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Scientific Program - Timetable

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GAMM 2015

Università del Salento

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S14: Applied analysis

This Session is devoted to the mathematical analysis of natural phenomena and engineering problems. In this area PDEs play a basic role. Therefore lectures discussing analytical aspects of PDE problems as well as problems in the Calculus of Variations are welcome. On account of the importance of applications in Materials Science, the Session will have a special focus on the following topics:

Shape memory polymers

Liquid crystals

Bio-materials

Ferromagnetic materials

Two-phase fluids

Gas storage in alloys and polymers

Phase transitions in smart materials

Damage and other dissipative processes in solids

The interest in such issues is witnessed by an intense research which requires a delicate interplay between experiments, modeling, theoretical analysis of PDEs and their numerical approximation. This Session is intended to be an occasion to discuss the various aspects of this interdisciplinary subject, taking advantage of the contributions of young researchers in the field.

Weak solution to certain problem in fluid mechanics

Eduard Feireisl
Czech Acad. Sci. Prague, Czech Republic

We discuss several concepts of weak, dissipative, very weak and/or measure valued solutions to various problems in fluid mechanics. We show that many equations modeling inviscid fluids may posses infinitely many weak solutions and even infinitely many dissipative weak solutions. Suitable admissibility criteria will be proposed.

A low volume-fraction limit for martensic microstructures in shape-memory alloys

Johannes Diermeier
University of Bonn

Shape-memory materials have the property that they recover their shapes under heating if they have been deformed at a low temperature. The reason for this behaviour is a solid-solid phase transition between different lattice structures. One can observe the occurence of microstructure around interfaces between different phases of the material.

We consider a variational model of the energy of a deformation and restrict ourselves to a scalar valued, two dimensional, geometrically linearized case with two variants of martensite, the low temperature state, in which one of the variants has a much smaller volume fraction than the other one. The energy is given as the sum of an elastic term and a surface term

$$E^{\varepsilon,\theta}(v) = \int_{(0,1)^2} \partial_x v^2 + \min\{|\partial_y v + \theta|^2, |\partial_y v - 1|^2\} \, \mathrm{d}\mathcal{L}^2 + \varepsilon ||D^2 v||((0,1)^2)$$

where $\theta \ll 1$ is the small volume fraction. The deformation $v \in W^{1,2}((0,1)^2)$ satisfies $Dv \in BV((0,1)^2)$ and zero-boundary values $v(0,y) = 0$ which force the microstructure to occur.

It is known that the energy follows the scaling law $E^{\varepsilon,\theta} \sim \varepsilon^{2/3}\theta^{2/3}$ if $\varepsilon \leq \theta^2$ and $E^{\varepsilon,\theta} \sim \theta^2$ otherwise. In the first case the upper bound is achieved by a self-similar twinning construction whereas one uses a single phase of martensite in the second case.

We are interested in a deeper analysis for $\varepsilon \sim \theta^2$. We set $\varepsilon = \sigma\theta^2$ for some $\sigma \in \mathbb{R}^+$ and determine the reduced model in the limt $\theta \to 0$ by means of Γ-convergence.

Homogenization of layered structures with rigid components in single-slip finite elastoplasticity

Fabian Christowiak, Carolin Kreisbeck
Faktultät für Mathematik, Universität Regensburg

In a first step towards a better understanding of the macroscopic response of elastoplastic composites, we consider materials with a bilayered structure. While one material component is rigid in the sense that it does not allow any plastic deformation, the other one is softer and has one active slip system with linear hardening. The model in two spatial dimensions is based on a time-discrete variational approach to single-slip finite plasticity, and is inspired by models for homogeneous single crystalline materials as studied for instance in [1, 2].

In this presentation, we characterize the asymptotic behavior of the energy in the first time increment as the layer thickness tends to zero in terms of Γ-convergence. In fact, due to the anisotropy of the setup, the resulting Γ-limit depends critically on the orientation of the slip system relative to the layers. This effect is reflected in the following two special cases, for which we provide explicit formulas. If the slip direction is parallel to the layers, the limit energy functional admits only deformations that are combinations of a global rotation and shear in slip direction. In contrast, for a slip direction orthogonal to the layers, one observes a complete blocking of the slip system, so that macroscopically the material sample is so rigid that it can only be rotated globally.

The main technical difficulty is to capture the rigidity introduced by the layered geometry in order to obtain a suitable lower bound. This requires a new rigidity result combining the Rešetnjak theorem [3] applied to each rigid layer with the process of decreasing layer thickness. For the upper bound a careful analysis of admissible microstructures is needed, which on a technical level means solving the differential inclusion induced by the variational models. Furthermore, we compare our findings to results of non-linear homogenization theory, in particular the common cell and multi-cell formulas [4].

