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UNIVERSITÀ DEL SALENTO

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Scientific Program - Timetable

Sunday 22	Time	Monday 23	Tuesday 24	Wednesday 25	Thursday 26	Friday 27
	9:15		Contributed sessions (15 in parallel)	Plenary Lecture Moritz Diehl	Contributed sessions (15 in parallel)	Contributed sessions (14 in parallel)
	9:30					
	9:45					
	10:15	Registration		von Mises prize lecture		
	10:30					
	10:45					
	11:15		Coffee Break	Coffee Break	Coffee Break	Coffee Break
	11:30					
	11:45					
	12:15		Plenary Lecture Thomas Böhlke	General Assembly	Plenary Lecture Ferdinando Auricchio	Contributed sessions (11 in parallel)
	12:30					
	12:45					
	13:15	Opening	Lunch	Lunch	Lunch	
	13:30					
	13:45	Univ. Chorus Performance				Closing
	14:15	Prandtl Lecture Keith Moffatt	Plenary Lecture Enrique Zuazua	Contributed sessions (15 in parallel)	Plenary Lecture Daniel Kressner	
	14:30					
	14:45					
	15:15	Plenary Lecture Giovanni Galdi	Plenary Lecture Nikolaus Adams		Plenary Lecture Stanislaw Stupkiewicz	
	15:30					
	15:45					
Registration pre-opening	16:15	Coffee Break	Coffee Break Poster session	Coffee Break	Coffee Break Poster session	
	16:30	Minisymposia & Young Reseachers' Minisymposia (10 in parallel)	Contributed sessions (14 in parallel)	Contributed sessions (15 in parallel)	Contributed sessions (15 in parallel)	
	16:45					
	17:15					
	17:30					
	17:45					
	18:15		Public lecture Francesco D'Andria			
	18:30					
	18:45					
	19:15	Opening reception at Castle of Charles V				
	19:30					
	19:45					
	20:15			Conference dinner at Hotel Tiziano		
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YRMS5: Topics in Low-rank Tensor Approximation

The recent years showed an emerging interest in low-rank tensor techniques in scientific computing for solving "high-dimensional" tensor structured problems to overcome the curse of dimension by using separation of variables. This includes eigenvalues problems, linear equations, stochastic and parametric PDEs, dynamical systems, or ground state calculations in quantum spin systems. At the same time, low-rank tensor techniques are successfully used in exploratory data analysis, signal processing, and statistics. Ideally, at least three aspects need to be addressed: justification of the low-rank approach by a-priori approximability results, design and efficient implementation of algorithms avoiding "high-dimensional" objects at any stage, and convergence analysis of the typically nonlinear methods. These and other challenging tasks make low-rank tensor techniques an interesting and many-sided field. This Young Researchers Symposium will highlight some of these aspects at the hand of the speaker's latest research results.

Finding low-rank bases of matrix subspaces

Yuji Nakatsukasa[1], Tasuku Soma[1], André Uschmajew[2]
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Given a d-dimensional matrix subspace spanned by matrices $M_1, M_2, \ldots, M_d \in \mathbb{R}^{m\times n}$, one can ask whether this subspace contains a basis of low-rank matrices. Having such a basis, it is easy to imagine useful applications, for instance a compressed storage of the initial matrices $M_1, M_2, \ldots, M_d$. It is interesting to note that the specific question of whether a basis of rank-one matrices exists is equivalent to the question, whether the $m \times n \times d$ tensor with slices $M_1, M_2, \ldots, M_d$ has canonical tensor rank d. In general, the task is twofold: if one knows the smallest ranks for which a basis exists, one can design many optimization procedures to hopefully find such a basis. For instance, if a rank-one basis exists, one may use tensor decomposition algorithms like alternating least squares. For the case of higher ranks, we propose a simple alternating projection strategy with hard singular value thresholding, and investigate its convergence to some extent. The hard task, however, is to estimate the smallest possible ranks a basis can have. We present a heuristic based on nuclear norm minimization (soft thresholding) and, again, alternating projections, which works astonishingly well.

Decoupling multivariate functions using tensor decompositions

Philippe Dreesen, Mariya Ishteva, Johan Schoukens

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We present a method to decompose a set of multivariate real functions into linear combinations of univariate functions in linear forms of the input variables [3]. The procedure collects first-order information by evaluating the Jacobian matrix of the function in a set of points. A tensor decomposition of the tensor constructed from the Jacobian matrices provides the decoupled representation.

