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**Lecce, Italy**

# Book of Abstracts - Extract
# 2015

UNIVERSITÀ DEL SALENTO

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# Scientific Program - Timetable

| Sunday 22 | Time | Monday 23 | Tuesday 24 | Wednesday 25 | Thursday 26 | Friday 27 |
|---|---|---|---|---|---|---|
| | 9: 15 30 45 | | Contributed sessions (15 in parallel) | Plenary Lecture Moritz Diehl | Contributed sessions (15 in parallel) | Contributed sessions (14 in parallel) |
| | 10: 15 30 45 | Registration | | von Mises prize lecture | | |
| | 11: 15 30 45 | | Coffee Break<br>Plenary Lecture Thomas Böhlke | Coffee Break<br>General Assembly | Coffee Break<br>Plenary Lecture Ferdinando Auricchio | Coffee Break<br>Contributed sessions (11 in parallel) |
| | 12: 15 30 45 | | Lunch | Lunch | Lunch | |
| | 13: 15 30 45 | Opening<br>Univ. Chorus Performance | | | | Closing |
| | 14: 15 30 45 | Prandtl Lecture Keith Moffatt | Plenary Lecture Enrique Zuazua | Contributed sessions (15 in parallel) | Plenary Lecture Daniel Kressner | |
| | 15: 15 30 45 | Plenary Lecture Giovanni Galdi | Plenary Lecture Nikolaus Adams | | Plenary Lecture Stanislaw Stupkiewicz | |
| Registration pre-opening | 16: 15 30 45 | Coffee Break<br>Minisymposia & Young Reseachers' Minisymposia (10 in parallel) | Coffee Break Poster session<br>Contributed sessions (14 in parallel) | Coffee Break<br>Contributed sessions (15 in parallel) | Coffee Break Poster session<br>Contributed sessions (15 in parallel) | |
| | 17: 15 30 45 | | | | | |
| | 18: 15 30 45 | | Public lecture Francesco D'Andria | | | |
| | 19: 15 30 45 | Opening reception at Castle of Charles V | | | | |
| | 20: 15 30 45 | | | Conference dinner at Hotel Tiziano | | |
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# GAMM 2015

Università del Salento

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# MS2: Applications of the Virtual Element Method

The Virtual Element Method is a brand new methodology, related to Mimetic Finite Differences (MFD), to XFEMS, and more generally to the extension of Finite Elements to polygonal and polyhedral decompositions. Unlike MFD, they are a natural Galerkin method, and unlike other extensions of FEM they provide the exact satisfaction of the Patch Test in a number of relevant problems.

The minisymposium would deal with the latest applications of the method, including mixed formulations, time dependent problems, convection or reaction dominated flows, topology optimization, fractured domains and possibly several others. The method is quite new, since the first paper appeared in 2013, and rapidly developing, so that we can expect several important novelties.

# Virtual Element Methods: an overview

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The talk will present a short overview of the Virtual Element Method (VEM) and of the results obtained by the authors in the last year. The first part of the talk will consist of a simple presentation of the general ideas that are at the basis of VEM's, especially suited for those, in the audience, that are not familiar with them. Taking the (simplest) Poisson problem in two dimensions, we will see the main features of Virtual Elements, and the general philosophy behind them. Then, in the second part, we shall move, at a swifter pace, through more advanced features, including three-dimensional problems, variable coefficients, mixed formulations, and some basic applications to simple problems like linear elasticity and Kirchhoff plates.

A particular effort will be done in trying to underline the strong points of VEM's as they stand now, and the aspects that still require an additional work. More generally we will present what are, in our opinion, the actual limits of the method and what are, always in our opinion, the next achievable goals.

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# The Virtual Element Method for large scale Discrete Fracture Network simulations: fracture-independent mesh generation

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Subsurface fluid flow has applications in a wide range of fields, including e.g. oil/gas recovery, gas storage, pollutant percolation, water resources monitoring. Underground fluid flow in fractured media is a heterogeneous multi-scale phenomenon that involves complex geological configurations; a possible approach for modeling the phenomenon is given by Discrete Fracture Networks (DFNs), which are complex sets of polygonal intersecting fractures. We focus on the resolution of the steady-state flow in large fracture networks. The quantity of interest is the hydraulic head in the whole network, which is the sum of pressure and elevation, and is evaluated by means of the Darcy law. We consider impervious rock matrix and fluid can only flow through fractures and through fracture intersections (called traces), but no longitudinal flow along the traces is allowed. Matching conditions need to be added in order to preserve hydraulic head continuity and flux balance at fracture intersections.

Geological fractured media are therefore characterized by a challenging geometrical complexity. This talk concerns the application of the Virtual Element Method [1, 2] to simulations in DFNs [3], in order to tackle this geometrical complexity. Indeed, a crucial issue in DFN flow simulations is the need to provide on each fracture a good quality mesh [4, 5]. Namely, if the mesh on the fractures are required to be conforming to the traces, and also conforming each other, the meshing process for each fracture is not independent of the others, thus yielding in practice a quite demanding computational effort for the mesh generation process. In some cases, the meshing process may even result infeasible.