References

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Homogenization for dislocation based gradient visco-plasticity

Sergiy Nesenenko
Universität Duisburg-Essen, Germany

In this work we study the homogenization for infinitesimal dislocation based gradient viscoplasticity with linear kinematic hardening and general non-associative monotone plastic flows. The constitutive equations in the models we study are assumed to be only of monotone type. Based on the generalized version of Korn's inequality for incompatible tensor fields (the non-symmetric plastic distortion) due to Neff/Pauly/Witch, we derive uniform estimates for the solutions of quasistatic initial-boundary value problems under consideration and then using a modified unfolding operator technique and a monotone operator method we obtain the homogenized system of equations. A new unfolding result for the Curl Curl-operator is presented in this work as well. The proof of the last result is based on the Helmholtz-Weyl decomposition for vector fields in general L^q-spaces.

On the use of potential fields in fluid mechanics

Florian Marner[1], Markus Scholle[1]
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For many systems of differential equations arising in classical physics the introduction of auxiliary potential fields turned out to be beneficial in various respects. For instance in classical fluid mechanics, potential fields have been employed to enable the integration of the equations of motion. As is well known, Bernoulli's equation is obtained as a first integral of Euler's equations in the absence of vorticity and viscosity if the velocity vector $\vec{u} = \nabla\Phi$ is perceived as the gradient of a scalar potential. The so-called Clebsch transformation

$$\vec{u} = \nabla\varphi + \alpha\nabla\beta \ ,$$

involving three scalar potentials allows for a further extension to flows with non-vanishing vorticity; the resulting set of equations together with the continuity equation turn out to be self-adjoint, allowing for a variational formulation. A generalization of the Clebsch transformation in the case of baroclinic flows has been formulated e.g. by [3]. All attempts in classic literature, however, are restricted to inviscid flows. The finding of a potential representation which makes possible the integration of the Navier-Stokes equations is therefore a very desirable aim.

Progress on this topic was reported by [1] who constructed a first integral of the two-dimensional incompressible Navier-Stokes equations by making use of an auxiliary potential field and a representation of the fields in terms of complex coordinates. Based on the alternative first integral formulation, a convenient representation of the dynamic boundary conditions has been established and an efficient finite element method for solving two-dimensional free surface flows developed [2]. Moreover, using a tensor potential rather then a scalar potential, the theory can be successfully generalized to encompass three-dimensional Navier-Stokes flow.

The search for potential fields allowing for the integration of the full Navier-Stokes equations is closely related to the construction of a Navier-Stokes Lagrangian. Such a Lagrangian can be extracted from the aforementioned tensor potential representation. However, a viable alternative results if the considerations are founded on and the formulation is derived from the basic physics directly. Since viscosity leads to dissipation and therefore to the irreversible transfer of mechanical energy to heat, thermal degrees of freedom have to be taken into account. Following this idea, a generalization of the Clebsch transformation to baroclinic flow can be found, as shown by [3]. The authors propose a further extension to viscous flow, which can be shown to fulfill the relevant symmetry criteria given in [4]. The resulting Lagrangian reproduces the Navier-Stokes equations except for an additional term, which may be controlled by an appropriate relaxation method.

References

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Well-Posedness and Stability for some Volume-Preserving Curvature Flows with Boundary Contact or Triple Junctions

Helmut Abels, Nasrin Arab, Harald Garcke, Lars Müller
Faculty of Mathematics, University of Regensburg

We will consider the dynamic stability of a spherical cap on a flat plane in three space dimensions with a dynamic boundary condition which is associated to an energy of the contact line between the spherical cap and the plane. Using a suitable parametrization we can apply results on maximal L^p-regularity for parabolic equations with dynamic boundary conditions and a generalized principle of linearized stability. To this end a careful analysis of the spectral properties of the linearized operator is needed. Moreover, we will briefly discuss the dynamic stability of a symmetric planar network evolving under curve diffusion.

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Nonlocal Cahn-Hilliard equation with a reaction term

E. Rocca[1], S. Melchionna[2]
[1]University of Vienna, Faculty of Mathematics
[2]Weierstrass Institute for Applied Analysis and Stochastics, Berlin

We prove existence, uniqueness, regularity and separation properties for a nonlocal Cahn-Hilliard equation with a reaction term. We deal here with the case of logarithmic potential and degenerate mobility as well as uniformly lipschitz in u reaction term $g(x,t,u)$.

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Well-posedness and optimal control of Allen-Cahn type equations with singular potentials and dynamic boundary conditions

Luca Calatroni, Pierluigi Colli
Cambridge Centre for Analysis, Univeristy of Cambridge, UK
Dipartimento di Matematica "F. Casorati", Universitá degli studi di Pavia, Italy.