Let $\mathbf{f} : \mathbb{R}^m \to \mathbb{R}^n$ be a given multivariate real function that allows a parameterization

$$\mathbf{f}(\mathbf{u}) = \mathbf{W}\mathbf{g}(\mathbf{V}^T\mathbf{u}), \tag{1}$$

where $\mathbf{W} \in \mathbb{R}^{n\times r}$ and $\mathbf{V} \in \mathbb{R}^{m\times r}$ are unknown linear transformations, and $\mathbf{g} : \mathbb{R}^r \to \mathbb{R}^r$ is an unknown vector function consisting of univariate functions $g_i(x_i)$, *i.e.*, every component of $\mathbf{g}$ only depends on a single variable x_i, which is the i-th component of $\mathbf{V}^T\mathbf{u}$, or $x_i = \mathbf{v}_i^T\mathbf{u}$.

The decoupling method proceeds by considering the first-order information of the functions $f_i(\mathbf{u})$, which is captured by the $n \times m$ Jacobian matrix $\mathbf{J}(\mathbf{u}) = [\partial f_i(\mathbf{u})/\partial u_j]$. By using the parameterization (1), the Jacobian matrix $\mathbf{J}(\mathbf{u})$ can be written as

$$\mathbf{J}(\mathbf{u}) = \mathbf{W}\,\mathrm{diag}(g_i'(\mathbf{v}_i^T\mathbf{u}))\mathbf{V}^T.$$

The Jacobian is evaluated in the points $\mathbf{u}^{(1)}, \ldots, \mathbf{u}^{(N)}$, resulting in the Jacobian matrices $\mathbf{J}(\mathbf{u}^{(1)}), \ldots, \mathbf{J}(\mathbf{u}^{(N)})$, which we stack into a three-way tensor $\mathcal{J}$ having dimensions $n \times m \times N$. Finding the unknowns in the parameterization (1) then amounts to solving a simultaneous matrix diagonalization problem, which is computed using the canonical polyadic decomposition [5]. Indeed, we can write the tensor of Jacobians as

$$\mathcal{J} = \sum_{i=1}^{r} \mathbf{w}_i \circ \mathbf{v}_i \circ \mathbf{h}_i, \tag{2}$$

where $\circ$ denotes the outer product. Decomposition (2) directly returns the unknowns $\mathbf{W}$ and $\mathbf{V}$, as well as the necessary information to reconstruct the univariate functions $g_i(x_i)$.

A variation of the decoupling method employs the block-term tensor decomposition [1, 2] instead of the canonical polyadic decomposition, making it possible to achieve partial decoupling. In a similar fashion, now a number of internal multivariate functions acts between smaller sets of internal partially coupled variables, rather than one-to-one univariate functions.

We will highlight applications in block-oriented system identification [4] where the above procedure is employed to decouple multiple-input-multiple-output static nonlinearities in order to recover physical interpretability and to reduce the number of parameters.

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Soft Thresholding of Hierarchical Tensors and Its Application in Iterative Methods

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TU Berlin

A widespread approach for solving high-dimensional problems using tensor representations is to modify a convergent standard iterative method by an additional rank reduction in each step. In the context of the hierarchical tensor format, this reduction is usually achieved by truncation of a higher-order singular value decomposition. In this talk, we consider an alternative type of rank reduction based on *soft thresholding* of tensors.

Whereas hard thresholding produces sparse approximations of a sequences by removing entries of absolute value below a certain threshold, soft thresholding also modifies all remaining entries. Applying soft thresholding to a sequence amounts to applying the function $s_\alpha(x) := \operatorname{sign}(x) \max\{|x| - \alpha, 0\}$ to each entry, and in contrast to the analogous function applied in hard thresholding, s_α is non-expansive, that is, Lipschitz continuous with constant one. It is well-known that non-expansiveness also holds for the resulting thresholding operation on sequences (with respect to the ℓ_2-norm) and on matrices (where it is applied to the singular values, and one has non-expansiveness with respect to the Frobenius norm).

In this talk, we describe a soft thresholding operation for hierarchical tensor representations that also preserves this property and which can serve as a substitute of truncated higher-order singular value decompositions (which would correspond to hard thresholding) for rank reduction in iterative methods. We give an analysis of the approximation properties of the thresholding for different types of hierarchical singular value decay and consider in detail its combination with convergent fixed point iterations, where non-expansiveness turns out to be surprisingly useful. We then focus on the treatment of linear elliptic operator equations based on a fixed discretization. We propose a convergent method with a posteriori choice of an appropriate sequence of thresholding parameters that only requires bounds on the spectrum of the operator. The favorable properties of this scheme, especially concerning the resulting hierarchical ranks of iterates, are also demonstrated in numerical tests.

