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Scientific Program - Timetable

Sun day 22	Time	Monday 23	Tuesday 24	Wednesday 25	Thursday 26	Friday 27
	9:15 30 45		Contributed sessions (15 in parallel)	Plenary Lecture Moritz Diehl	Contributed sessions (15 in parallel)	Contributed sessions (14 in parallel)
	10:15 30 45	Registration		von Mises prize lecture		
	11:15 30 45		Coffee Break	Coffee Break	Coffee Break	Coffee Break
	12:15 30 45		Plenary Lecture Thomas Böhlke	General Assembly	Plenary Lecture Ferdinando Auricchio	Contributed sessions (11 in parallel)
	13:15 30 45		Lunch	Lunch	Lunch	
		Opening				
		Univ. Chorus Performance				Closing
	14:15 30 45	Prandtl Lecture Keith Moffatt	Plenary Lecture Enrique Zuazua	Contributed sessions (15 in parallel)	Plenary Lecture Daniel Kressner	
	15:15 30 45	Plenary Lecture Giovanni Galdi	Plenary Lecture Nikolaus Adams		Plenary Lecture Stanislaw Stupkiewicz	
Registration pre-opening	16:15 30 45	Coffee Break	Coffee Break Poster session	Coffee Break	Coffee Break Poster session	
	17:15 30 45	Minisymposia & Young Reseachers' Minisymposia (10 in parallel)	Contributed sessions (14 in parallel)	Contributed sessions (15 in parallel)	Contributed sessions (15 in parallel)	
	18:15 30 45		Public lecture Francesco D'Andria			
	19:15 30 45	Opening reception at Castle of Charles V				
	20:15 30 45			Conference dinner at Hotel Tiziano		
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S12: Waves and acoustics

Waves are a ubiquitous natural phenomenon and acoustics are, besides surface water waves, the most obvious representatives, familiar to anybody and quantitatively known to any student of mathematics, physics or a technical subject. To this corresponds a long mathematical tradition, continuing today in the accurate numerical computation of linear and nonlinear wave phenomena.

Our session is devoted to the simulation and understanding of waves and wave interactions. The range of applications is thus very broad, while the focus is meant to be on the unifying physical phenomenon. In the past years we had numerous contributions from solid mechanics, porous media flow, turbulence and aeroacoustics, from crack detection to explosions.

Propagation of acoustic waves in a turbulent boundary layer

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Excitation of the walls of a vehicle by turbulent boundary layers constitutes a major source of interior noise, and can be dominant for cruising trips. The random character of the pressure distribution induced by the boundary layer has a major effect on the noise transmitted through the fuselage and its cross-spectral density plays a major role in determining the effective force causing motion to the structure. In the present study, flat-plate turbulent boundary layers with zero pressure gradient are computed using direct noise computation, i.e. solving the compressible Navier-Stokes equations. These numerical experiments allows the determination of the direct aeroacoustic contribution, of low intensity but high efficiency, together with the hydrodynamic pressure due to the turbulence, which can excite structural modes [1].

Noise intensities radiated in the freestream indicate that the sources are turbulence quadrupoles, explaining the low efficiency. In particular, a U^8 power-law dependence is found confirming the quadrupolar nature of wall turbulence, in accordance with Powell's reflection principle [2]. For Mach numbers ranging between 0.3 and 0.9, we have seen that the acoustic wavefronts have a large wavelength and an upward-oriented directivity. The acoustic domain for the frequency-wavenumber representation of the wall pressure, which is the imprint of acoustic waves on the wall, is analysed. At the low frequencies of interest, it can not be unambiguously separated from the convective ridge due to the relatively high Mach numbers considered, but the acoustic activity is clearly visible. Unlike the far-field noise, the high frequency range is dominated by acoustic waves propagating in the downstream direction, whereas counterflow waves are almost absent.

