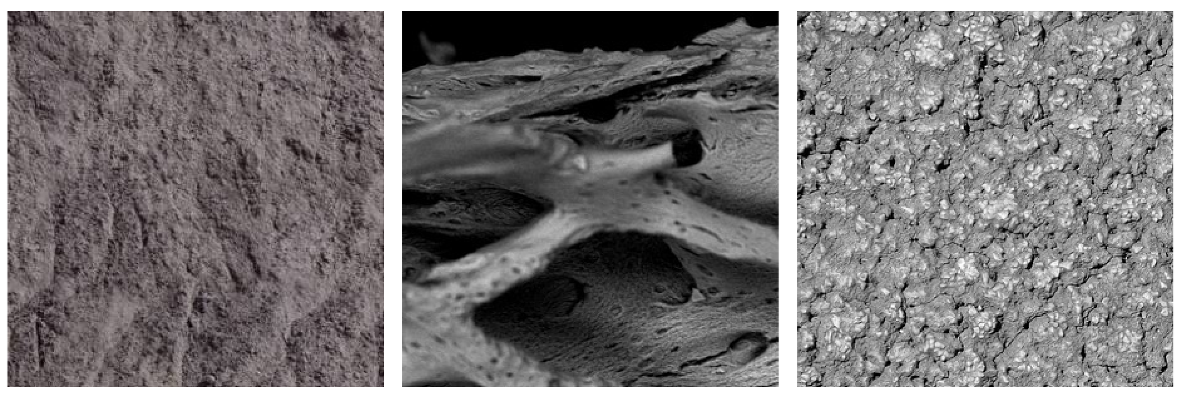
GESELLSCHAFT für
ANGEWANDTE MATHEMATIK und MECHANIK e.V.
INTERNATIONAL ASSOCIATION of APPLIED MATHEMATICS and MECHANICS

Stochastic elastoplastic modelling

Introduction

Materials such as soil, rock, concrete, powder, and bone cannot be described by deterministic models due to heterogeneities at the microstructural level. Additionally, deterministic analysis fails to accurately predict the response of structures exposed to uncertain external excitations, e.g. seismic force, traffic loading, etc.

