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Scientific Program - Timetable

Time	Sunday 22	Monday 23	Tuesday 24	Wednesday 25	Thursday 26	Friday 27
9:00			Contributed sessions (15 in parallel)	Plenary Lecture Moritz Diehl	Contributed sessions (15 in parallel)	Contributed sessions (14 in parallel)
10:00		Registration		von Mises prize lecture		
11:00			Coffee Break	Coffee Break	Coffee Break	Coffee Break
11:30			Plenary Lecture Thomas Böhlke	General Assembly	Plenary Lecture Ferdinando Auricchio	Contributed sessions (11 in parallel)
12:30			Lunch	Lunch	Lunch	
13:00		Opening				
13:30		Univ. Chorus Performance				Closing
14:00		Prandtl Lecture Keith Moffatt	Plenary Lecture Enrique Zuazua	Contributed sessions (15 in parallel)	Plenary Lecture Daniel Kressner	
15:00		Plenary Lecture Giovanni Galdi	Plenary Lecture Nikolaus Adams		Plenary Lecture Stanislaw Stupkiewicz	
16:00	Registration pre-opening	Coffee Break	Coffee Break Poster session	Coffee Break	Coffee Break Poster session	
16:30		Minisymposia & Young Reseachers' Minisymposia (10 in parallel)	Contributed sessions (14 in parallel)	Contributed sessions (15 in parallel)	Contributed sessions (15 in parallel)	
18:30			Public lecture Francesco D'Andria			
19:00		Opening reception at Castle of Charles V				
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MS4: Optimal Control and Hybrid Systems

On the one hand, optimal control of real physical processes is of crucial importance in all modern technological sciences. On the other hand, extensive research has been made in the field of hybrid systems in the last decades, since it is well known that the behavior of modern technical systems can only be modeled appropriately by a combination of continuous dynamics and discrete events models. Relevant applications are for example mechanical systems with impacts, power and electronic systems with switches, or complex networks whose coupling structure changes at discrete events. A great interest lies in the optimal control of hybrid systems since this includes not only the computation of optimal control trajectories for the continuous parts but also an optimization of the discrete variables.

The topics in this minisymposium are the analysis and control of hybrid and switched systems, switching-time optimization, mixed-integer optimization for ODE and PDE systems as well as applications in engineering systems and robotics such as bipedal walking.

Sequential Action Control for Nonlinear and Hybrid Systems

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Sequential Action Control (SAC) [1] is a control technique based on ideas from hybrid control theory. The idea behind SAC is that one wants to optimize the next control *action* (defined by a control vector u at a specified time t for a specified duration λ) that improves some cost J by a specified amount $\alpha \in \mathcal{R}$. These actions can then be sequenced to form a discrete-time feedback law, applicable to any nonlinear control system. Surprisingly, for any smooth objective function $J = \int_{t_0}^{t_f} \ell(x,t)dt + m(x(t_f))$ and dynamic constraint $\dot{x} = g(x) + h(x)u$ (almost everywhere differentiable with respect to x), optimizing $J_2 = \int_{t_0}^{t_f} \frac{1}{2}[\frac{dJ}{d\lambda} - \alpha]^2 + ||u||^2 dt$ with respect to u has a unique solution that depends only on the evaluation of $x(t)$ over the time horizon and evaluation of the adjoint variable $\rho(t)$ over the same time horizon. In J_2, $\frac{dJ}{d\lambda}$ is the needle variation that arises in optimal control and hybrid control, so optimizing this expression essentially sets the needle variation equal to α (regularized by the norm on u). Interestingly, the optimal choice of u is a Tikonov regularization; as a consequence it scales nicely with respect to state. The resulting control is a new type of "local" controller, in the sense that J_2 is a local model of J that one optimizes to obtain a control value. Typically local controllers are local with respect to a trajectory $x(t)$ (e.g., linear quadratic regulators), whereas SAC is local both with respect to the state and the duration of application, but it is global with respect to the time horizon t_f and does not locally approximate the dynamics (in contrast to linear quadratic controllers). To obtain a control signal, one may compute these control actions in a sequence, updating as the state updates, so the control calculation is naturally of the form of a feedback law. Moreover, one may optimize over *when* the next control action is taken, since the next control action is well defined for all time t up to t_f. Lastly, since the control is optimal for a infinitesimal duration λ, a backtracking line search may be used to determine the finite duration of the action.