To explain this paradox, ray tracing [3] is used to characterize propagation effects due to the mean boundary layer profile. By neglecting the slow thickening of the TBL in the flow direction, the effect of the mean flow on the acoustic propagation is well-described by the Pridmore-Brown operator for a simple shear flow [4]. A shadow zone is formed upstream of the source because of the refraction effects, and for sufficiently high frequencies, the refraction of acoustic rays lead channeled waves trapped inside the boundary layer downstream of the source. The latter phenomenon, commonly observed in ocean acoustics, is referred to as channeling or wave guide effect. According to [5], the multiple trapped channeled waves can form caustics, leading to localised regions of intense pressure. They argue that the caustics can thus participate to the mechanisms sustaining coherent structures in wall turbulence. The range of acoustic frequencies must be compatible with the turbulent characteristic frequencies such as the bursting and ejection events.

The ray tracing is performed for the different Mach numbers of the large-eddy simulation databases. As the freestream Mach number decreases, the critical angle for the formation of a shadow zone becomes higher. A large amount of acoustic energy is focused near the critical angle for higher Mach numbers. Channelling effect is also reinforced. As a consequence, in the high-frequency limit, the wall acoustics is essentially in the downstream direction due to the channeled waves, and the acoustic peak in the rearward direction is hard to observe because of refraction effect. Only turbulent scattering can provide acoustic energy in this direction.

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Non-destructive testing of porous media by means of sound wave analysis

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The theoretical investigation of sound waves in porous materials is motivated by the possible construction of non-destructive testing methods. Here, geotechnical applications are of interest where low frequencies from zero to 100 Hz occur but also systems with higher frequency ranges may be tested non-invasively. Examples are concrete and other construction materials, road surfaces and pavements (some MHz), biological materials like bones and soft tissues, e.g. the surface of the heart (up to 3 MHz) right up to surface coatings by nanomaterials (approximately 100 MHz).

In the geotechnical area the wave propagation analysis may lead to the construction of several non-destructive testing methods.

The analysis of surface waves (see e.g. [1] or [2]) may help to develop a method for soil characterization. By use of the SASW-technique conclusions about building grounds can be drawn from the measurement of sound wave speeds. I.e. no expensive and invasive acoustical measurements in boreholes or laboratory tests are necessary to characterize the soil prior to a building project.

In problems like seepage processes in road and dam constructions or tunneling in rocks different permeability in different directions plays an important practical role. The analysis of monochromatic waves in two-component poroelastic materials described by a model with anisotropic permeability yields two different shear modes [3]. Due to their appearance one can construct a device for measuring the anisotropy of the permeability. It would have to induce shear waves of different polarization and different directions of propagation. Then, one could measure principal values and directions of the tortuosity by comparing the amplitudes of arrivals for different polarization of signals.

Also the analysis of sound waves in partially saturated porous media [4] provides the hope for the construction of non-destructive testing methods. Namely, the velocity of the P1-wave which is the fastest wave and thus the first arrival on an oscillogram, increases abruptly to nearly the double of its value for a high degree of saturation. This provides the hope for the development of a monitoring method to warn against land slides.

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Calculation of the effective speed of sound in corrugated pipes by multiple scales

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Corrugated pipes are commonly used in many engineering and industrial applications because they combine local rigidity with global flexibility. In many cases, the flow of air through such pipes can cause a loud whistling at specific frequencies. In most industrial cases, such whistling is an undesired effect as it may result in severe noise and structural vibration problems (see [1] for a recent review, covering industrial applications and available work on this problem). On the other hand, the ability of such ducts to generate clear and tonal sound is also used in a musical toy called the "hummer" or "Voice of the dragon" [2].

The mechanism responsible to whistling is known to combine two ingredients. First, vortex shedding occurs at the upstream edge of the cavities forming the wall of the pipe. Small disturbances are amplified along the shear layer of the cavities by the Kelvin–Helmholtz instability and impinge on the downstream edges of the cavities, producing a fluctuating axial force which behaves like an acoustic source. Secondly, this synchronized vortex shedding leads to a global resonance if the shedding frequency coincides with the natural acoustical frequencies of the tube.

