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Scientific Program - Timetable

Time	Sunday 22	Monday 23	Tuesday 24	Wednesday 25	Thursday 26	Friday 27
9:00			Contributed sessions (15 in parallel)	Plenary Lecture Moritz Diehl	Contributed sessions (15 in parallel)	Contributed sessions (14 in parallel)
10:00		Registration		von Mises prize lecture		
11:00			Coffee Break	Coffee Break	Coffee Break	Coffee Break
11:30			Plenary Lecture Thomas Böhlke	General Assembly	Plenary Lecture Ferdinando Auricchio	Contributed sessions (11 in parallel)
12:30			Lunch	Lunch	Lunch	
13:00		Opening				
13:30		Univ. Chorus Performance				Closing
14:00		Prandtl Lecture Keith Moffatt	Plenary Lecture Enrique Zuazua	Contributed sessions (15 in parallel)	Plenary Lecture Daniel Kressner	
15:00		Plenary Lecture Giovanni Galdi	Plenary Lecture Nikolaus Adams		Plenary Lecture Stanislaw Stupkiewicz	
16:00	Registration pre-opening	Coffee Break	Coffee Break Poster session	Coffee Break	Coffee Break Poster session	
16:30		Minisymposia & Young Reseachers' Minisymposia (10 in parallel)	Contributed sessions (14 in parallel)	Contributed sessions (15 in parallel)	Contributed sessions (15 in parallel)	
18:30			Public lecture Francesco D'Andria			
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20:30				Conference dinner at Hotel Tiziano		
21:00						

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S09: Flows and transition

This section will focus on the analysis and modeling of transition from laminar to turbulent flow using DNS, LES, RANS equations, and experiments.

Contributions are expected in, but not limited to, the following topics: stability of incompressible and compressible flows, fundamental study of the dynamics of transition, influence of the wall roughness on transition, transition modelling for LES and RANS equations, transition in flows with complex geometries, subcritical transition.

Stability analysis of the flow past miniature vortex generators in a Blasius boundary layer

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Transition delay from laminar to turbulent regime in a boundary layer (BL) is a subject of fundamental importance for its direct connection with the reduction of friction drag. Considering a Blasius BL, for external disturbances with low intensity, the transition scenario is dominated by exponentially growing plane waves, known as Tollmien-Schlichting (TS) waves. In the literature, it has been shown that TS waves can be damped and transition delayed by means of a proper modulation of the streamwise velocity in the BL, commonly called *streaks* (see, for instance, [1] and [2]). The streaks are low- and high-speed regions inside the BL, typically obtained with surface mounted vortex generators in order to activate the lift-up mechanism.
Among various configurations for obtaining streaks in a BL, the use of small winglets, commonly referred to as *miniature vortex generators* (MVGs), has shown to lead to the formation of streaks with good characteristics, as documented in [3]. This type of BL control is experimentally investigated in [4] (see also [5]), where a spanwise array of MVGs is installed on a flat plate in order to quantify its stabilizing effect on the evolution of the TS waves, generated upstream of the MVGs. For all the experimental configurations documented in [4], the TS waves undergo an amplification in the near wake of the MVGs but, moving further downstream, the TS wave amplitude shows different behaviours depending on the experimental set-up. For some configurations (e.g. C01 and C11 in [4]), the experiments show a monotonic decrease of the TS wave amplitude along the flat plate. For other configurations (e.g. C09 and C05 in [4]), the damping of the TS waves ends at a given distance from the MVGs and, further downstream, the amplitude of the TS wave starts to grow again. The aim of this work is to propose an interpretation of these different behaviours based on the investigation of the stability properties of the controlled BL.
Some representative experimental set-up are selected among all the configurations documented in [4] and simulated by Direct Numerical Simulations (DNSs) in order to have highly resolved velocity fields in space. The DNS results, validated against the experimental data, are then used in the Bi-Global stability analysis in order to estimate the neutral stability curves for the controlled cases. The stability curves are discussed in comparison with the one of the uncontrolled Blasius BL and they are shown to be in good agreement with the available experimental observations, thus providing an interpretation for the behaviour of TS waves. Finally, the results from the stability analysis are used to characterize further the stabilization mechanism induced by the MVGs.

