PMNP 2017

Università del Salento

Scientific Program

Posters

Soliton amoebas

Boris G. Konopelchenko, Mario Angelelli
University of Salento and Sezione INFN, Lecce

Singular sectors of a family of solutions of the KP II equation are revisited. The structure of the zero loci of Wronskian τ-functions for a subclass of solutions with different combinations of signs of pre-exponential factors is analyzed. It is shown that the zero loci have an amoeba-type form and a particolar inclusion property. Such *soliton amoebas* are connected with a particular class of statistical amoebas associated to a family of partition functions in statistical physics. Constraints on the possible choices of signs preserving the Wronskian form and the relation with general (unconstrained) statistical amoebas are explored.

Generalizations of the semi- and fully discrete Lotka-Volterra lattice

Xiaomin Chen[1], Xingbiao Hu[2]
[1] Max Planck Institute for Dynamics and Self-Organization, 37073 Göttingen, Germany
[2] ICMSEC AMSS, Chinese Academy of Sciences, Beijing 100190, PR China

It is well known that the integrable semi-discrete and fully discrete Lotka-Volterra equations possess special Lax pairs involving symmetric orthogonal polynomials. We construct a nonisospectral semi-discrete Lotka-Volterra equation and a generalized fully discrete Lotka-Volterra equation. The key point is introducing a more general evolution relation for the moments of symmetric orthogonal polynomials, which involves a "convolution term", of the kind that appeared in [1, 2]. Furthermore, we generate corresponding exact ("molecule") solutions, expressed in terms of Hankel-type determinants. Our approach makes use of Hirota's bilinear method and determinant techniques.

References

[1] K. Kajiwara, T. Masuda, M. Noumi, Y. Ohta, Y. Yamada. Determinant Formulas for the Toda and Discrete Toda Equations. Funkcialaj Ekvacioj 44 (2001) 291–307.

[2] F. Peherstorfer, V. P. Spiridonov, A. S. Zhedanov. Toda chain, Stieltjes function, and orthogonal polynomials. Theoretical and Mathematical Physics, 151 (2007) 505–528.

Higher-order interaction induced effects for strong dispersion-managed optical solitons with dissimilar powers

Francisco J. Diaz-Otero, Omar Guillan-Lorenzo
Laura Pedrosa-Rodriguez
University of Vigo, EI Telecommunication, Spain
OHB System AG, Pre-development and Studies, Germany

We address one of the major penalties associated with the transmission of optical solitons in fiber-optic networks, the intrachannel interactions between pulses in a single channel strong dispersion-managed (DM) transmission link. As we are moving towards narrower pulse widths, up to femtosecond time slots, higher-order effects such as third-order dispersion (TOD), stimulated Raman scattering (SRS) and self-steepening must be considered. All of them determine a significant enhancement of the Gordon-Haus effect and the timing jitter and also induce crosstalk [1, 2].

We calculate the interaction length between sets of two and three pulses as a function of the ratio of dissimilar peak powers [3] of interacting solitons by means of a variational approach to the Generalized Nonlinear Schrodinger Equation describing the evolution of optical soliton pulses [4]. We find a maximum of this colliding length for specific values of pulse energies. The effect of higher-order perturbation terms shows an enhancement of the interaction distance in a broad range of dispersion difference values. We conclude that the transmission characteristics of soliton trains may be improved by means of using specific values of unequal energies and taking into account higher-order correction terms.

References

[1] T. Lakoba and G. Agrawal. Effects of third-order dispersion on dispersion-managed solitons. J. Opt. Soc. Am. B 16 (1999), 1332.

[2] S. Mookherjea and A. Yariv. Hamiltonian dynamics of breathers with third-order dispersion. J. Opt. Soc. Am. B 18 (2001), 1150.

[3] O. Istrin and B. A. Malomed. Interactions between dispersion managed solitons with unequal powers. J. Opt. Soc. Am. B 27 (2010), 844–851.

[4] F. J. Diaz-Otero, P. Chamorro-Posada and J.C. Garcia-Escartin. Dispersion-managed soliton interactions in the presence of third-order dispersion. II International Conference on Advanced Optoelectronics and Lasers, CAOL Proceedings (2005), 1531–1555.

On solutions of equations of incompressible liquids

V.S.Dryuma
IMI AS of Rep.Moldova

In the report, an examples of exact solutions of a system of equations describing the flow of an incompressible fluid will be presented.