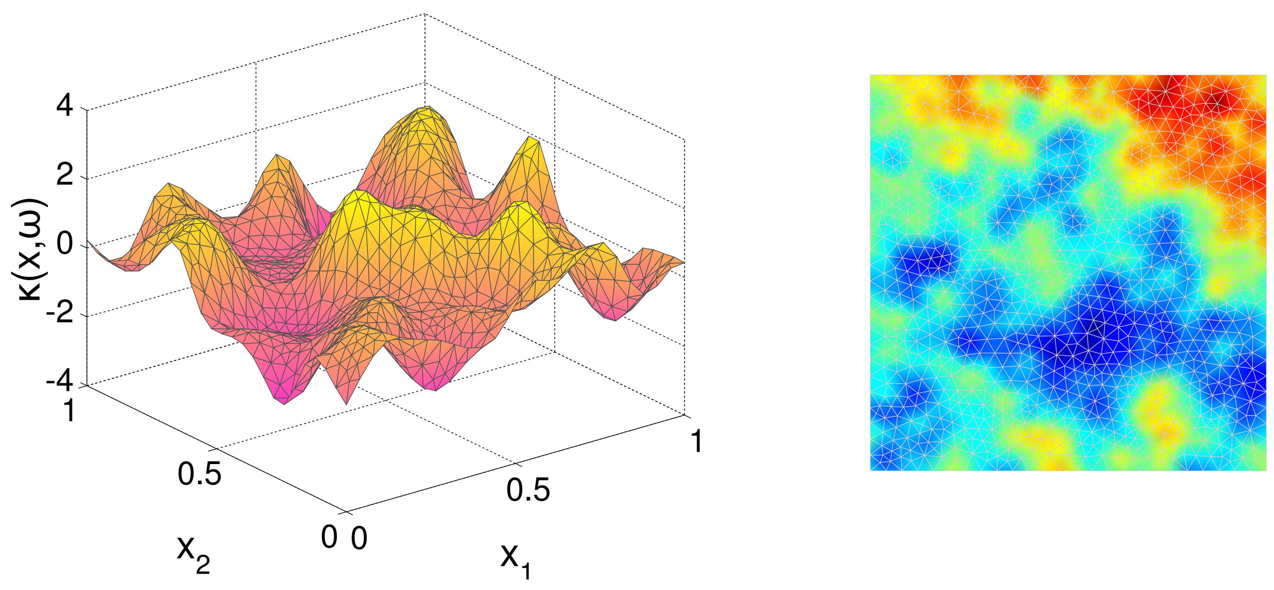


a) concrete *b) bone tissue* *c) soil*

Assuming that the heterogeneous material behaves according to an elastoplastic model, one may consider experimental data and identify the probability distributions of unknown quantities describing both the elastic/reversible behaviour as well as inelastic/irreversible behaviour. Mathematically speaking, this further means that the material parameters can be modelled as spatial random fields:

$$\kappa(x,\omega): \quad \mathcal{G}\times\Omega\to\mathcal{V}$$

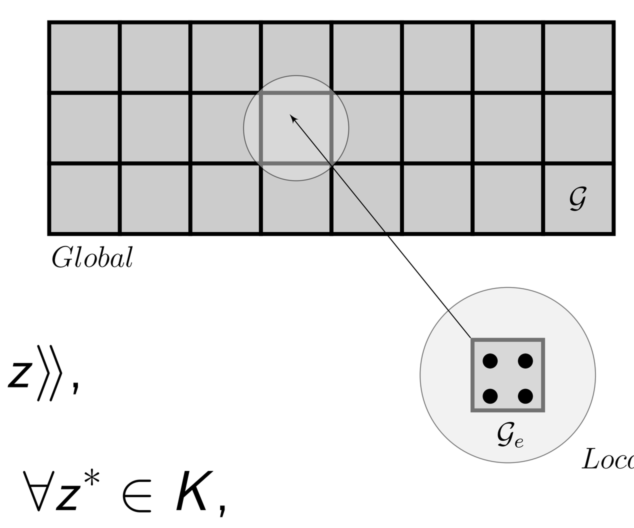
in a probability space $(\Omega,\mathcal{B},\mathbb{P})$.



One realisation of a random field

Elastoplasticity

The mathematical formulation of a stochastic elastoplastic material behaviour consists of the equilibrium equation and the evolutionary inequality



$$1)\quad a(w(t),z)+\langle\!\langle \frac{dw}{dt}(t),z\rangle\!\rangle=\langle\!\langle f,z\rangle\!\rangle,$$

$$2)\quad \langle\!\langle \frac{dw}{dt}(t),z^*-w^*\rangle\!\rangle\le 0,\quad \forall z^*\in K,$$

respectively, in which $\langle\!\langle\cdot,\cdot\rangle\!\rangle$ denotes the duality pairing defined as

$$\langle\!\langle v_1,v_2\rangle\!\rangle:=\mathbb{E}(\langle v_1,v_2\rangle)=\int_\Omega\int_{\mathcal{G}} v_1:v_2\,\mathrm{d}x\,\mathbb{P}(\mathrm{d}\omega).$$