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P-norm optimal 3D perturbations in the Poiseuille flow

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The fastest route to turbulence in a subcritical ($Re = 4000 \leq Re_c = 5772.2$) plane Poiseuille flow is investigated by means of a Lagrange-multiplier direct-adjoint optimization process in order to find the three-dimensional perturbative velocity field that provides the largest perturbation growth at a given time horizon (T_{opt}). In particular, spatially localized three-dimensional optimal perturbations are sought, like the ones that can be found including non-linear terms in the optimization process [1, 2]. In this work, we aim at obtaining a similar localization also in a linear framework by a suitable choice of the objective function, as proposed in [3] for the same flow case in a two-dimensional framework. Instead of the classical L^1 norm of the perturbative energy density, we use as objective function a general L^p norm that provides localization of the optimal perturbation even in a linear framework.

We focus our attention on the small time scale (typical of the Orr mechanism), choosing a short optimization time, and on moderate initial energies E_0, in order to avoid the triggering of strong non-linear effects, and thus allow a comparison between the non-linear 1-norm optimisation and the linear p-norm one. For $T_{opt} = 10$ and $E_0 < 2 \cdot 10^{-6}$ the 1-norm non-linear optimal shows a weaker localization than the corresponding linear p-norm one. However, the linear optimal vortices are streamwise-aligned, in contrast with the non-linear ones [1]. For a given initial energy, the linear p-norm optimization at $T_{opt} = 10$ provides a disturbance with higher values of the three components of the velocity, which is thus more efficient than the non-linear optimal in inducing transition.

Increasing the target time, the effects of non-linearity on the shape of the optimal perturbations, in terms of streamwise/spanwise modulation and spatial localization of the vortices, begin to be more and more relevant, at high enough energy levels. On the other hand, the p-norm linear optimal appears to be not affected by an increase of the target time. Therefore, for higher target times ($T_{opt} \geq 20$), the 1-norm non-linear optimal provides the most efficient route to transition in terms of time to transition and initial energy.

Finally, the transition mechanisms of the optimal perturbations discussed above are studied by direct numerical simulations. In particular, the p-norm optimal follows a transition path similar to the oblique transition scenario [4], with slightly oscillating streaks, which saturate and then experience secondary instability. Concerning the non-linear optimal, at short times, it is also characterized by a modal composition resembling an oblique wave. However, its time evolution differs from an oblique wave one, since it rapidly forms bent streaks with large amplitude instead of streamwise aligned ones. Non-linear optimals are thus able to provide a faster route to transition by skipping the phase of streak saturation. This study thus provides an analysis of the potential and the limits of p-norm optimisations for obtaining localized optimal structures able to lead to transition even in a linear framework, avoiding the hard computational and storage costs typical of a non-linear optimization approach.

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Unified description of bifurcation processes associated with laminar boundary-layer separation

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The presentation will concentrate on flows where no steady state exists if an appropriately defined controlling parameter exceeds a critical value while non-uniqueness is observed for subcritical values of this parameter. Special attention is placed on flow phenomena which are associated with the passage through criticality. Based on a triple deck analysis it found that they can be decribed as solutions of differential equations of Fisher type which are better known from evolution studies of gene populations. Special examples which will be discussed include 2D marginally separated flows, weakly 3D transonic flows in slender channels and fully 3D subsonic flow past expansion ramps.

Investigation of the roughness-induced transition: linear and non-linear optimal perturbations

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For small amplitude disturbances and supercritical Reynolds numbers, linear stability theory predicts the slow transition of flat plate boundary layer flows as a result of the generation, amplification and secondary instability of Tollmien-Schlichting (TS) waves. Fransson *et al.* [1] have shown that the streaks induced by a three-dimensional roughness element placed on the flat plate can stabilize these TS waves. Unfortunately, beyond a given amplitude of the roughness element, the resulting flow can transition right downstream the roughness element. Such transition past cylindrical roughness elements has been investigated by Loiseau *et al.* [2] in the framework of *global stability* where it has been shown that transition could be explained by a global instability of the flow. Despite the plausible explanation it provides, this study does not rule out the possibility for the flow to experience *bypass transition* wherein the flow transitions to turbulence despite all the modes of the linearised Navier-Stokes operator being linearly stable. Such transition is related to the non-normality of the linearised operator: small disturbances can experience a large transient amplification due to constructive interferences of non-orthogonal stable modes. If the growth is sufficiently large, such disturbances can trigger non-linear effects allowing them to self-sustain, resulting in bypass transition to turbulence. It is the computation and investigation of such perturbations that we are addressing in the present work using the framework of linear and non-linear *optimal perturbation analysis*.