In the case of a corrugated pipe, the presence of cavities at the walls modifies the effective speed of sound c_{eff} with respect to the case of straight pipe (c_0), so the resonance frequencies are given by $f_m = mf_1 = mc_{eff}/2L$, where $m = 1, 2, 3, \dots$ and L is the length of the pipe. The knowledge of c_{eff} is thus crucial to predict the whistling frequencies of such devices.

The purpose of this paper is to accurately compute the effective velocity of sound based on a numerical solution of the Helmholtz equation in a periodic domain. It is derived on the assumption that the acoustical wavelength is large compared to the periodicity of the pipe which is verified in most applications. The results will be compared with those derived from a simpler model proposed by Cummings and Elliott [3], that gives an estimation of the effective speed of sound as function of the geometry of the duct

$$\left(\frac{c_{eff}}{c_0}\right)^2 = \frac{V_{in}}{V_{tot}},$$

where V_{in} is the volume of the inner part of the duct (excluding the cavities) and V_{out} is the total volume (including the cavities). The results are presented for a variety of geometries used in the literature, considering both axisymmetric and 2D cases: the Cummings–Elliott model reproduces well the results derived with our approach for short cavities but significant disagreement is found for longer cavities.

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Numerical Calculation of Acoustic Sources for the Landing Gear of Aeroplane during Take-off and Landing

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The sound which is generated from the aircraft during the take-off and landing is one of the main problems for the people who live in the areas near the airport. It is very important to allocate and accurately calculate acoustic sources generated from turbulent flow produced by the aerodynamics components of the aircraft. This is done in order to calculate inhomogeneous term of Helmholtz equation which serves as a prediction tool of sound propagation in the domain. It is used subgrid-scale stabilized (SGS) finite element method for solving incompressible Navier-Stokes equation which simulate turbulent flow. Afterwards is done double divergence of Litghill's tensor in order to calculate acoustics sources. Further, the transformation from time domain to frequency domain is used with Direct Fourier Transform which leads to smaller memory usage and computational cost.

The aim of the article is to show that previously mention method lead to better and richer representation of the spectrum of frequencies obtained from turbulent flow. Good representation of spectrum will give better inhomogeneous term of Helmholtz equation. Better prediction and calculation of acoustics sources will lead to reduction of sound generation through design of aerodynamics components on the aircraft.

Forward and Inverse Viscoacoustic Modelling in a Tunnel Environment

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In this work, the goal is to model forward acoustic waves in a tunnel environment with attenuation and to do full waveform inversion. In reality, there is no material without attenuation. Some materials, such as rocks, have so low attenuation that, in a small domain, the waves are almost not damped at all. At the same time, there are materials with high attenuation. In an environment with such materials, the attenuation has to be taken into account in order to model the waves properly. In this study, attenuation effect is integrated into acoustic equation by using Kolsky-Futterman model ([1], [2]) which only replaces velocity field with a complex-valued field in frequency domain. Apart from attenuation, another objective is to consider an inhomogeneous density field. Mainly, acoustic equation with a constant density field is referred to in many studies. In many cases, it may suffice to model waves appropriately. However, in reality, the density field of ground can be highly inhomogeneous. The objective is to investigate the effect of the inhomogeneity in waves, and to search for density field ρ and attenuation parameter Q as well as pressure wave velocity c using full waveform inversion.

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On numerical simulation of tsunami run-up on shore

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The numerical modelling of tsunami run-up on shore is done using the allocation of waterfront (adaptive grid method), i.e. in the domain with moving boundary. The domain is covered with a moving grid, and one of the boundary coordinate lines coincides with waterfront. The waterfront position is obtained using the exact analytical solutions of the shallow water equations in the vicinity of waterfront within the time period, which is equal to the time step of the used finite difference predictor-corrector scheme [1, 2]. The dynamically adaptive grid is constructed on each time step, thus, the time expenses can become larger than those for the solution of difference equations. Two grids are used to avoid that. The basic grid is generated before the solution of the problem and covers the water area and the part of adjoining land. The boundary coordinate line of this grid is located on the land and well spaced from the initial waterfront position, guaranteeing that the wave will never reach this position. The computational grid is generated on each time step and adapted to the numerical solution and moving waterfront. This grid is “1D”, because its nodes move along one family of coordinate lines of the basic grid only. The grid nodes are defined by 1D equidistribution method for plane curves.

