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Scientific Program - Timetable

Sunday 22	Time	Monday 23	Tuesday 24	Wednesday 25	Thursday 26	Friday 27
	9: 15 30 45		Contributed sessions (15 in parallel)	Plenary Lecture Moritz Diehl	Contributed sessions (15 in parallel)	Contributed sessions (14 in parallel)
	10: 15 30 45	Registration		von Mises prize lecture		
	11: 15 30 45		Coffee Break Plenary Lecture Thomas Böhlke	Coffee Break General Assembly	Coffee Break Plenary Lecture Ferdinando Auricchio	Coffee Break Contributed sessions (11 in parallel)
	12: 15 30 45		Lunch	Lunch	Lunch	
	13: 15 30 45	Opening Univ. Chorus Performance				Closing
	14: 15 30 45	Prandtl Lecture Keith Moffatt	Plenary Lecture Enrique Zuazua	Contributed sessions (15 in parallel)	Plenary Lecture Daniel Kressner	
	15: 15 30 45	Plenary Lecture Giovanni Galdi	Plenary Lecture Nikolaus Adams		Plenary Lecture Stanislaw Stupkiewicz	
Registration pre-opening	16: 15 30 45	Coffee Break Minisymposia & Young Reseachers' Minisymposia (10 in parallel)	Coffee Break Poster session Contributed sessions (14 in parallel)	Coffee Break Contributed sessions (15 in parallel)	Coffee Break Poster session Contributed sessions (15 in parallel)	
	17: 15 30 45					
	18: 15 30 45		Public lecture Francesco D'Andria			
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S24: History of mechanics

This section will provide a forum for the presentation of historical and/or speculative studies on mechanics focusing on the relations between concepts from antiquity up to now, which could be of interest to historians of mechanics and physics as well as researchers in mechanics.

The contributions of those authors who have benefited from the study of old theories for developing current models in the field of applied mechanics and mathematics will be also considered. This in order to highlight the importance of historical and epistemological perspective setting for the current and future developments in science.

With these aims in mind, topics of applications will include (but are not limited to):

Virtual work laws

Variational methods and conservation principles

Molecular foundation for continuum mechanics

Coarse graining processes and the use of Cauchy-Born rule

Continuum and statistical thermodynamics

Material deformation theories and experimental results in solid mechanics

Inelastic deformations, damage and fracture

Development and use of the Ricci's tensor calculus in deformation theories of solids, shells and fluids

Mechanical models for ancient constructions

The change of perspective with the advent of Quantum Mechanics

S. Esposito
I.N.F.N. - Naples' Unit
Complesso Universitario di M. S. Angelo, Via Cinthia, 80126 Naples, Italy
(Salvatore.Esposito@na.infn.it)

Differently from other previous generalizations, such as Einstein's Theory of Relativity for example, Quantum Mechanics has introduced a radical change of physical perspective in the study of the microscopic world with respect to Classical Mechanics. The Newtonian determinism, centered about the initial position and velocity (or momentum), has abdicated in favor of quantum determinism, based on Heisenberg's uncertainty relations and centered about the initial wave-function of the system.

Also, the recovery of the "classical limit" showed to be not at all obvious, even just from a philosophical point of view: the mathematical limit of vanishing Planck constant $h \to 0$ requires an interpretation that is completely dissimilar, for instance, from that natural of small velocities $v \ll c$ of the Theory of Relativity, or any other generalization handled by Physics. The mathematical formalism has changed accordingly, and assumed a foundational character that cannot be disjointed by the physical interpretation.

In this talk I will review the main conceptual steps that have changed our view of the mechanical world in the XX century, by focussing on the issues mentioned above, as well as on applications of Quantum Mechanics that have had a direct consequence also on the macroscopic world. The strict relationship between micro- and macro-world, indeed, is a peculiar feature of the modern technological applications, which have affected at a large extent – not at all limited to philosophical issues – the applied mechanics behind daily life.