To the Euler system of equations

$$\vec{V}_t + (\vec{V} \cdot \nabla)\vec{V} + \nabla P = 0, \quad \nabla \cdot \vec{V} = 0,$$

where $\vec{V} = (U(\vec{x},t), V(\vec{x},t), W(\vec{x},t))$ are the components of velocity, $P = P(\vec{x},t)$ is the pressure of fluid, $x \in R^3$, nonsingular and localized solutions with constant pressure $P(\vec{x},t)$=const was obtained by author in [1] and have the form

$$W(\vec{x},t) = c + \frac{e^{\alpha(x-at)+\beta(y-bt)+\delta(z-ct)}}{1+e^{\alpha(x-at)+\beta(y-bt)+\delta(z-ct)}}, \; V(\vec{x},t) = b + \frac{e^{\alpha(x-at)+\beta(y-bt)+\delta(z-ct)}}{1+e^{\alpha(x-at)+\beta(y-bt)+\delta(z-ct)}},$$

$$U(\vec{x},t) = a - \frac{(\beta+\delta)\, e^{\alpha(x-at)+\beta(y-bt)+\delta(z-ct)}}{\alpha\left(1+e^{\alpha(x-at)+\beta(y-bt)+\delta(z-ct)}\right)},$$

$$U(\vec{x},t) = a + \frac{e^{-(\beta+1)(x-at)+\beta(y-bt)+z-ct} + e^{-(\beta+1)(x-at)+(\beta-1)(y-bt)+2(z-ct)}}{1+e^{-(\beta+1)(x-at)+\beta(y-bt)+z-ct}\left(1+e^{-y+bt+z-ct}\right)},$$

$$V(\vec{x},t) = b + \frac{e^{-(\beta+1)(x-at)+\beta(y-bt)+z-ct} + e^{-(\beta+1)(x-at)+(\beta-1)(y-bt)+2(z-ct)}}{1+e^{-(\beta+1)(x-at)+\beta(y-bt)+z-ct}\left(1+e^{-y+bt+z-ct}\right)},$$

$$W(\vec{x},t) = c + \frac{e^{-(\beta+1)(x-at)+\beta(y-bt)+z-ct} + e^{-(\beta+1)(x-at)+(\beta-1)(y-bt)+2(z-ct)}}{1+e^{-(\beta+1)(x-at)+\beta(y-bt)+z-ct}\left(1+e^{-y+bt+z-ct}\right)}.$$

$$U(x,y,z,t) = \frac{\beta\,\delta\, e^{\alpha x+\beta y+\delta z}}{\left(1+e^{\alpha x+\beta y+\delta z}\right)^2}, \quad V(x,y,z,t) = \frac{\alpha\,\delta\, e^{\alpha x+\beta y+\delta z}}{\left(1+e^{\alpha x+\beta y+\delta z}\right)^2},$$

$$W(x,y,z,t) = -2\,\frac{\alpha\,\beta\, e^{\alpha x+\beta y+\delta z}}{\left(1+e^{\alpha x+\beta y+\delta z}\right)^2}.$$

The examples of solutions with condition $P(\vec{x},t) \neq const$ are

$$U(\vec{x},t) = a + A\sin(z-ct) + C\cos(y-bt), \; V(\vec{x},t) = b + B\sin(x-at) + A\cos(z-ct),$$