Here, the VEM will be used in conjunction with a newly conceived PDE-constrained optimization approach for dealing with DFN simulations without the need of mesh conformity [6, 7, 8], as well in conjunction with a mortar approach. Indeed, taking advantage from the great flexibility of VEM in allowing the use of rather general polygonal mesh elements, a suitable mesh for representing the solution can be easily obtained starting from an arbitrary triangular mesh independently built on each fracture, and independent of the trace disposition.

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# The Plane Wave Virtual Element Method for the Helmholtz Problem

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Concerned with the time-harmonic wave propagation governed by the Helmholtz equation, we present a novel Galerkin approximation that can deal with general polygonal partitions. Virtual element methods have been recently introduced [1, 2] as extension of finite elements to general polygonal decompositions for different classes of definite and semidefinite problems. Here we design and analyse a method for an indefinite problem.

Because of the oscillatory behavior of solutions to the Helmholtz equation, methods that incorporate information about the solution in the form of plane waves have received attention in the last years. Our virtual element method for the Helmholtz problem in two dimensions introduces modulated plane wave basis functions.

## References

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# Virtual Element Methods for parabolic problems on polygonal meshes

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The Virtual Element Method (in short, VEM) is a recent technology introduced in [2] as a generalization of the finite element method, that can make use of very general polygonal/polyhedral meshes without the need to integrate complex non-polynomial functions on the elements and preserving an optimal order of convergence. Making use of polygonal meshes brings forth a range of advantages, including for instance better domain meshing capabilities, automatic use of nonconforming grids, more efficient approximation of geometric data features, more efficient and easier adaptivity, more robustness to mesh deformation.

The main idea behind VEM is to use approximated discrete bilinear forms that require only integration of polynomials on the (polygonal) element in order to be computed. The ensuing discrete solution is conforming and the accuracy granted by such discrete bilinear forms turns out to be sufficient to recover the correct order of convergence. Following such approach, VEM is able to make use of very general polygonal/polyhedral meshes without the need to integrate complex non-polynomial functions on the elements and without loss of accuracy. As a consequence, VEM is not restricted to low order converge and can be easily applied to three dimensions and use non convex (even non simply connected) elements.

The talk describes the Virtual Element Methods for parabolic problems. It is considered as a model problem the classical time-dependent diffusion equation. The discretization of the problem requires the introduction of two discrete bilinear forms, one being the approximated grad-grad form of the stationary case [2] and the other being a discrete counterpart of the $L^2$ scalar product. The latter is built making use of the the enhancements techniques of [3]. In the talk we develop a full theoretical analysis, first focusing on the error among the semi-discrete and the continuous problems and later giving an example of error analysis involving the fully discrete case. A large range of numerical tests in accordance with the theoretical derivations is presented. In particular, we study also the possibility to use a simpler but non-coercive $L^2$ discrete bilinear form. Since the low modes are expected to dominate the problem, this choice leads to good results in a lot of situations, as shown by the numerical tests.

This is a joint work with Lourenço Beirão da Veiga from the University of Milano.

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# A $C^1$ virtual element method for the Cahn-Hilliard problem

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Due to the wide spectrum of applications (e.g. phase separation in binary alloys, tumor growth, galaxy formation, foam formation, solidification processes and image processing) the study of efficient numerical methods for the approximate solution of Cahn-Hilliard equations has been the object of an intensive research activity. As the Cahn-Hilliard problem is a fourth order nonlinear equation, a natural numerical approach is to resort to the use of $C^1$ finite elements. However, the well known difficulties related to the practical implementation of $C^1$ finite elements have represented so far an important obstruction that has drastically limited their use in practical applications, thus paving the road to the use of mixed methods (with an increase of the numbers of degrees of freedom, and thus of the computational cost).

In this talk we introduce and analyze a $C^1$ virtual element method (VEM) for the approximate solution of the following Cahn-Hilliard problem: find $u : \Omega \times [0, T] \to \mathbb{R}$ such that:

$$\begin{cases} \partial_t u - \Delta\big(\phi(u) - \gamma^2 \Delta u(t)\big) = 0 & \text{in } \Omega \times [0, T] \\ u(\cdot, 0) = u_0(\cdot) & \text{in } \Omega \\ \partial_{\mathbf{n}} u = \partial_n\big(\phi(u) - \gamma^2 \Delta u(t)\big) = 0 & \text{on } \partial\Omega \times [0, T], \end{cases}$$

where $\partial_{\mathbf{n}}$ denotes the (outward) normal derivative, the function $\phi(x) = \psi'(x)$ with $\psi(x) = (1 - x^2)^2/4$ and $\gamma \in \mathbb{R}^+$, $0 < \gamma << 1$, represents the interface parameter. The virtual element method (see, e.g., [2] for an introduction to the method and [3] for the details of its practical implementation) is characterized by the capability of dealing with very general polygonal/polyedral meshes and the possibility of easily implementing highly regular discrete spaces [5, 4]. In the present contribution we develop an evolution of the $C^1$ virtual elements (of minimal degree) of [4] for the approximation of the Cahn-Hilliard equations. Also taking inspiration from the enhancement techniques of [1], we define the virtual space in order to be able to compute three different projection operators, that are used for the construction of the discrete scheme. Afterwards, we show convergence of the approximation scheme and investigate its performance numerically. We underline that, on our knowledge, this is the first application of the newborn virtual element technology to a nonlinear problem.

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