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Walter Noll and the Bourbakization of Mechanics

Gianpietro Del Piero
Dipartimento di Ingegneria, Università di Ferrara, Italy
International Research Center M&MoCS, Cisterna di Latina, Italy

There was a moment, in the second half of the 19th century, in which a strong need for axiomatization was felt by the mathematical community. This goal was pursued by a number of eminent scientists, and continued in the subsequent century, up to the monumental work of the Bourbaki group. Thanks to this, many basic definitions, concepts and notations of mathematics were fixed in the way they are now universally known and accepted. The same did not happen for mechanics, for which a systematic apparatus was developed only for discrete systems. For continuum mechanics, the pioneering work of the fathers was not followed by an adequate axiomatic settling. So it can be claimed that, as far as continuum mechanics is concerned, Hilbert's *sixth problem* on the axiomatization of physical sciences is still open.

Since long time, there has been a diffuse feeling that the giants of the past did the whole job, that what they did is definitive, and that further investigation on the principles would be rewardless. And yet, just after the half of the 20th century, a live interest in the fundamentals of continuum mechanics unexpectedly arose. Roughly, it went along three main directions. The first, stimulated by the many technical problems raised during the 2nd wold war, was addressed to theories of inelastic behavior, such as plasticity, damage, fracture, and fatigue. The second was a renewed interest in generalized continua, resuscitated after the oblivion which followed the appearing of the Cosserat's book in 1909. And, finally, there was a reborn attention the foundations. Initially, this was carried on by a single man, W. Noll. In the years between the end of the 1950's and the half of the 1970's he produced a series of papers, in which the whole corpus of the foundations was revisited. The list of his main achievements includes

- a neat distinction between general field equations and *constitutive equations* valid for specific classes of materials (1958),
- the *principle of indifference* of the constitutive equations to changes of observers (1958),
- the *principle of local action*, discriminating between *local* and *non-local* material behavior (1958),
- a mathematical definition of the *material symmetries* (1958),
- the proof of of Cauchy's conjecture on the *dependence of the contact force on the normal* to the contact surface (1959),
- the theory of *materials with fading memory* (with B.D. Coleman, 1960),
- the deduction of Newton and Euler's general laws from the principle of indifference of power (1963),
- the *new theory of simple materials*, explicitly designed to include plasticity and damage in a general scheme for material behavior (1972),
- some basic statements on the level of regularity to be required to the regions occupied by a continuous body (with E.G. Virga, 1988),
- the removal of inertia, of linear and angular momentum, and of kinetic energy from the basic concepts of mechanics (1995).

Though most of these items are today common heritage, some did not receive adequate attention. The subsequent research activity involved a number of researchers, and split into several directions, not all of great moment. But, surprisingly enough, very little progress was made in some strategic esearch lines, such as multiscale kinematics or the mechanics of fractal bodies. A remarkable result, coming from an idea of M.E. Gurtin and L.C. Martins (1976), subsequently developed by M. Šilhavỳ (1985, 1991), is that Cauchy's *tetrahedron theorem*, rather than a consequence of the balance of the linear momentum, is a regularity property of the system of the contact actions. The enormous potential impact of this result on the axiomatization of continuum mechanics, and especially of the mechanics of generalized continua, has not yet been realized. In the proposed communication, following in part the contents of my paper [1], I will try to give an idea of the relevance of this result, of some immediate consequences, and of possible future developments.

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Beltrami and mathematical physics in non-Euclidean spaces

Danilo Capecchi, Giuseppe Ruta
"Sapienza" università di Roma, Dipartimento di ingegneria strutturale e geotecnica