Sequential Action Control is effective across a relatively wide variety of nonlinear and nonsmooth problems. It solves a variety of benchmark problems, including cart-pendulum swing up [2], acrobot swing up [3], pendubot swing up [4], and the minimum time parking problem. In these examples, SAC outperforms best-case solutions in the literature. Compared to direct optimization techniques based on sequential dynamic programming (SQP), SAC finds a solution to the acrobot swing up problem 10^7 times faster than SQP. A change of seven orders of magnitude in computational requirement changes what types of control problems are tractable, particularly as the dimension of an optimal control problem increases.

Other typical requirements for optimal control calculations are also easier to compute using SAC than in many methods for nonlinear and hybrid control. For instance, SAC natively admits control saturation, and incorporates unilateral state constraint easily. Moreover, SAC is guaranteed to be linear near equilibria, so local stability can be assessed using standard techniques (e.g., using sum of squares (SOS) techniques).

Lastly, SAC is as easy to compute for hybrid systems as it is for smooth systems. This is because the adjoint variable $\rho(t)$ can be computed for a given trajectory so long as there are only a finite number of discontinuities in the dynamics. Applying SAC to hopping/running control for SLIP models of human and animal locomotion [5] leads to reliable motion control including nonlinear dynamics and challenging terrain that includes both nonlinearities and nonsmooth characteristics.

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Output Regulation in Differential Variational Inequalities using Internal Model Principle and Passivity-Based Approach

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In this talk, we consider the problem of designing feedback control laws for output regulation in systems with differential variational inequalities (DVIs). In addition to ordinary differential equations, DVIs comprise variational inequalities (VIs) which describe algebraic constraints and relations on the state trajectories [1]. The class of DVIs considered in this paper are described as follows:

$$\dot{x}(t) = f(t,x) + G\lambda(t) \tag{1a}$$
$$v(t) = Hx(t) + J\lambda(t), \quad v(t) \in \mathcal{S}(t), \tag{1b}$$
$$\langle v' - v(t), \lambda(t)\rangle \geq 0, \quad \forall\, v' \in \mathcal{S}(t). \tag{1c}$$

where $\mathcal{S} : [0,\infty) \rightrightarrows \mathbb{R}^{d_s}$ is a set-valued mapping, and it is assumed that $\mathcal{S}(t)$ is closed, convex, and nonempty, for each $t \geq 0$. The variational inequalities expressed in (1c) find utility across many applications [2] and may be used to express optimality conditions, mechanical systems with impacts, and electrical circuits with diodes. While VIs can also be written using the notion of (set-valued) normal cone operators so that the differential inclusion resulting from (1) has maximal monotone operators, the introduction of matrices G, H, J and the vector field $f(\cdot,\cdot)$ may not preserve monotonicity which makes it difficult to study the solution of such systems. From control point of view, the discontinuities in the description of such systems are state-dependent which introduces several complexities in designing feedback control laws.

To address these issues [3], we first derive conditions to study the existence and uniqueness of solutions in such systems. It is shown that under certain conditions on the system data, the given system could be equivalently written as a differential inclusion where the right-hand side is the sum of a multivalued maximal monotone operator and a Lipschitz-continuous function. Such inclusions are then shown to possess unique solutions. For the output regulation problem, we let $f(t,x) := Ax + Bu(t) + Fx_r(t)$, where x_r is an exogenous signal generated by an exosystem, and our objective is to design feedback control u such that the resulting closed-loop system is well-posed and the state x asymptotically tracks x_r. The derivation of control laws is based on the use of internal model principle, and two cases are treated: first, a static feedback control law is derived when full state measurements are available; In the second case, only the error to be regulated is assumed to be available for measurement and a dynamic compensator is designed. Under the condition that the closed-loop system can be rendered passive, we show that the overall system is indeed well-posed and the desired error variable indeed converges to the origin. As applications, we demonstrate how control input resulting from the solution of a variational inequality results in regulating the output of the system while maintaining polyhedral state constraints. Another application is seen in designing switching signals for regulation in power converters.