Tensor-structured approximation for the solution of differential equations

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In d dimensions, we consider problems with linear second-order elliptic differential operators of the form

$$\mathcal{L} = -\nabla^\top A \nabla + b^\top \nabla + c,$$

where the coefficients A, b and c are sufficiently smooth. A textbook approach based on the low-order finite-difference or finite-element approximations constructed on a uniform, tensor-product grid of size $n \times \ldots \times n$ seems infeasible due to the rapid growth of the complexity of both the representation of the discretizations and the whole solution algorithm, which are at least $\mathcal{O}(n^d)$. This very approach, however, turns out to be highly efficient when the vectors and matrices involved in the numerical algorithm are parametrized using tensor decompositions [1, 2], of which we consider here the *tensor train* (TT) [3, 4] and *quantized tensor train* (QTT) [5, 6] representations of tensors.

If a d-dimensional $n \times \ldots \times n$-vector $\boldsymbol{u}$ satisfies the equation

$$\boldsymbol{u}_{j_1,\ldots,j_d} = \sum_{\alpha_1=1}^{r_1} \ldots \sum_{\alpha_{d-1}=1}^{r_{d-1}} U_1(j_1,\alpha_1) \cdot U_2(\alpha_1,j_2,\alpha_2) \cdot \ldots \cdot U_{d-1}(\alpha_{d-2},j_{d-1},\alpha_{d-1}) \cdot U_d(\alpha_{d-1},j_d)$$

for $1 \le j_k \le n$ and $1 \le k \le d$ with two- and three-dimensional arrays $U_1, U_2, \ldots, U_d$, then $\boldsymbol{u}$ is said to be represented in the TT decomposition in terms of the *core tensors* $U_1, U_2, \ldots, U_d$. The summation limits $r_1, \ldots, r_{d-1}$ are called *ranks* of the TT representation. The quantization of the kth dimension relies on the positional representation of the "physical" indices in terms of "virtual" indices, e.g., for $n = 2^l$, on the binary encoding:

$$j_k \mapsto \overline{j_{k,1}, \ldots, j_{k,l}} = 1 + \sum_{\mu=1}^{l_k} 2^{l-\mu}(j_{k,\mu} - 1) \quad \text{for} \quad 1 \le k \le d.$$

The QTT representation is then a combination of the quantization of the "physical" dimensions with the TT decomposition, in which the latter separates the "virtual" dimensions produced by the former. The number of parameters TT and QTT representations involve is bounded from above by dnR_{TT}^2 and $2dlR_{\mathrm{QTT}}^2$ respectively, where R_{TT} and R_{QTT} are upper bounds on the TT and QTT ranks. These complexity bounds may be dramatically smaller than n^d, the formal number of degrees of freedom, provided that the dependence of the rank bounds R_{TT} and R_{QTT} on n and d is moderate. This is often observed experimentally for the data involved in the numerical solution of PDEs, see [2, 7, 8, 9, 10] and references therein. The talk will present recent theoretical results on the rank bounds obtained jointly with Ch. Schwab (SAM ETH Zurich).

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Riemannian BFGS on the Tensor Train component space using an inherited tensor metric

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We apply existing concepts of Riemannian geometry to the Tensor Train (TT) component space, which is a quotient space of the full component space

$$\mathcal{C} = \prod_{i=1}^{d} \mathbb{R}_*^{r_{i-1} \times n_i \times r_i}.$$

It has been established that the TT component space is a smooth manifold that is related to a product of Grassmannians [1]. The according tangent space is obtained by posing a gauge condition. We use a non-canonical Riemannian metric and introduce projections, retractions and vector transport in order to be able to perform second order optimization [2]. The metric simulates the TT manifold of tensors of fixed TT rank,

$$\mathcal{M}_{\mathbf{r}} = \{\mathcal{U} \in \mathbb{R}^{n_1 \times \cdots \times n_d} : \mathrm{rank}_{\mathrm{TT}}(\mathcal{U}) = \mathbf{r}\},$$

and as such, it allows us to break down minimization problems on this space to the component space. The Riemannian BFGS is defined and performed in local coordinates, which ensures the optimality of the BFGS update. The possibility of a limited memory method is explored [3].

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Preconditioned Riemannian optimization for low-rank tensor equations

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The solution of very large linear systems is a challenging task often encountered as a core ingredient when solving partial differential equations on high-dimensional domains. In these cases, the degrees of freedom in the linear system grow exponentially with the number of dimensions, making classic approaches unfeasible. Approximation of the solution by low-rank tensor formats often allows us to avoid this *curse of dimensionality* by exploiting the underlying structure of the linear operator. We propose a new algorithm that performs a preconditioned gradient method on the manifold of tensors of fixed rank. In particular, we focus on tensors represented in the *Tensor Train* (TT) / *Matrix Product States* (MPS) format. We demonstrate the flexibility of our algorithm by comparing different approximations of the Riemannian Hessian as preconditioners for the gradient directions. Finally, we compare the efficiency of our algorithm with other tensor-based approaches such as the *Alternating Linear Scheme* (ALS).