Provided a linearly stable base flow $\mathbf{U}_b$, the aim of this work is to compute the linear and non-linear velocity perturbations $\mathbf{u}$ optimizing the objective functional $\mathcal{J}(\mathbf{u}) = \frac{1}{2}\int_V \mathbf{u}(T)\cdot\mathbf{u}(T)\ \mathrm{dV}$. That is, we look for the perturbation at time $t = 0$ which provides the maximum value of the objective functional at a given target time $t = T$. The optimization problem is subject to a set of partial differential constraints, *i.e.* the perturbative (linear or non-linear) Navier-Stokes equations, that must be verified at each time and each point of the computational domain. A constraint on the initial value of the perturbation's kinetic energy is also imposed, *i.e.* $E(0) = E_0$. These optimal perturbations are computed using a *Lagrange multipliers technique* which involves the adjoint Navier-Stokes equations. For the particular flow configuration considered, once spatially discretised, the optimisation problem involves almost 100 millions of degrees of freedom. To solve it, a rotation-update gradient-based method [3] is used.

The linearly stable flow investigated is the same as in [1] and [2]. The base flow consists in a horseshoe vortex wrapped around the cylindrical roughness element, whose legs give birth further downstream to streamwise velocity streaks. A central low-speed region is also generated in the wake of the roughness element due to the blockage it induces. Linear optimal perturbation analyses have shown that, despite the flow being linearly stable, infinitesimal perturbations can have their energy amplified by a factor 10^6 over a time interval $T \simeq 55$ before decaying due to the linearly stable nature of the flow. While the optimal perturbation consists in alternated patches of velocity oriented against the base flow shear and localized in the vicinity of the roughness element, the optimal response is a wavepacket of velocity patches, now oriented along the base flow shear, that has travelled further downstream. The underlying amplification mechanisms have been investigated using the Reynolds-Orr equation which governs the transfer of energy between the base flow $\mathbf{U}_b$ and the perturbation $\mathbf{u}$. This analysis has revealed that the perturbation bases its transient amplification on two different mechanisms: (*i*) the Orr mechanism at short times ($t < 40$), and (*ii*) the lift-up effect at larger times ($t > 40$). How non-linearity influences the optimal perturbation and associated response, as well as the energy extraction process, is currently under investigation with the computation of the corresponding non-linear optimal perturbations.

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Extreme event detection in near-wall turbulence using reflection-encoded readout of micropillar arrays

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This contribution describes a method to detect specific events in the near-wall turbulence with micropillar arrays. Such events are e.g. the recently discovered rare backflow events, which have been so far not been detected using state-of-the art techniques to measure the velocity close to the wall such as hot-wire or LDA probes [1]. This is because the sensing volume needs to be located away from the wall, and the frequency of occurrence drastically reduces with distance from the wall. Second, in order to capture the strong velocity gradients within the viscous sublayer, the diameter of the measurement volumes has to be as small as the viscous length scale, which brings along a drastic reduction of the sampling rate [1]. In comparison, flexible micropillar structures tailored for the specific flow conditions can provide distributed quantitative measurements of the WSS in sufficient spatial and temporal resolution [2]. So far, the recordings are interpreted in a post-processing routine to obtain the WSS field and no direct read-out is possible. The present paper uses a new technique to detect these events in a quasi online manner. Therefore, the pillars are tilted in streamwise direction and have a rectangular front-wall which is reflective coated. If the array is illuminated with parallel light from the side, the reflected light spots are of maximum intensity at certain bending loads. The recording camera can be set in such a position that these reflections disappear at zero load which signals the appearance of critical points in the flow [3]. A camera records these dark spots on-line and allows to record these rare backflow events. Since those events are correlated with strong vortices in the log-layer the sensors can be used for flow control purposes. The calibration of the system is done in a planar Couette flow facility at different shear rates. Measurements are shown for well defined disturbance patterns in the flow and are compared with theoretical response behavior in unsteady flow conditions.