Eugenio Beltrami (Cremona 1836 - Roma 1900) in some of his works investigated the behaviour of elastic continua embedded in non-Euclidean spaces. In these papers, Beltrami used his great skills in pure mathematics with the aim of understanding the geometrical nature of physical space, where natural phenomena occur. In particular, starting from his thorough knowledge of geometry and his mastery in continuum mechanics, he asked himself if the actual space, where elastic, electric, thermal, and magnetic phenomena occur, were Euclidean or not. Such a theme was well present among the mathematical scientific community at the end of the 19th century, especially after the contributions by the Russian Nikolaj I. Lobacevskij (1793-1856), the Hungarian Janos Bolyai (1802-1860), the German Bernhard Riemann (1826-1866), and Beltrami himself in the development of non-Euclidean geometry. The first two, in the first decades of the 19th century, searched for a geometry which could be built independent of Euclid's fifth postulate, hence worked on a purely mathematical basis. Beltrami and Riemann, who were long in touch in Pisa, on the other hand were interested not only on the basics of non-Euclidean geometry, but also on how non-Euclidean geometry could affect the field equations of the great problems of mathematical physics of their time, that is, elasticity and electro-magnetism.

Beltrami's works on the field opened for sure the path for a new way of intending mathematical physics. Some of Beltrami's pupils followed him in this enterprise, among them Ernesto Cesaro, who devoted some investigation to elasticity in hyper-spaces. In this contribution, we sketch Beltrami's original results, basing on his papers, and propose a critical discussion on them and on their influence.

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Gustav R. Kirchhoff and the dynamics of tapered beams

Antonio M. Cazzani,
University of Cagliari

The dynamics of straight tapered beams (within the framework of Euler-Bernoulli beam theory) was first studied by Gustav Kirchhoff in a seminal paper dating 1879[1]. The importance of this contribution was already recognized by Todhunter and Pearson who cited it in their monumental work[3].

Kirchhoff applied for the first time an approach based on what is now known as Forbenius method to the solution of the fourth order partial differential equation which is now generally written in the form:

$$E\frac{\partial^2}{\partial x^2}\left(J(x)\frac{\partial^2 y}{\partial x^2}\right)+A(x)\rho\frac{\partial^2 y}{\partial t^2},$$

where x is the beam axis, y its deflection, t represents time and E, ρ, A, J are respectively the Young's modulus, the density, the cross-section area and the corresponding area moment of inertia.

He showed the general solution and then take explicitly into account the two cases of a tapered beam having a wedge and a cone/pyramid shape with a linear taper.

This result was subsequently generalized in the following decades by many authors to other kind of taper law and also to truncated cones, pyramids etc. [3].

The present contribution is concerned with an analysis of the original paper and of the solution method applied there and only roughly described by the author, which allowed him (see [3]) to reduce the solution of the fourth order equation to that of two second order equations.

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On the Derivation of the Equations of Hydrodynamics 65 years after Irving&Kirkwood

Paolo Podio-Guidugli
Accademia Nazionale dei Lincei
and
Department of Mathematics, University of Rome TorVergata

In their landmark paper [1], Irving and Kirkwood demonstrated that certain basic equations central to all of continuum mechanics (in their terminology, 'the equations of hydrodynamic') can be derived from a statistical setting of the Newtonian motion problem for a system of mass points. The equations they derived are the standard balances of mass, momentum, and energy. Certain mathematical shortcut in [1] were later corrected by Noll [2], who returned to the subject much later [3], when his 1955 paper was translated into English. Noll's main contribution is a constructive formula for the stress tensor that corresponds to the multifold particle interactions one deals with in a discrete framework; important recent contributions to the subject are, among others, those in [4, 5].

In this talk, I plan to discuss along the path of [1] other concepts and laws of importance in continuum mechanics, such as the entropy concept and the balance of moment of momentum.

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How does dynamic complexity contribute to the advancement of mechanics

Giuseppe Rega
Department of Structural and Geotechnical Engineering,
Sapienza University of Rome, Italy,
giuseppe.rega@uniroma1.it

Starting with the pioneering studies of Henry Poincaré, and being nurtured by the outstanding contributions of a number of giants in modern science, the dynamics of complex systems has become a revolutionary area of research in the last few decades of the 20th century and around the turn of the millennium [1]. The theoretical achievements earlier obtained within the applied mathematics and physics communities have later on entailed meaningful outcomes also within the mechanics community, as soon as the importance of nonlinear phenomena in view of technical applications has been realized. Based on sophisticated analytical, geometrical, and computational techniques employing powerful concepts and tools of dynamical systems, bifurcation, and chaos theory, a wide variety of applications to mechanical and structural systems has flourished, properly updating and complementing them with meaningful experimental verifications and with a view to engineering aims.