$$W(\vec{x},t) = c + C\sin(y-bt) + B\cos(x-at),$$

$$P(\vec{x},t) = -1/2\,CB\sin(x-at+y-bt) + 1/2\,CB\sin(x-at-y+bt) -$$

$$-1/2\,CA\sin(z-ct+y-bt)+1/2\,CA\sin(y-bt-z+ct)-1/2\,BA\sin(x-at+z-ct)-$$
$$-1/2\,BA\sin(x-at-z+ct)+F_3(t),$$
$$U(\vec{x},t)=a-1/2\,C_1\,\sin(-y+bt)+1/2\,E_1\,\cos(-y+bt)+1/2\,F_1\,\cos(-z+ct)-$$
$$-1/2\,H_1\,\sin(-z+ct),$$
$$V(\vec{x},t)=b-1/2\,F_1\,\sin(-z+ct)-1/2\,H_1\,\cos(-z+ct)-A_1\,\sin(x-at)+$$
$$+B_3\,\cos(x-at),$$
$$W(\vec{x},t)=c+B_3\,\sin(x-at)+A_1\,\cos(x-at)+1/2\,C_1\,\cos(-y+bt)+$$
$$+1/2\,E_1\,\sin(-y+bt),$$
$$P(\vec{x},t)=1/2\,F_1\,B_3\,\cos(x-at)\sin(-z+ct)-1/2\,F_1\,A_1\,\sin(x-at)\sin(-z+ct)+$$
$$+1/2\,H_1\,B_3\,\cos(x-at)\cos(-z+ct)-1/2\,H_1\,A_1\,\sin(x-at)\cos(-z+ct)+$$
$$+1/4\,F_1\,C_1\,\sin(-y+bt)\cos(-z+ct)-1/4\,F_1\,E_1\,\cos(-y+bt)\cos(-z+ct)-$$
$$-1/4\,H_1\,C_1\,\sin(-y+bt)\sin(-z+ct)+1/4\,H_1\,E_1\,\cos(-y+bt)\sin(-z+ct)-$$
$$-1/2\,A_1\,\cos(x-at)C_1\,\cos(-y+bt)-1/2\,A_1\,\cos(x-at)E_1\,\sin(-y+bt)-$$
$$-1/2\,B_3\,\sin(x-at)C_1\,\cos(-y+bt)-1/2\,B_3\,\sin(x-at)E_1\,\sin(-y+bt)+K(t).$$

$$U(\vec{x},t)=-\frac{\cos(t)\beta}{\alpha}-\frac{\sin(t)\beta}{\alpha}-2\,\frac{\beta\,e^{\alpha\,x+\beta\,y+\beta\,z}}{\alpha\,(1+e^{\alpha\,x+\beta\,y+\beta\,z})},$$
$$V(\vec{x},t)=1/2\,\cos(t)+1/2\,\sin(t)+\frac{e^{\alpha\,x+\beta\,y+\beta\,z}}{1+e^{\alpha\,x+\beta\,y+\beta\,z}},$$
$$W(\vec{x},t)=1/2\,\cos(t)+1/2\,\sin(t)+\frac{e^{\alpha\,x+\beta\,y+\beta\,z}}{1+e^{\alpha\,x+\beta\,y+\beta\,z}},$$
$$P(\vec{x},t)=\frac{(-\sin(t)+\cos(t))\,\beta\,x}{\alpha}-1/2\,(y+z)\cos(t)+1/2\,(y+z)\sin(t)+P_0.$$

To the NS-system of equations

$$\vec{V}_t+(\vec{V}\cdot\nabla)\vec{V}-\mu\Delta\vec{V}+\nabla P=0,\qquad \nabla\cdot\vec{V}=0,$$

where μ is viscosity of liquid we find the solution

$$U(\vec{x},t)=-e^{-2\,\mu\,t}\cos(x)\cos(z),$$
$$V(\vec{x},t)=-2\,e^{-2\,\mu\,t}\left(\cos(x)\sin(z)+C\cos(\sqrt{2}x)\right),$$
$$W(\vec{x},t)=-e^{-2\,\mu\,t}\left(\sin(x)\sin(z)+C\sqrt{2}\sin(\sqrt{2}x)\right),$$
$$P(\vec{x},t)=-2\,e^{-4\,\mu\,t}\cos(x)C\cos(\sqrt{2}x)\sin(z)+1/2\,e^{-4\,\mu\,t}\left(\cos(z)\right)^2-$$
$$-e^{-4\,\mu\,t}\sin(x)C\sqrt{2}\sin(\sqrt{2}x)\sin(z)-1/2\,e^{-4\,\mu\,t}\left(\cos(x)\right)^2+F(t).$$

References

[1] V.S. Dryuma. Towards the theory of solutions of flow equations of uncompressible liquid. Internationl Conference "Anosov Systems and Modern Dynamics", Moscow, December 19-23, 2016, Abstracts, p.28-31.

On the first quantization of $SU(2)$ $\mathcal{N} = 2^*$ Seiberg-Witten theory: an integrable perspective

Alberto Fachechi, Davide Fioravanti
Dipartimento di Matematica e Fisica E. De Giorgi,
Università del Salento & INFN
Dipartimento di Fisica e Astronomia,
Università di Bologna & INFN

Since the work by Seiberg and Witten [1], low energy effective $\mathcal{N} = 2$ gauge theories have gained a great interest due to their intrinsic richness. The main reason is that the exact solution is encoded in the monodromy properties of certain hyper-elliptic curves which are mapped to spectral curves of suitable classical integrable systems. Therefore, in SW theories both perturbative and non-perturbative effects are fully under control by using the electric-magnetic ($\mathcal{S}-$)duality.