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Spreading of Linear Disturbances in Boundary Layers at Mach Number Five and the Effect of Wall Cooling

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In this contribution we focus on inviscid linear stability eigensolutions of zero pressure gradient flat plate boundary layers at Mach number $M = 5$ without and with wall cooling. The spreading of linear disturbances in these boundary layers is investigated and its relation to the transition process is discussed.

The undisturbed stationary, two-dimensional base flow is obtained from the boundary layer equations for a perfect gas with a ratio of specific heats $\gamma = 1.4$ and a constant Prandtl number of $Pr = 0.72$. Sutherland's law for viscosity with a constant $Su = Su^*/T^*_\infty = 110K/293K$ is applied where T^*_∞ is the freestream temperature and $*$ denotes dimensional values. The Dorodnitsyn-Howarth transformation is applied to the boundary layer equations and the resulting ordinary differential equation system is solved with a shooting method.

At $M = 5$ inviscid instabilities are present in the boundary layer [1] which we compute with a local eigenvalue solver based on a shooting method. For the singularity (inflection point) a contour deformation in the complex plane is made. The contour deformation is also applied to the baseflow where the same shooting method is employed.

The resulting dispersion relation $\omega(\alpha)$, where ω is the frequency and α is the wavelength, is evaluated at $M = 5$. The first mode − equivalent to the Tollmien-Schlichting mode in incompressible flow − shows its largest growth in the oblique direction. Higher two-dimensional modes are present as well. Two-dimensional second-mode instabilities show the largest growth rates. Wall cooling damps the first mode and amplifies higher modes. For a wall temperature T_w that equals the freestream temperature, the first mode is no longer present and higher modes have merged [1].

We use the method of steepest descent [2] to determine the spreading of local disturbances. The solutions are wave packets described by rays $x/t = \partial\omega/\partial\alpha$ ($\partial^2\omega/\partial\alpha^2 \neq 0$) of constant growth $\alpha_i x/t - \omega_i$ in the limit of time $t \to \infty$. Since the method provides the solution for this limit and inviscid instabilities are eigensolutions far downstream in the boundary layer (also in the limit $t \to \infty$) the method appears to be well suited for the problem.

In the case of the adiabatic wall we analyse modes 1 and 2 separately with the method of steepest descent. For every ray of the resulting wave packet we take the solution with the largest growth. For the streamwise direction, the solution of mode 2 shows higher growth rates throughout than the packet of mode 1. Thus mode 1 is negligible compared to mode 2, and mode 2 dominates the wave packet.

The amplification of higher modes by wall cooling with $T_w = T_\infty$ results in wave packets of larger extent. The wave packet has the special property of very high wave numbers at the trailing edge which propagates slowly.

Although the results clearly show that the boundary layers are convectively unstable, we interpret the slow trailing edge of the packet for wall cooling as a trend towards absolute instability. It appears that for the cooled wall the transition properties become close to those resulting from an absolute instability. This can be seen in turbulent spots which become highly elongated in the streamwise direction for cooled walls [3]. Since the propagation velocity of the turbulence is always larger or equal to the spreading of linear disturbances [2], we see the strong elongation of the spots as a result of the linear stability properties and the large streamwise spreading rates. Long elongated structures are known from absolutely unstable flows like the rotating disc boundary layer where stationary streamwise vortical structures are observed in the transitional zone.

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Receptivity and non-uniqueness of turbulent boundary layer flows

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This contribution ties in with the established asymptotic theory of boundary layer stability in the limit of large globally formed Reynolds numbers, Re, surveyed e.g. in [1]. In particular, the asymptotically correct description of the propagation and excitation of Tollmien–Schlichting (TS) waves in a (two-dimensional) laminar boundary layer by spatially isolated events, as typically an oscillating surface protuberances, and/or acoustic and vortex waves in the exterior flow lays the foundation of what has been referred to as receptivity theory.