Without attempting at prematurely interpret the underlying, and still unsteady, process of mutual influence between different scientific areas, it appears fully legitimate to wonder about what dynamic complexity has contributed to the advancement of mechanics and, even more, about how can further contribute to its future. For the sake of an accessible introductory treatment of a difficult matter, one can schematically distinguish between the involved phenomenological and methodological aspects, with also possible technological consequences, though being all of them somehow interconnected.

From a phenomenological viewpoint, the unquestionable enrichment brought to mechanics by the knowledge of dynamical systems stands in the detection, description and understanding of a multitude of nonlinear events of theoretical and practical interest (e.g., [2]). To name just a few concepts and ensuing practical tools, one can think of the local and, mostly, global bifurcations determining strong, and often sharp, changes of system response, of the highly non-regular, and seemingly exoteric, features which characterize chaotic behaviors, of the structural stability issues providing a well–founded theoretical framework for the interpretation of still unexplained engineering phenomena (such as flutter or galloping).

From the methodological viewpoint, it is worth distinguishing between general and specific aspects. General aspects are concerned with making the need of a cross-disciplinary approach to science definitely clear. Indeed, in this respect, complexity and nonlinear dynamics represent updated paradigms which allow to consciously address a variety of disciplines sharing common, though technically distinct, phenomenological aspects [1]. They govern a huge amount of events in both hard and soft scientific areas, which encompass physics, chemistry, biology, and engineering, with all of their varied branches, as well as medicine, economy, arts, and architecture, with even possibly undue significance and misleading interpretations. At the same time, modern mechanics is fruitfully contaminated by a variety of other disciplines, which include not only nonlinear dynamics and complexity but also, e.g., control and optimization.

More specific methodological aspects are concerned with an expected change of perspective in the analysis and control of engineering systems, and in the ensuing assessment of their safe design [3]. Consideration of global nonlinear phenomena entails revisiting the classical concept of theoretical stability, as earlier formulated (Euler) and later on enriched under different perspectives (Lyapunov, Koiter, and others), and accounting for the potentially dramatic effects of the small – yet finite – dynamical perturbations always occurring in real systems, which have to be properly framed within a practical stability perspective capable to guarantee the achievement of the needed load carrying capacity. Global phenomena govern the robustness of static/dynamic solutions against variations of initial conditions and/or control parameters, and affect the dynamic integrity of mechanical/structural systems in applications ranging from the macro- to the micro/nano-scale. Dynamic integrity concepts and tools allow to reliably analyse and control global dynamics, to effectively interpret and predict unexpected experimental behaviors, to provide important hints towards safe and aware engineering design. Pursuing practical stability means abandoning the merely local perspective traditionally assumed in the analysis and design of systems and structures, and moving to a global one where the whole dynamic behaviour of the system is considered, even if being actually interested in only a small (but finite) neighbourhood of a given solution. In spite of its conceptual simplicity, this is a paramount enhancement, full of theoretical and practical implications.

As a matter of fact, the great potential of nonlinear dynamics to significantly enhance performance, effectiveness, reliability and safety of systems has not yet been exploited to the aim of conceiving/developing novel design criteria. Yet, fundamental understanding of various nonlinear physical phenomena producing bifurcations and complex response has now reached such a critical mass that it's definitely time to develop a novel design philosophy, as well as the basic technologies, capable to take full advantage of the natural richness of behavior offered by the inherently nonlinear systems of mechanics.

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On the role of virtual work in Levi-Civita's parallel transport

Giuseppe Iurato[1], Giuseppe Ruta[2]
[1]University of Palermo, IT, [2]Sapienza University, Rome, IT

In this contribution we point out the role of the virtual work principle in the origin of the notion of parallel transport by Tullio Levi-Civita. For a history of the virtual work principle, we may quote [1].