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Gene regulatory networks: equivalence between Utkin's and sigmoidal approach

Del Buono N., Elia C., Lopez L.
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Mathematical models of gene regulatory networks have received increasing attention in the literature in the past decades. One of the pioneering approaches, due to Glass and Kauffman ([1]), proposes piecewise linear control systems to model gene networks. However, a major concern with such models is that the vector field is not defined on (the intersection of) discontinuity hyperplanes. To overcome this issue, Utkin's and Filippov's approaches have been employed in literature, even though they are in general both ambiguous on the intersection of discontinuity hyperplanes. An alternative continuous model of gene networks, based on steep sigmoidal functions, has been given more recently by Plahte and Kjoglum in [2]. Our purpose in this talk is twofold: show that Utkin's approach gives a unique vector field on the intersection Σ of two discontinuity hyperplanes (under assumptions of attractivity), and investigate when Utkin's approach and the steep sigmoidal approach are equivalent, i.e., when the corresponding solutions on Σ are the same. This allows one to study the piecewise dynamical system, and hence the gene regulatory network it models, with no ambiguity.

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On the value function of mixed-integer optimal control problems

Martin Gugat, Falk M. Hante

We present sensitivity analysis for the optimal value $\nu = \inf \Phi(y)$ with y given by a control problem that involves both continuous and discrete control decisions u and v, respectively:

$$\dot{y} = Ay + f(y, u, v),\ u \in \mathcal{U}(v) \text{ on } (t_0, t_f), \quad y(t_0) = y_0,$$

where the pair (A, f) represents an abstract semilinear evolution system on a Banach space and the sets $\mathcal{U}(v)$ represents control constraints.

For parametric initial data $y_0 = y_0(\lambda)$, we obtain local Lipschitz continuity of the optimal value function $\nu(\lambda)$ for rather mild regularity assumptions on the pair (A, f). The result ist based on a characterization of the optimal value function using a reduction to operational differential inclusions using the theory of [1]. For parametric control constraints $\mathcal{U}(v) = \mathcal{U}(v, \lambda)$ given in terms of inequalities, we obtain the same regularity for convex programs satisfying a Slater-type constraint qualification. In this case, we can give an explicit representation of the one-sided derivatives of the optimal value function $\nu(\lambda)$. The result is based on a strong duality result for parametric disjunctive programming [2].

Our analysis can be applied to assess the robustness of mixed-integer optimal controls even for certain control problems involving partial differential equations and being generated for example by relaxation methods [3].

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Relaxing mixed integer optimal control problems using a time transformation

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Nonlinear control systems with instantly changing dynamic behavior can be described by differential equations $\dot{x} = F(x, u, v)$ that depend on an integer valued control function $v \in \mathcal{L}^\infty(I, \mathcal{V})$, mapping the time interval $I = [t_0, t_f]$ to the integer values $\mathcal{V} = \{1, \ldots, n_\mathcal{V}\}$. Such systems occur for example in the optimal control of a driving car with different gears [1], or a subway ride with different operation modes [2], leading to a mixed integer optimal control problem (MIOCP). A discretize-then-optimize approach leads to a mixed integer optimization problem that is not differentiable with respect to the integer variables, such that gradient based optimization methods can not be applied. Differentiability with respect to all optimization variables can be achieved by reformulating the MIOCP, e.g. by using a relaxed binary control function [2], or by introducing a fixed integer control function $\bar{v}_{N,n} \in \mathcal{L}^\infty(I, \mathcal{V})$ and a time transformation $t_w \in \mathcal{W}^{1,\infty}(I, I)$ that allows to partially deactivate the fixed integer control function [1]. The latter approach is presented here, while the main focus lies on new theoretical and numerical results that take different functions $\bar{v}_{N,n}$ into account.