As an important result of the analysis, the boundary layer is highly susceptible to downstream-growing instabilities that originate in weakly unstable modes referring to the vicinity of the lower branch of the neutral curve in the Re-wavenumber plane. Hence, these modes finally trigger the classical scenario of laminar–turbulent transition. Upstream of this event, the aforementioned rigorous stability and receptivity theory applies in its currently available, advanced form, which could explain many interesting phenomena with great success [1].

Interestingly, the initiation of TS waves and finally their impact on transition by distributed rather than localised disturbances has not gained much awareness, probably due to the associated mathematical difficulties. Here surface roughness forms the, from an engineering point of view, probably most important and typical example. By the same token, one may ask how a more-or-less developed and nominally two-dimensional and steady turbulent boundary layer reacts on such perturbations. This question is even more interesting as it has been shown quite recently that a fully developed turbulent boundary layer manifests itself in two distinctly different structures, depending on the driving pressure gradient and upstream conditions, and the transition between those is an issue of their stability against small disturbances. In [2] the associated question of non-uniqueness was clarified for the specific case of self-preserving flows under the strongest adverse pressure gradients found possible; a preliminary study sheds some light on their temporal evolution for relatively long times and associated stability. The asymptotic approach requires to extend the classical Reynolds-averaging to a conditional sampling in an asymptotic sense, i.e. by starting with a time filter of a width depending on the global reference quantities and then suitably straining the non-dimensional time, t, in dependence on Re. Let x denote the accordingly non-dimensional streamwise coordinate and $DO(1) = O(1)$ be a measure for the streamwise velocity deficit in the core of the boundary layer. The asymptotic analysis of the accordingly averaged Navier–Stokes equation then allows to express non-unique states of the base flow and their stability and reorganisation by common secular-term elimination for correspondingly long fractions of time in form of the evolution equation

$$D\,\partial D/\partial\theta = 1 - m(\theta)D^2 + nD^3\,, \quad D = D(\theta,\xi)\,, \quad \theta := \ln t/(\ln Re)^{2/3}\,, \quad \xi := x - t^{3/4}\,. \tag{1}$$

Here the function m accounts for the imposed variation of the external driving pressure and the coefficient n for nonlinearities due to inertia; it might depend on x if surface curvature is taken into account. The distortion of the time is necessitated by the self-similar structure of the base flow. For constant and positive values of m, the notations upper/lower branch now refer to the curve in the (m, D)-plane that describes unstable/stable steady-state solutions of (1) for accordingly large/small velocity deficits. Near its turning point, including effects of weak spanwise variations of the flow modifies (1) to the well-known Fisher's equation having exciting properties.

We extend the analysis by considering perturbations with wavelengths much smaller than the reference length of the x-direction, like the representative TS waves in laminar flow. We then study the receptivity of a turbulent boundary layer not only against pressure disturbances but also small-scale surface waviness. Due to the predominantly inviscid nature of turbulent flow on the scales considered, the analysis differs distinctly from its counterpart applying to laminar boundary layers. Most important, no specific turbulence model is needed.

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Axisymmetric flow over a sudden expansion in an annular pipe

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Many investigations of the flow over a backward-facing step have been carried out for plane channel flows, see e.g. [1, 2, 3]. Here, we investigate the flow over a sudden expansion in an annular pipe in which the radius of the inner cylinder R_i^{in} decreases suddenly to R_i^{out}, whereas the radius of the outer cylinder R_o remains constant. The steady axisymmetric flow is computed using primitive variables discretized by a second-order finite-volume method. The resulting algebraic equations are solved by Newton–Raphson iteration, as in [3]. The problem under consideration is an extension of the plane-channel sudden-expansion problem which is recovered in the limit of small radius ratios, i.e. for $\eta^{\text{out}} = R_i^{\text{out}}/R_o \to 1$ with a fixed non-zero step height $R_i^{\text{in}} - R_i^{\text{out}} > 0$.