The current historical literature usually reports that parallel transport was motivated by Levi-Civita's attempt to give a geometrical interpretation to the covariant derivative of absolute differential calculus. A careful reading of the Introduction to [2], however, shows that Levi-Civita searched to simplify the computation of the curvature of a manifold. In Section 1 of [2] he considers a metric on a finite-dimensional manifold V_n

$$ds^2 = \sum_{ik} a_{ik} dx_i dx_k \tag{1}$$

and embeds V_n into an ordinary Euclidean space S_N with a sufficiently great dimension, so that it may be described by the system (corresponding to equation (1) of [2], Section 1)

$$y_\nu = y_\nu(x_1, ..., x_n) \qquad \nu = 1, 2, ..., N \tag{2}$$

formally coinciding with a smooth holonomic system subjected to (invertible) virtual displacements. In Section 2 Levi-Civita considers an arbitrary direction of S_N with unit vector $\vec{f}$ of direction cosines f_ν, and another arbitrary direction at a point P of V_n. This last has unit vector $\vec{\alpha}$ with direction cosines α_ν with respect to S_N. The point P may be thought varying on a given smooth curve $\mathcal{C}$ lying on V_n and naturally parameterized by the abscissa s in equation (1), thus α_ν depend on s.

Considering an infinitesimal variation ds of s, the cosine of the angle between $\vec{f}$ and $\vec{\alpha}$, i.e., $\sum_{\nu=1}^N \alpha_\nu(s) f_\nu$, undergoes the variation $ds \sum_{\nu=1}^N \alpha'_\nu(s) f_\nu$. Then, ordinary parallelism between $\vec{\alpha}$ and $\vec{f}$ would require this variation to vanish as $\vec{f}$ varies, thus implying α_ν to be constant. Levi-Civita, however, imposes the weaker condition that the angle between $\vec{\alpha}$ and $\vec{f}$ is constant when $\vec{f}$ varies on V_n. That is, he supposes the variation $ds \sum_{\nu=1}^N \alpha'_\nu(s) f_\nu$ to vanish only for the directions tangent to V_n as P varies along the curve $\mathcal{C}$. Hence, Levi-Civita claims that these directions are compatible with the constraints (2), so that, replacing f_ν with quantities proportional to them, his definition of parallelism implies

$$\sum_{\nu=1}^N \alpha'_\nu(s) \delta y_\nu = 0 \tag{3}$$

for any displacement δy_ν compatible with the constraints (2). With a suitable interpretation of $\vec{\alpha}$ in the first formulas of Section 3 in [2], equation (3) is but a formulation of the virtual work principle for the smooth bilateral holonomic system of equation (2). Hence, Levi-Civita defines his notion of parallel transport and applies it to a Riemannian manifold, in particular to the computation of its curvature. Precisely, he deduces equation (8) of [2], Section 2

$$\sum_{\nu=1}^N \alpha'_\nu(s) \frac{\partial y_\nu}{\partial x_k} = 0 \qquad (k = 1, 2, ..., n), \tag{4}$$

then equation (1_a) of [2], Section 3, where he provides the exact formal intrinsic conditions characterizing the parallel transport of the direction $\vec{\alpha}$ along $\mathcal{C}$ as a function of its directional parameters $\xi_1, ..., \xi_n$ with respect to V_n, within the framework of absolute differential calculus. Furthermore, in the following sections of [2], Levi-Civita does not make any explicit mention to covariant derivatives, except for a hint to Ricci's rotation coefficients in Section 13.

In conclusion, from a historical standpoint, the virtual work principle played a key role in the origin of the idea of connection in Riemannian geometry.