The computation of the inundation zones after tsunami run-up on the shore with weakly curved waterfront line is done. For a plane slope the results are compared with the known analytical solutions and experimental data. A good correspondence of results and analytical solutions is obtained for the range of parameters, for which those solutions are valid. The numerical experiments have shown that the substitution of irregular slope with plane one can lead to the large errors in estimation of inundation zones. Also, the pictures of wave interaction with shore differ greatly. After the wave runs up to a plane slope, it runs down with the generation of the principal reflected wave. For an irregular slope, the reflected wave is generated in the moment when the run-up still continues on the shore; and, while the water is present on low gradient irregular slope, several reflected waves are generated, which is not so for a flat slope.

The method does not work for a highly irregular shoreline, in particular, when simply connected water domain becomes multiply connected due to the formation of separate islands. Then a combined method [3] is used, based on two approximations of the shallow water model: 1D and 2D. First, the 2D model with the reflecting boundary condition on the shore is used for modelling of wave motion from its source to the shore. The continuous record of wave parameters on a given isobathic line is done. This isobathic line is located well spaced from the shore, sufficient for the minimization of the influence of reflected waves on the form of the main part of in-going wave. The continuous record of wave parameters on a given isobathic line is done during the computation. This isobathic line is located at a certain distance from a shore, sufficient for the minimization of the influence of reflected waves on the form of the main part of in-going wave. Then, this data is used as boundary conditions for 1D computations along cross-sections, drawn from this given isobathic line in the shore direction. On the basis of these 1D computations the vertical and horizontal run-ups are computed along the chosen cross-sections. The procedure is developed for the reconstruction of inundation zone boundary using the detailed digital land relief. The method of allocation of moving waterfront point (adaptive grid method) is used for run-up computation in 1D cross-sections. The modelling of waterfront point motion is done on the basis of the analytically obtained motion law for this point. The developed methodology is applied for computation of inundation zones and has shown [3] the satisfactory correlation of the numerical results and field data.

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Potential flow simulations of Peregrine-type deep water surface gravity wave packets

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Applying a higher order spectral method [1] for potential flow we numerically determine the time evolution of non-linear deep water wave packets originating from initial conditions derived from the Peregrine breather [2] solution of the weakly non-linear Schrödinger equation (NLS) [3]. A number of experimental studies on this NLS solution confirm its role for deep water oceanic rogue waves, see e.g. [4, 5].

In this contribution we modulate the non-linear Stokes wave solution [6] according to early stages of the Peregrine breather solution of the NLS and then follow the evolution of the resulting wave packet within the potential flow approximation.

The spatio-temporal evolution of the wave packets qualitatively agrees well with what would be expected from the NLS solution. However some quantitative discrepancies do exist: the propagation speed of the packets depends on the wave steepness of the background waves, and the maximum wave envelope amplification occurs slightly later than predicted from the NLS. The amplification factor itself exceeds slightly the factor known from the NLS solution.

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Application of variational-asymptotic method to modulation theory

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This work presents the application of the variational-asymptotic method to the Whitham's modulation theory for differential equations of nonlinear dispersive waves. The Whitham averaging method is based on two important assumptions: the differential equation possesses the solution of steady profile and the existence of a number of associated local conservation laws [1]. Shortly later he developed an equivalent method based on an average variational problem [2]. Since it requires the knowledge about the uniform solution or its first integrals of the corresponding differential equation in one wavelength, it cannot be extended to systems where such solutions are not known in advance. In contrary, the variational-asymptotic method develops the multi-scale approach which enables one to reduce systematically variational problems containing a small parameter to the average variational problems via the solution of the so-called strip problems [3, 4, 5]. By applying the variational-asymptotic method, we obtain the modulation theory for several differential equations such as the Korteweg-de Vries (KdV) equation [6], the scalar Boussinesq equation [7, 8], the one- and two-dimensional sine-Gordon equations [9, 10]. Depending on the type of the differential equations we may derive the theories of either amplitude or slope modulation. In the limit of trains of solitons or positons, the asymptotic solutions of the modulation equations can be obtained.