We prove well-posedness results for the solution to an initial and boundary value problem for an Allen-Cahn type equation describing the phenomenon of phase transitions for a material contained in a bounded and regular domain. The dynamic boundary conditions for the order parameter account for interactions with the walls. Our results are showed by using regularisations of the nonlinearities of the problem and performing some a priori estimates which allow us to pass to the limit thanks to compactness and monotonicity arguments [1]. Moreover, some results on the optimal control problem will be reviewed for a class of singular potentials including the derivatives of logarithmic potentials confined in $[-1, 1]$ and subdifferentials of the indicator function of the interval $[-1, +1]$ up to concave perturbations [2, 3].

References

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Relaxation in nonlinear elasticity with constraints on the determinant

Sergio Conti, Georg Dolzmann
Universität Bonn
Universität Regensburg

Consider the variational problem in nonlinear elasticity of the form $E[u] = \int W(Du)dx$. Here $u : \mathbb{R}^n \to \mathbb{R}^n$ the elastic deformation and W represents the free energy density which is defined on $n \times n$-matrices with positive determinant and and satisfies $W(F_n) \to \infty$ along sequences $(F_n)_{n\in\mathbb{N}}$ of matrices with $\det F_n > 0$ and $\det F_n \to 0$ for $n \to \infty$. It is shown that under suitable growth assumptions the functional $\int W^{\mathrm{qc}}(Du)dx$ is an upper bound for the relaxation of E, and that it coincides with the relaxation if the quasiconvex envelope W^{qc} of W is polyconvex and has p-growth from below with $p \geq n$. This includes several physically relevant examples.

References

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A Linear Scale-Space Theory for Continuous Nonlocal Evolutions

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Most scale-space evolutions are described in terms of partial differential equations. In recent years, however, nonlocal processes have become an important research topic in image analysis. The goal of our paper is to establish well-posedness and scale-space properties for a class of nonlocal evolutions. They are given by linear integro-differential equations with measures. In analogy to Weickert's diffusion theory (1998), we prove existence and uniqueness, preservation of the average grey value, a maximum–minimum principle, image simplification properties in terms of Lyapunov functionals, and we establish convergence to a constant steady state. We show that our nonlocal scale-space theory covers nonlocal variants of linear diffusion. Moreover, by choosing specific discrete measures, the classical semidiscrete diffusion framework is identified as a special case of our continuous theory. Last but not least, we introduce two modifications of bilateral filtering. In contrast to previous bilateral filters, our variants create nonlocal scale-spaces that preserve the average grey value and that can be highly robust under noise. While these filters are linear, they can achieve a similar performance as nonlinear and even anisotropic diffusion equations.

Rate-independent systems with viscosity and inertia – Evolutionary Gamma-convergence

Riccarda Rossi, Marita Thomas
Università di Brescia
Weierstrass Institute, Berlin

This talk is the second part of two contributions dealing with the analysis of rate-independent processes coupled with rate-dependent ones and inertial effects in a general framework. It focuses on Evolutionary Gamma-convergence results based on variational convergence in the spirit generalized gradient flows. The abstract results are substantiated with problems from applications, such as damage, delamination, and plasticity.

Analysis of a gradient enhanced damage model

Christian Meyer, Livia Susu
TU Dortmund

The talk is concerned with a damage model including two damage variables, a local and a non-local one, which are coupled through a penalty term in the free energy functional in the spirit of [1]. After introducing the precise model, we prove existence and uniqueness for the viscous regularization thereof. Moreover, we rigorously study the limit for penalization parameter tending to infinity. It turns out that in the limit both damage variables coincide and satisfy the classical viscous damage model discussed in [2].

References

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Mixed-growth plasticity and generalized rigidity

Janusz Ginster
University of Bonn

Under large stresses metals generate dislocations which are local defects of the crystalline lattice and can be interpreted as line-singularities of the elastic strain field. Straight parallel edge dislocations can be described by a variational model in the orthogonal plane of the singularities. The dislocations are then point singularities of the strain field β, i. e.

$$\operatorname{curl}\beta = \sum_{i}^{N} b_i \delta_{x_i} \tag{1}$$

for some dislocation points $x_i \in \mathbb{R}^2$ and Burgers' vectors $b_i \in \mathbb{R}^2$. The set of feasible dislocation points and Burgers' vectors is determined by the interatomic spacing $\varepsilon > 0$.

The energy for a strain $\beta : \mathbb{R}^2 \supseteq \Omega \to \mathbb{R}^{2\times 2}$ and a corresponding singularity distribution $\mu = \sum_i^N b_i \delta_{x_i}$ connected to β through (1) is of the form

$$E_\varepsilon(\mu, \beta) = \int_\Omega W(\beta)\, dx.$$

Near singularities the strain field typically diverges as $\frac{1}{|x|}$, so in order to obtain a finite energy we consider energy densities W with quadratic growth near $SO(2)$ and subquadratic growth at infinity.