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Nineteenth century molecular models with a glance at modern discrete–continuum theories

Patrizia Trovalusci
Department of Structural and Geotechnical Engineering,
Sapienza University of Rome, Italy,
patrizia.trovalusci@uniroma1.it

This contribution focuses on the genesis of constitutive theories for continuous models originated from discontinuous descriptions of materials which in turn historically coincides with the genesis of continuum mechanics. The molecular theory of elasticity, as developed by Navier, Cauchy and Poisson [4, 3, 6] in the 19^{th} century, represents an attempt to give explanations 'per causas' of elasticity, which were presumed to stem from the natural attractive or repulsive properties of elementary particles ('molecules'), as in the original idea of Newton, specified later by Boscovich, Coulomb and others. In these mechanistic descriptions the molecules, or atoms, are perceived as ultimate particles without extension inside which no forces are accounted for. These particles interact in pairs through forces, depending on their mutual distance, directed along the line connecting their centres ('central–force' scheme). Macroscopic quantities, like stress, elastic moduli, etc., were then derived as averages of molecular material quantities over a convenient volume element, called 'molecular sphere of action', outside which intermolecular forces are negligible.

However, this scheme led to experimental discrepancies concerning the number of elastic constants needed to represent material symmetry classes. Successively, Voigt and Poincaré introduced mixed energetic/mechanistic approaches [9, 10, 5] providing a refined descriptions of the classical molecular model that solve the problem of the underestimation of the number of the material constants related to the central–force scheme. In particular, Voigt introduced potentials of force and moment interactions exerted between pairs of rigid bodies, while Poincaré proposed a multibody potential description [8, 1, 2]. Even if both Voigt and Poincaré, removing the central–force scheme, offered a good solution to the controversy about the elastic constants, the mechanistic–molecular approach, which was longer supported also by Saint Venant, was abandoned in favour of the energetic–continuum approach by Green, and their works have been neglected for long time.

Now these ideas found a renewed interest with reference to the problem of constitutive modelling of complex materials. Current researches in solid state physics, as well as in mechanics of materials, show that energy–equivalent continua obtained by defining direct links with lattice systems are still among the most promising approaches in material science. The mechanical behaviour of complex materials, characterised at finer scales by the presence of heterogeneities of significant size and texture, strongly depends in fact on their microstructural discrete nature. By lacking in material internal scale parameters, moreover, the classical continuum does not always seem appropriate to describe the macroscopic behaviour of such materials taking into account the size, the orientation and the disposition of the micro heterogeneities. This calls for the need of non–classical continuum descriptions obtained through multiscale approaches aimed at deducing properties and relations by bridging information at proper underlying discrete micro–levels via energy equivalence criteria. The circumstances in which the inadequacy of the classical hypothesis of lattice mechanics (lattice homogeneous deformations and central–force scheme), not very differently than in the past, still call for the need of improved constitutive models will be discussed and the suitability of adopting discrete–continuous Voigt–like models, based on a generalization of the so-called Cauchy–Born rule used in crystal elasticity as in classical molecular theory of elasticity, will be pointed out. It will be shown that this approach allow us to identify continua with additional degrees of freedom (micromorphic, multifield, etc.), which are essentially non–local models with internal lengths and dispersive properties [7].

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The discovery of the vector representation of moments and angular velocity (1750 - 1830)

Sandro Caparrini,
Dipartimento di Matematica, Università di Torino

The introduction of vectors in mechanics falls naturally into two distinct periods. In fact, force and velocity belong to the elements of point mechanics, while moment of forces and angular velocity are mainly used to describe the motion of rigid bodies. Therefore, the vector representation of the latter kind of quantities must be traced in papers written after 1750.

The first statement and proof that the infinitesimal rotations about concurrent axes can be composed according to the parallelogram law are due to Paolo Frisi (1759). Contemporarily with Frisi, another Italian mathematician, Tommaso Perelli, obtained the same theorem, but did not publish his work.

Lagrange gave a purely analytical formulation of this theorem in the first edition of his Méchanique analitique (1788). He came close to establishing the vectorial character of small rotations, yet he failed to do so. In the second edition (1811) of his treatise, whose title was now changed to Mécanique analytique, Lagrange added a completely new analysis of the decomposition of a given infinitesimal rotation along the coordinate axes of two different systems of rectangular Cartesian coordinates, demonstrating that the partial rotations around the coordinate axes transform as the components of a displacement. Thus he succeeded at last in formulating the vectorial representation of small rotations.