The time interval I is partitioned into N major intervals I_j, each I_j into n minor intervals I_j^i and $\bar{v}_{N,n}$ is defined to be constant on each of the minor intervals I_j^i. The combination of $\bar{v}_{N,n}$ and a time control $w \in \mathcal{L}^\infty(I, \mathbb{R})$ with $w(\tau) = \frac{dt_w}{d\tau}(\tau) = t'_w(\tau)$ for a.e. $\tau \in I$ results in the following time transformed MIOCP (TMIOCP):

$$
\begin{aligned}
\min_{x,u,w} \quad & J^*(x, u, w) = \int_I w(\tau) B(x(\tau), u(\tau), \bar{v}_{N,n}(\tau))\, d\tau \\
\text{s. t.} \quad & \dot{x}(\tau) = w(\tau) F(x(\tau), u(\tau), \bar{v}_{N,n}(\tau)) && \text{for a.e. } \tau \in I \\
& g_0(x(\tau), u(\tau)) \leq 0 && \text{for a.e. } \tau \in I \\
& w(\tau) g(x(\tau), u(\tau), \bar{v}_{N,n}(\tau)) \leq 0 && \text{for a.e. } \tau \in I \\
& r(x(t_0), x(t_N)) = 0 && \\
& w(\tau) \geq 0 && \text{for a.e. } \tau \in I \\
& \Delta I_j = \int_{I_j} w(s) ds &&
\end{aligned}
$$

Here, J is the objective functional, g_0 and g are inequality constraints and r represents the boundary conditions. A fixed integer control function $\bar{v}_{N,n}$ is called control consistent (CC), if a switch of v at any time in a major interval I_j from a value $l_1 \in \mathcal{V}$ to a value $l_2 \in \mathcal{V}$ can be achieved with $\bar{v}_{N,n}$ by scaling the minor intervals I_j^i, in particular some minor intervals can be deactivated by scaling to zero. The optimal trajectory resulting from the TMIOCP using a CC fixed integer function $\bar{v}_{N,n}$ can coincide with the optimal trajectory of the MIOCP. In contrast to that, the optimal trajectory resulting from the TMIOCP in the case that $\bar{v}_{N,n}$ is not CC (NCC) can result in a higher objective value. Examples demonstrate this and furthermore that solving a TMIOCP using a CC fixed integer control function $\bar{v}_{N,n}$ can lead to a lower number of discretization variables as a TMIOCP that utilizes a fixed integer function that is NCC. The number of necessary discretization variables depends on the total number of used minor intervals and it is shown that the total number can be unbounded in the NCC case and is bounded in the CC case, even though the number of minor intervals for each major interval can be lower in the NCC case. An extension to bipedal walking models is planed in future works.

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Optimal Energy-Based Control of Hybrid Systems with Applications to Robotic Walking

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Over the last fifty years, researchers have addressed the problem of bipedal robotic locomotion using a range of approaches with varying degrees of formality [3]. In this presentation, we are interested in one particular approach known as *energy shaping*. The main idea behind energy shaping approaches involves using the structure of energy to create stabilizing controllers for periodic behaviors in dynamical systems. The primary focus is on recent results [5] which demonstrate an optimal controller for stabilizing the energy dynamics of periodic behaviors in hybrid mechanical system while maintaining exponential stability of the overall hybrid systems.

The development of energy shaping approaches spans the past few decades although energy-based approaches to analytical mechanics date back over 200 years to the work of Lagrange and the principles of stability involved were first presented by Lyapunov over 100 years ago. The ideas herein also build on more recent results spread throughout the literature. Over the last few decades, researchers have presented control schemes which seek to achieve periodic behaviors in dynamical systems and bipedal walking in particular by formulating their control objectives in terms of the energy of a system. Based on McGeer's observation that compass-gait bipeds[1] with appropriate mass distributions can walk down shallow slopes without actuation [4], Spong presented controlled symmetries [8] as a method for obtaining walking on a compass-gait biped on flat ground by injecting energy in a such a way that the shaped potential energy of the robot's gait on flat ground mimicked the potential energy of a passive biped walking down a slope. Based on this idea, Spong later provided a controller which could shape the total energy of a compass-gait biped and showed that the controller would guarantee asymptotic stability of the energy dynamics to a reference level, E_{ref}, through the continuous dynamics although nothing was formally established with regards to stability of the overall system. He later demonstrated how the ideas could be extended to non-conservative systems [8] by considering energy storage functions. While the compass-gait biped is of interest as a low-dimensional example of walking, the same method of controlled symmetries has been used as a basis for achieving walking in more complex models with knees and non-trivial foot action [6] and has been extended to three-dimensional bipeds using functional Routhian reduction [7].