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A novel approach to mode - tracing in SBFEM - Simulations: Higher-order Taylor- and Padeapproximation

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This contribution presents the study of strange phenomena in wave mode representations of waveguides. For this study the waveguides are computed by means of Scaled Boundary Finite Element Methods (SBFEM). Different approaches of mode tracing are used to identify the characteristics of the resulting wave modes. Higher order differentials of the underlying eigenvalue problem are the basis for these approaches.

The main idea behind mode tracing approaches is to reduce the cubic computation time to solve the eigenvalue problem for each frequency of interest. There exist several algorithms for mode tracing from semi-analytical approaches [1] to approximation methods [2]. Although these algorithms show great performances for a variety of cases, under certain unfavorable circumstances these methods run into problems - mainly expressed in the loss of solutions or producing a small number of wrong solutions. Luckily these problems are reproducible and occur at certain frequencies only.

This contributions goal is to present and discuss the underlying cause for these problems. The fascinating effects at critical frequencies are displayed and a suggestion for a stabilization for the solution process is made. This study bases its conclusion on a numerical viewpoint. A different way of identifying the problem and phenomena through an analytical viewpoint can be found in [3].

Using the model developed by Hauke Gravenkamp [4] the modeling of waveguides is based on a second order eigenvalue problem:

$$(k^2 \mathbf{E}_0 - ik(\mathbf{E}_1^\top - \mathbf{E}_1) - \mathbf{E}_2) \Psi = \mathbf{M}_0 \Psi \Omega$$

with $\mathbf{E}_0, \mathbf{E}_1, \mathbf{E}_2, \mathbf{M}_0 \in \mathbb{C}$, k being the wave number and Ψ, Ω corresponding frequencies and displacements. The main focus lies on frequencies Ω at which certain modes show strongly coupled behavior.

Main aspects in this study include high order differentials of the eigenvalue problem and the corresponding Taylor and Pade approximations for the eigenvalue problem as a whole.

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Numerical modeling of a sub-sonic roll-over load along a rods skin

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To adequately describe the wave propagation in rod-like structures is a matter of enormous practical relevance. Practical applications involve, but are not restricted to, pile-driving for off-shore wind turbines and dynamic pull-out of reinforcement elements in composite materials, for example [1, 2, 3]. As can readily be seen these problems give significant challenges for both the analytical modeling and the numerical implementation. These challenges arise e.g. from the interaction of the traveling wave with the host in which the rod is embedded in. For example, the dynamic pull-out of a reinforcement element is characterized by a relative slip between the reinforcement and the embedding matrix. But before relative slip at a differential element of the reinforcement can occur, the bond of the respective element (of differential length) to the surrounding matrix has to be completely damaged which equals a crack. Thus, a crack front will propagate along the skin of the rod. Whereas the rod initially had a perfect bond to the surrounding matrix it is now subdivided into two parts: one with a completely damaged bond and another with a perfect bond. However, the boundary between these two parts will travel with the speed of the crack front which itself follows the stress wave propagating through the rod.

To cope with such and other phenomena in a first step a rather simple example will be investigated. It consists of a semi-infinite rod at whose skin a constant longitudinal load is traveling from the free end to infinity. Herein, the speed c_q this load propagates with is lower than the longitudinal wave speed c_L within the rod so that a sub-sonic problem is at hand. The material behaviour of the homogeneous rod is characterized as linearly elastic and isotropic. Within this contribution firstly an analytical model describing the wave propagation is presented. Afterwards, the results of this analytical approach are compared to the results obtained by standard FEM using 8-node-solid-elements. Afterwards, the results of this analytical approach are further analyzed by numerical calculations. The adequate description of the specific boundary conditions is obtained by special surface elements that allow for the roll-over load.

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