After 1811, this result was taken up by Jacques Frédéric Français (1812), Jacques Philippe Marie Binet (1814) and Poisson (1833). A detailed study of the angular velocity vector appeared in Poinsot's Théorie nouvelle de la rotation des corps (1834). Poinsot made full use of the vectorial properties of the angular velocity in his classic theory of the motion of a rigid body.

The discovery that moments of forces are vectors was made by Euler and can be found in two papers written in1780 but published only in 1793. He obtained this result as a corollary to a formula for the moment of force about a point, expressed in terms of components. Euler saw that his formula indicates that moments of forces can be represented by a directed line segment and can be decomposed by means of the parallelogram law.

Unaware of Euler's results, a few years later Laplace published two articles based essentially on the law of transformation of moments in passing from one system of coordinates to another (1798). His discovery of the invariable plane is roughly equivalent to the vectorial representation of moment of momentum. The connection between Laplace's invariable plane and Euler's formula was established by Gaspard de Prony in his Mécanique philosophique (1800).

A purely geometrical approach to the theory of moments of forces was first obtained by Poinsot in his Éléments de statique (1803). Here he created the concept of a couple of forces, and demonstrated that if we represent a couple with a segment perpendicular to its plane we can compound two couples by means of the parallelogram law. In 1808 Poisson represented the moment of a force by means of a parallelogram.

These new discoveries were further developed. In 1814 Binet wrote the law of moment of momentum in a form which takes into account the geometric representation of moments; in 1818 he created the concept of areal velocity and derived the principle of moment of momentum for a system of mass points from the law $F = ma$ and the equality of action and reaction (this result is usually called "Poisson's theorem" in modern textbooks). Antonio Bordoni generalized Euler's formula to non-orthogonal Cartesian axes (1820).

In 1826 Cauchy wrote a sequence of five papers in which he brought the theory of moments to its final form. Cauchy's treatment does not differ in essence from the theory that we find today - in vector formulation - in every textbook.

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Domenico Fontana and the entrance of mechanics in architecture

R. Masiani, C. Tocci
University of Rome La Sapienza, Department of Structural and Geotechnical Engineering

Compared to that of his contemporaries, the work of Domenico Fontana takes its place in the late sixteenth century architecture with connotations of significant originality. Not so much for the quality of the outcomes, "constantly poised between elegance and obviousness" according to the sharp synthesis of Paolo Portoghesi [1], but for the greater importance attached to the structural and constructional problems of architecture that seems to reveal, in some cases, a not superficial knowledge of the basic concepts of mechanics and the awareness of the possibility to use them in practice.

Beginning from the reading of archive's documents relating to the static restoration of the abacus of Marcus Aurelius column made in 1589 [3], and, for comparison with the description of the translation of the Vatican obelisk contained in the famous book published the year after [2], the contribution attempts to outline as a preliminary the relationship of Domenico Fontana with the new technical literature of the second half of the sixteenth century (Federico Commandino, Guidobaldo del Monte, Bernardino Baldi).

The analysis of the building site processes, concisely described for the Marcus Aurelius column and meticulously illustrated for the Vatican obelisk, allows illustrating – not by chance on two occasions in which structural and constructional tasks are prevailing , if not exclusive, compared to architectural concerns – not only the complete mastery that Fontana exhibits about the statics of elementary machines (superbly described, for example, in Guidobaldo del Monte's Mechanicorum Liber) but also, and above all, the ability to exploit such a mastery as a concrete support in the phase of structural invention.

From this point of view, Domenico Fontana unequivocally heralds an attitude that would become shared approach more than a century later, representing one of the first examples in history in which mechanics is consciously employed for the structural control of architecture.