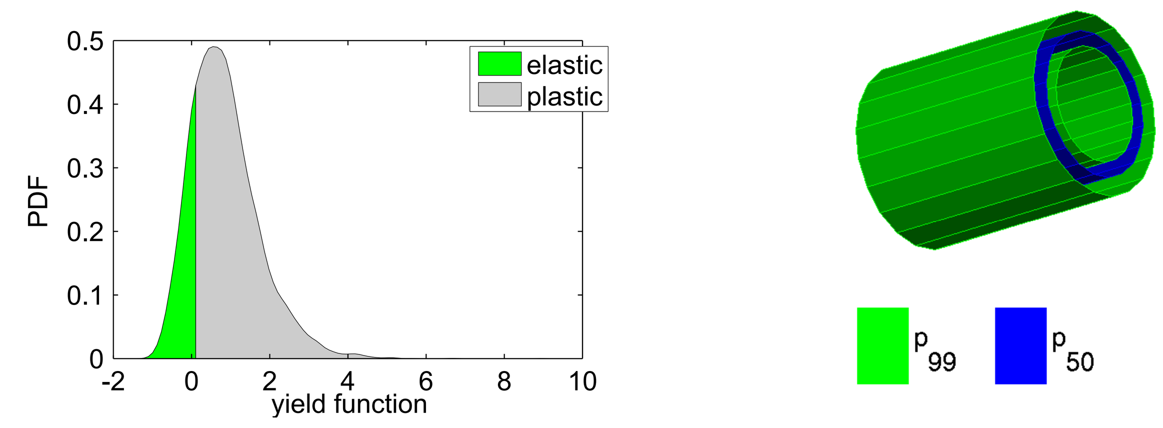
Eq. (1) is solved globally with the help of the stochastic Galerkin method, whereas Eq. (2) requires local convex minimisation, see [1].

Closest point projection

After the time discretisation by implicit Euler, the goal of the local analysis step is to solve

$$\Sigma_n=\arg\min_{\Sigma\in K}\frac{1}{2}\langle\!\langle\Sigma^{\mathrm{trial}}-\Sigma,A^{-1}(\Sigma^{\mathrm{trial}}-\Sigma)\rangle\!\rangle=P_K(\Sigma)$$

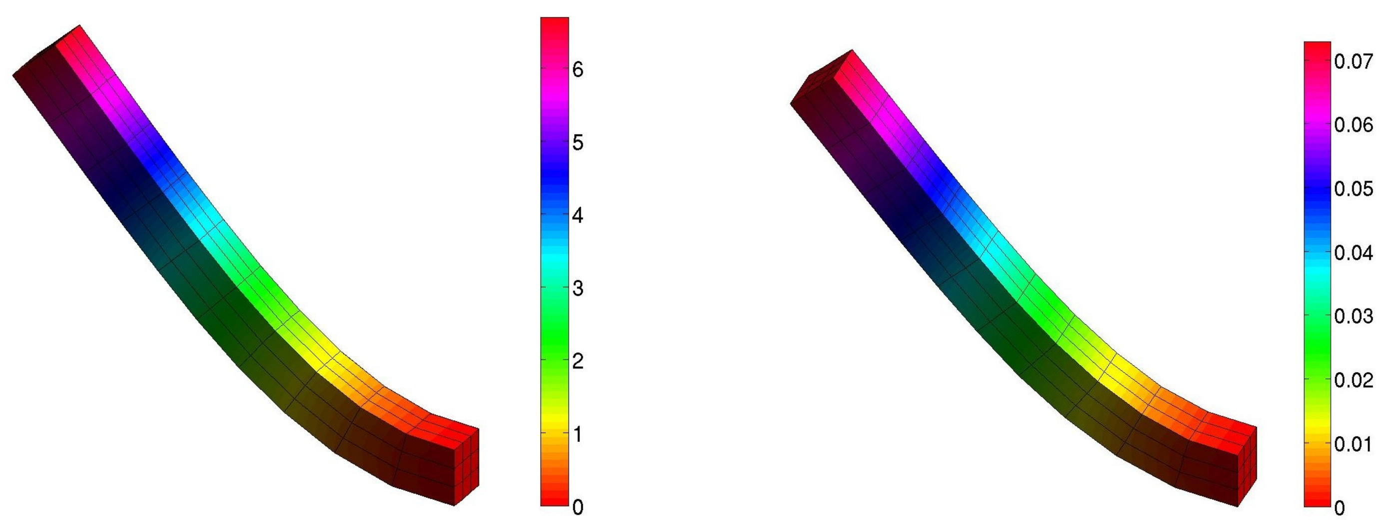
in which $K(\omega)=\{\Sigma(\omega):\ \phi_K\le 0\quad \mathbb{P}-\text{ almost surely}\}$. The projection can be done for each integration point independently if the convex set K is approximated in one of the following ways: a) mean value approach $K^*:=\mathbb{E}(K)$, b) probabilistic approach $K^*=\{\Sigma:\ \Pr(\phi_K\le 0)\ge p_r\}$ c) set of deterministic constraints $K^*=\{\Sigma(\boldsymbol{\theta}_j):\ \phi_K(\boldsymbol{\theta}_j)\le 0\}$.