We will identify the limit as $\varepsilon \to 0$ of a suitably scaled version of the energy E_ε by means of Γ-convergence. As a key ingredient we will present a rigidity estimate for fields whose curl is a bounded measure.

New results on Cahn-Hilliard-Navier-Stokes systems with nonlocal interactions

Sergio Frigeri
Weierstrass Institute for Analysis and Stochastics, WIAS, Berlin

In this contribution we shall consider the nonlocal Cahn/Hilliard-Navier-Stokes system describing flow and phase separation of a binary mixture of incompressible viscous isothermal fluids. We shall first briefly recall some basic mathematical results concerning well-posedness, regularity and asymptotic behavior under different assumptions on mobility and double-well potentials (cf. [1, 2, 3, 4, 5, 6]). Then, we shall turn to the new results concerning optimal control (cf. [7]) and concerning the existence of weak solutions for the case of the nonlocal Cahn-Hilliard/Navier-Stokes system with unmatched densities (nonlocal Abels-Garcke-Grün model).

References

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Wulff shape and isoperimetric characterization of crystals

Paolo Piovano
Department of Mathematics, University of Vienna, Austria

In this talk the problem of analytically explaining why particles at low temperature arrange in periodic lattices will be considered and the emergence of the Wulff shape of crystals will be investigated. Ground states of phenomenological energies accounting for two-body (and three-body) short-ranged interactions will be shown to be connected subsets of the reference lattice, and their energies will be exactly quantified.

As the energy favors particle bonding and 'boundary' particles have in general less bonds, ground states are intuitively expected to have minimal 'perimeter', or maximal 'area'. This intuition will be verified by introducing a suitable notion of perimeter and area of configurations, and by showing that ground states are characterized as those configurations which realize equality in a discrete isoperimetric inequality.

In view of this characterization the emergence of a macroscopic Wulff shape as the number of particles grows will be established, and ground states will be shown to deviate from the asymptotic Wulff shape at most by $O(n^{3/4})$ particles. This result nicely reflects the inherent multiscale nature of the crystallization phenomenon.

Effective Maxwell equations in a geometry with flat rings of arbitrary shape

Ben Schweizer, Agnes Lamacz
Technische Universität Dortmund, Fakultät für Mathematik

Propagation of light in heterogeneous media is a complex subject of research. It has received renewed interest in recent years, since technical progress demands smaller devices and offers new possibilities. At the same time, theoretical ideas inspired further research. Key research areas are photonic crystals, negative index metamaterials, perfect imaging, and cloaking. In the field of negative index materials, which we want to focus on in this contribution, two very influential works are the theoretical study by Veselago [3] and the approach of Pendry and others to the actual construction of such materials; see, e.g., [1].

The mathematical analysis of negative index materials is connected to a study of singular limits in Maxwell's equations. In this contribution, see [2], we present a result on homogenization of the time harmonic Maxwell's equations in a complex geometry. The homogenization process is performed in the case that many (order η^{-3}) small (order η^1), thin (order η^2), and highly conductive (order η^{-3}) metallic objects are distributed in a domain $\Omega \subset \mathbb{R}^3$. We determine the effective behavior of this meta-material in the limit $\eta \searrow 0$. For $\eta > 0$, each single conductor occupies a simply connected domain, but the conductor closes to a ring in the limit $\eta \searrow 0$. This change of topology allows for an extra dimension in the solution space of the corresponding cell-problem. Even though both original materials (metal and void) have the same positive magnetic permeability $\mu_0 > 0$, we show that the effective Maxwell system exhibits, depending on the frequency, a negative magnetic response. This magnetic activity is the key feature of a negative index meta-material.

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A line-tension energy for dislocation networks on several slip planes

Peter Gladbach

Universität Bonn

We study dislocation networks contained in finitely many parallel slip planes starting from the model by Koslowski, Cuiti no, and Ortiz, which couples a nonconvex Peierls potential acting on the slip fields with long-range elastic stress.

In the limit for small lattice spacing, in the sense of Γ-convergence, we obtain a line-tension functional depending on the Burger's vector and direction of the dislocation lines. For certain configurations we observe that oscillatory and splitting microstructure achieves a lower energy than straight dislocations.

As both the lattice parameter and separation of the slip planes tend to zero simultaneously, parallel dislocation lines in different planes interact. For certain configurations a coupling is favorable. In intermediate scaling regimes we show that a two-scale microstructure produces a lower energy than any single-scale one and that some two-scale microstructure is in fact optimal.

References

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