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On the graphic statics for the analysis of masonry domes

N. Cavalagli, V. Gusella
Department of Civil and Environmental Engineering, University of Perugia, Perugia, Italy

During the second half of the 19th-century the graphic statics widely disseminated for the analysis of structures, from the academic to the architectural and engineering circles ([1],[2]). In the studying of masonry domes, the crucial aspect of the graphic statics is the analysis of a three-dimensional structural behaviour in a bi-dimensional space. The "calculus problem" is well exposed by Camillo Guidi in his treatise, in which he wants to underline that «staticamente indeterminato è il tracciamento della curva delle pressioni; né la teoria dell'elasticità fornisce di questo problema una soluzione semplice e pratica; conviene perciò accontentarsi di una soluzione approssimata, o piuttosto di una specie di calcolo di verifica» [3]. Eddy [4] was the first to propose a graphical method for the study of a slice of dome interacting with neighbours, by considering, in the balance of a block, both the meridian and hoop forces. In addition to Eddy's method, very complex for a practical application, other solutions can be found in literature, among them the contributions of Schwedler [5], Lévy [6], Wolfe [7], Guidi [3] and another one shown in an italian practical manual for architects [8]. Owing to the indeterminateness of the problem, all these methods differ in the initial hypotheses made on the position of the thrust surface.

The aim of this paper is to compare the results obtained by the application of several methods to an hemispherical dome with the analytical and numerical results given by the membrane theory and the Finite Element Method respectively.

In addition, a specific case study will be analysed, the dome of the Basilica of Santa Maria degli Angeli in Assisi [9]. This case consists of a single shell masonry dome with a lantern at the top, which generates a concentrated external load on the dome slice analysed by means of graphic statics. The results obtained by the application of several graphical methods will be discussed, taking also into account the numerical solution obtained by a finite element model.

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G. W. Leibniz's *"Machina Deciphratoria"*, the first described cypher machine from the 17$^{\text{th}}$ century, constructed and built in 2013/14 for the Leibniz Exhibition of the Leibniz Universität Hannover

Erwin Stein
Professor of Mechanics and Computational Mechanics, Leibniz Universität Hannover (LUH), Curator of the Leibniz Exhibition of the LUH

According to Leibniz's major postulate *Theoria cum praxi*, his plan of general progress in the 2$^{\text{nd}}$ half of the 17$^{\text{th}}$ century, the beginning of the age of enlightenment, was to combine his new *Scientia generalis* with the *Ars inveniendi* and the *Ars combinatoria*, both in connection with his *Calculus logicus*.

Important technical outputs were his famous *Machina arithmetica*, the first *Four-species-calculating-machine* with a uniform logic for the gear mechanisms (especially the decimal carries), and moreover, the description of a *Machina deciphratoria*, a cypher machine for six-fold coding and decoding of messages via monoalphabetic substitutions of letters in 2 x 6 = 12 alphabet strips, each with 26 letters. This gear-driven machine was designed according to Leibniz's proposals by Prof. Nicholas Rescher, University of Pittsburgh, PA, constructed in detail and mounted by Klaus Badur and built by Gerald Rottstedt, both Garbsen near Hannover, Germany, in 2013/14. This machine became part of the Leibniz Exhibition of our University as a permanent loan of the Fritz Behrens Stiftung Hannover.

It is remarkable that in both very different machines the so-called *stepped drum*, a cogwheel with linearly decreasing tooth lengths in five planes (from 12 to 2 cogs), plays an important role. In the cypher machine it is attached at the left end of the transport drum which can be fixed in six axial positions of the stepped drum. Coding (substituting) of a letter is realized by pressing the related key, by which the transport drum is rotated by 60°. The teeth of the stepped drum are rotating the cogwheel of the display drum (beneath the transport drum) on which the coded letter is visible on a strip of 26 alphabet letters. It is obvious that the six possible axial positions of stepped drum enormously increase the number of possible encryptions.

Leibniz's invention of very different new products - using the same key components - shows his modern holistic and system-oriented (not product-oriented) thinking within his *Ars inveniendi*, because *a single general method and its solutions yields very many single inventions* (from a letter by Leibniz to Duke Ernst August of Hannover in 1685/87).

The lecture also touches the exciting history of cryptography from ancient times until today with undeclared electronic cypher wars.