The energy shaping problem, as formulated herein, falls under a class of problems involving stability of systems with zero dynamics. In [1], a similar problem was considered in which a stabilizing control law was constructed using control Lyapunov functions to stabilize to a zero dynamics which exhibited hybrid invariance; that is, for initial conditions on the intersection of the switching surface and the hybrid zero dynamics manifold, application of the reset map will result in a state which is still on the hybrid zero dynamics. This was a key assumption underlying [1] but this assumption does not hold for energy shaping as energy is generally not invariant through impact, though there may be pathological examples which demonstrate this property. In fact, for certain conservative systems like the compass-gait biped, which exhibits local exponential stability, energy change can only occur through discrete transitions and so impacts actually act as a stabilizing influence.

In order to apply energy shaping, one begins with an autonomous hybrid mechanical system with position and velocity coordinates $(q, \dot{q})$ which contains a locally exponentially stable periodic orbit, possibly induced by some feedback control law. For systems of this type, a conserved energy quantity exists through the continuous dynamics and it comprises kinetic and potential energy as well as an additional term. This additional term can be treated with a storage function which tracks the energy flowing out of the system; such energy flow occurs due to non-conservative forcing. For periodic behaviors, this conserved quantity, expressed as

$$E_c = T(q, \dot{q}) + U(q) - \int_0^t F_{nc}(q(\tau), \dot{q}(\tau)) \cdot \frac{dq(\tau)}{d\tau}\, d\tau$$

with the terms respectively representing kinetic energy, potential energy, and stored energy, is also periodic and, by resetting the storage function appropriately through the discrete dynamics, can be made to be a constant. Using the conserved quantity, which is a function of the system state, and its constant reference level, one can design an energy shaping controller to drive the conserved energy of the system to the reference level, thereby stabilizing the system's energy dynamics. The particular approach discussed achieves this energy stabilization through the use of a *rapidly exponentially stabilizing control Lyapunov function (RES-CLF)* [1, 2]. Roughly

[1] Video of a compass-gait biped available online http://youtu.be/mwugDGGhPmI.

speaking, one must choose control inputs such that the energy of the system is exponentially stable through the continuous dynamics. This is done by constructing the Lyapunov candidate function $V(q,\dot{q}) = (E_c(q,\dot{q}) - E_{ref})^2$. Because $\dot{V}(q,\dot{q},u) = L_f V(q,\dot{q}) + L_g V(q,\dot{q})\,u$ is affine with respect to the control input, the problem can be posed as a quadratic program operating on a convex set:

$$\mu_\varepsilon(q,\dot{q}) = \underset{u\in\mathbb{R}^m}{\arg\min}\ u^T u \\ \text{s.t. } L_f V(q,\dot{q}) + L_g V(q,\dot{q})\,u + \frac{c_3}{\varepsilon} V(q,\dot{q}) \le 0.$$

This results in an optimal controller that can be evaluated with relatively low computational cost. In addition to being easy to solve, this formulation confers the added benefit of allowing torque limitations as well as constraints on friction and on ground reaction forces, although doing so may destroy feasibility. With no constraints, the solution is feasible and is known in closed form, often referred to as the pointwise min-norm controller [2].

In contrast to its predecessors, it has been formally shown that the presented controller maintains exponential stability of the hybrid system to which it is applied for small enough control gains. Although formal results regarding the robustness of the shaped system to perturbations in initial conditions have not yet been established, simulations show that the method can result in a larger domain of attraction and shorter stabilization times for limit cycles of certain systems. Indeed simulation results are provided for a cart–spring system to demonstrate improved convergence properties and for a compass-gait biped to demonstrate the robustness improvement that can be conferred by energy shaping. Despite the lack of formal claims with respect to global properties, the local stability properties are practically useful when using energy shaping as a stabilizing controller in operating regions around the desired behavior. Ultimately, the goal will be to apply these methods to complex biped and humanoid robots with a view toward robotic–assistive devices.

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