Streamline patterns and properties of the separation zones are presented as functions of the outlet radius ratio η^{out} for a fixed expansion ratio $H/h = 2$ and Reynolds numbers in the range $10^{-2} \leq \text{Re} \leq 800$. The Reynolds number at which the flow starts to separate from the outer cylinder decreases as the outlet radius ratio η^{out} decreases. This indicates the flow separation on the outer cylinder is stronger the smaller inner radius R_i^{out} of the outlet is. In fact, for $\eta^{\text{out}} < 0.3$ the jet emerging from the narrow upstream annulus into the wider downstream annulus remains attached to the inner cylinder for a long distance downstream of the step. This is accompanied with the separation zone on the outer cylinder being elongated in downstream direction. As result further separation zones arise nested inside the separation zone defined by the streamline separating first from the outer cylinder. Moreover, owing to the intense flow inside the stretched separation zone on the outer wall co-rotating vortices resembling Kelvin's cat's eyes arise inside the separation zone.

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The Clebsch transformation and its capabilities towards fluid and solid mechanics

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In fluid dynamics, Clebsch made use of the representation for the velocity field $\vec{u}$ in terms of three potentials Φ, α, β as

$$\vec{u} = \nabla\Phi + \alpha\nabla\beta \tag{1}$$

in order to construct a first integral of the equations of motion in case of an inviscid flow with vortices [1]. Apart from this, he recieved a self-adjoint form of the equations allowing for deriving them from a variational formulation. In later times the Clebsch transformation has been extended towards more general cases, for instance to baroclinic flow [2] by including thermal degrees of freedom. The capability of this method, however, is not restricted to fluid dynamics: Wagner [3] applied this methodical approach to the Maxwell equations in classical electrodynamics and Schoenberg [4] made use of it in quantum theory within the context of a quantization of vortex tubes. He also gave a detailed description of the symmetry group of gauge transformations of the Clebsch potentials. For Calkin [5] the use of Clebsch potential was requiered in order to formulate an variational principle in Magnetohydrodynamics and, finally, by Zuckerwar and Ash [5] Clebsch potentials have been used for the analysis of compressible flow with volume viscosity. Shear viscosity, however, has not been part of their investigations and has, to our best knowledge, not yet been formulated in terms of Clebsch variables.

In his paper [6] Scholle found general rules for the construction of Lagrangians in continuums theory based on symmetry consideration. In this context the mathematical structure of the Clebsch transformations appears as a natural outcome.

It is the aim of this paper to demonstrate how Clebsch variables can be applied to viscous flow on the one hand, leading to a first integral of Navier-Stokes equations as a first example.

As a second example, solid mechanics is considered: by making use of an analogy between vortices in fluid flow on the one hand and dislocations in crystals on the other hand, a dynamic theory of dislocations can be established by using a certain modification of the Clebsch transformation.

Examples are given and further perpectives of the method are discussed.

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Potential theory application to superfluidic multiphase flows

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Last decades superfluidic flows attract constantly growing interest of investigators in the field of fluid mechanics and cryogenic technologies. Mathematical models of superfluidic flows are relatively simple and belong to well-known and good-investigated ideal fluid flow theory. However there is a sufficient difference between traditional ideal fluid flow and superfluidic flow in plane case, concerning circulation and correspondent lifting force. According to L. Prandtl viscous forces, acting during long time in boundary layer, generate vortices that are moved by flow on enough far distance. At last, flow with zero separated vortex sheet is realized asymptotically in time. However there is not any viscous force in superfluidic flow and, consequently, there is not any mechanism of circulation generation. Thus zero-circulation condition must be prescribed for any plane superfluidic flow instead additional conditions for determination of circulation. Most of usual mathematical models of multiphase flows belong to Eulerian models and are based on some homogenization principle for phase interaction. In contrast to them, only hydrodynamic interaction forces and possibly dynamical forces in non-stationary flows may take place in the considered case. Dynamical forces cannot be so sufficient due to specific constrictions to superfluidic flow exists. Hydrodynamic interaction forces can be analyzed only by Lagrangian approach. Stationary multiphase superfluidic flow is unique case completely driven by hydrodynamic interaction forces. The problem is formulated using velocity potential as Neumann problem for Laplace equation. Due to extremely complex shape of solution domain only boundary element method can be effectively used in this case. Dependences of hydrodynamic interaction force on shape of bodies, their sizes and distances between them are calculated numerically in plane and space cases. The most interesting was problem about superfluidic flow around regular grid of identical bodies. Stability of this system is investigated numerically for the plane case of regularly situated circles. The considered system is stable in longitudinal direction and unstable in transversal direction.