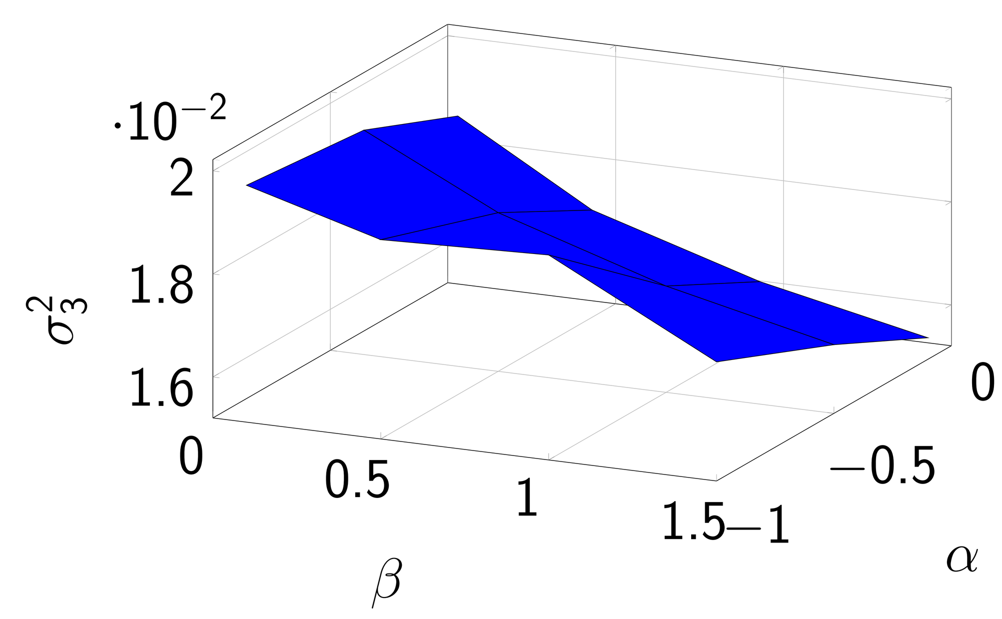
where $B = GG^T$ is the so called diffusion matrix. Since the coefficients in (4) are nonlinear, it is not possible to find an exact solution for p. Thus, the exact solution p_0 of the linearized problem is used together with a polynomial correction term to calculate an approximate solution. Introducing the ansatz

$$p = p_0 \sum_{i=1}^{m} c_i \phi_i \qquad (5)$$

in (4), the unknown coefficents c_i can be calculated in a GALERKIN type scheme.
The resulting probability density function is used to calcualte the voltage variance σ_3^2 which is proportional to the expected value of the output power and therefore can be used to design an optimal energy harvester.

Numerical Example

The figure below shows the voltage variance σ_3^2 for varying values of α and β (for parameter values consult Ref.(2)).



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- Research interests
  - Nonlinear Dynamics in Energy Harvesting
  - Mechanical Systems under Random Excitation

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