a) Uncertain yield function *b) von Mises surface*

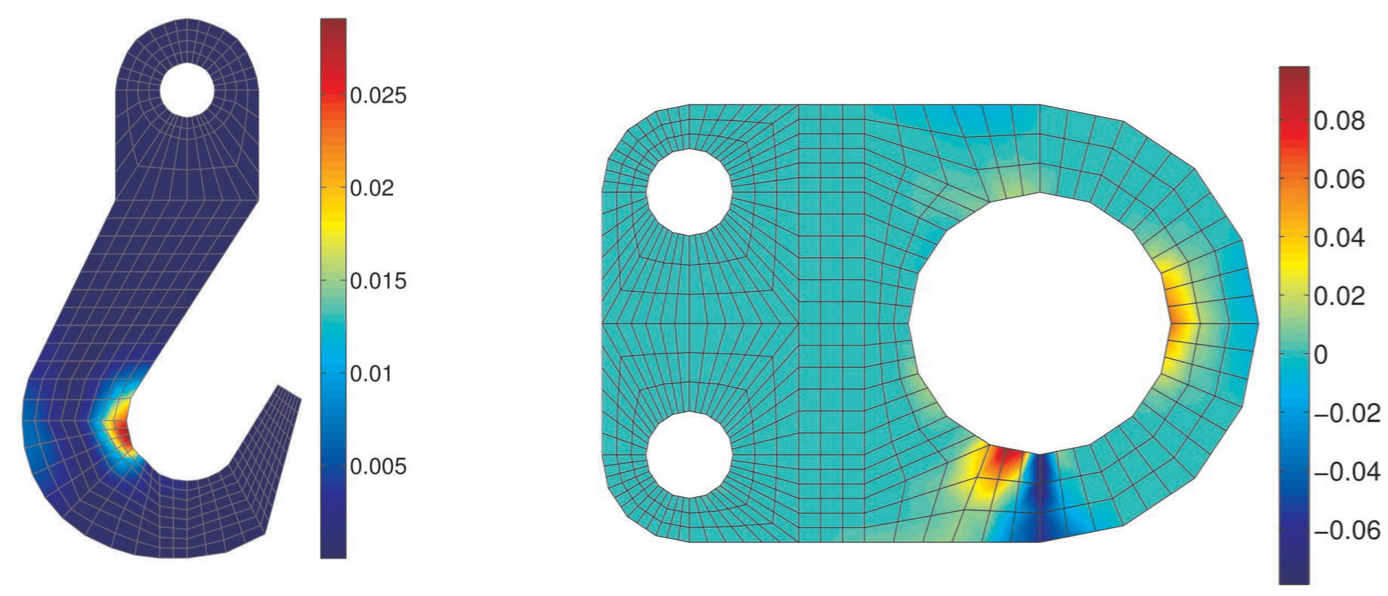
Uncertainty Quantification

Uniformly loaded beam described by uncertain Young modulus:



a) Mean value of displacement *b) Standard deviation of displacement*

Plastic strain ϵ_{pyy} for uncertain bulk and shear moduli:



a) Variance *b) 99% quantile*

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  - Inverse problems, Elastoplasticity

[1] Rosić et al. Variational Theory and Computations in Stochastic Plasticity. *Archives of Computational Methods in Engineering*, available online
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