Altought Seiberg-Witten curve encodes the exact solution of low-energy theory, a direct systematical check was made possible only with the introduction of the so-called Ω-deformation by Nekrasov [2], a 2-parametric (ϵ_1, ϵ_2) regularization of the instanton moduli space curing its non-compactnesses. Recently, Ω-deformed theories also gained a renewed interest going beyond the merely computational task. In fact, there are strong indications [3] that Ω-background preserves the integrable structure of Seiberg-Witten theories. This would therefore generalize the above relation to quantum versions of the "classical" systems, replacing the original spectral curve with a new structure encoding quantum integrability. In this sense, the Nekrasov-Shatashvili (NS) limit $\epsilon_2 = 0$ can be regarded as a first quantization of the model.

We considered the NS limit of the $\mathcal{N} = 2^*$ $SU(2)$ gauge theory, for which the Seiberg-Witten curve is deformed in a TQ-like equation with infinite shifts [4]. Furthermore, the deformed prepotential of the theory plays the role of energy in a stationary Schrödinger problem with Lamé potential [5], while the Floquet exponent of the wave-function of the spatial variable are proportional to the vevs of the gauge theory. We considered the large energy expansion of Lamé equation and give a novel and purely algebraic description of the monodromy problem. Then, implementing $\mathcal{S}$-duality directly into the Schrödinger picture, we obtain a simple expression for the deformed prepotential in terms of the KdV charges and extract non-perturbative informations of the theory like modular anomaly equation.

References

[1] N. Seiberg and E. Witten, "Electric - magnetic duality, monopole condensation, and confinement in N=2 supersymmetric Yang-Mills theory," Nucl. Phys. B **426** (1994) 19 Erratum: [Nucl. Phys. B **430** (1994) 485] doi:10.1016/0550-3213(94)90124-4, 10.1016/0550-3213(94)00449-8 [hep-th/9407087].

[2] N. A. Nekrasov, "Seiberg-Witten prepotential from instanton counting," Adv. Theor. Math. Phys. **7** (2003) no.5, 831 doi:10.4310/ATMP.2003.v7.n5.a4 [hep-th/0206161].

[3] N. A. Nekrasov and S. L. Shatashvili, "Quantization of Integrable Systems and Four Dimensional Gauge Theories," arXiv:0908.4052 [hep-th].

[4] F. Fucito, J. F. Morales, D. R. Pacifici and R. Poghossian, "Gauge theories on Ω-backgrounds from non commutative Seiberg-Witten curves," JHEP **1105** (2011) 098 doi:10.1007/JHEP05(2011)098 [arXiv:1103.4495 [hep-th]].

[5] K. Maruyoshi and M. Taki, "Deformed Prepotential, Quantum Integrable System and Liouville Field Theory," Nucl. Phys. B **841** (2010) 388 doi:10.1016/j.nuclphysb.2010.08.008 [arXiv:1006.4505 [hep-th]].

New multisoliton solutions of the complex KdV equation.

A. Laptev, M. Levitin, F. Ferrulli
Imperial College London
Reading University

We present a class of multi-soliton solution for the complex KdV equation. To the best of our knowledge, this new class seems to have not been noticed in literature. The solutions are obtained by means of an extension of the Darboux method to complex wavenumbers.

Consider complex wavenumbers $k_1, k_2, \dots, k_i$, then our solutions are of the form

$$U_i(x,t) = \frac{d^2}{dx^2}\Big(\ln(W(f(x,t,k_1), g(x,t,\bar{k}_1), \dots, f(x,t,k_i), g(x,t,\bar{k}_i))\Big)$$

$$f(x,t,k) = \cosh\left(\frac{k}{2}(x-k^2t)\right), \quad g(x,t,k) = \sinh\left(\frac{\bar{k}}{2}(x-\bar{k}^2t)\right)$$

where $W(\cdot,\dots,\cdot)$ denotes the usual Wronskian.

These complex solutions exist for all real times t, and their real and imaginary parts are both bounded for all real x. Depending on the localisation of the spectral parameter in the complex plane, these waves exhibit different qualitative behaviour: they can travel in both directions along the real line with different speeds. Interesting phenomena like phase-shift and formation of extreme peaks can be observed during the interaction time. In the special case $\mathrm{Re}(k) = \pm\sqrt{3}\,\mathrm{Im}(k)$, the waves are stationary.

Regarded as potential in the spectral problem at time zero in the Lax's formulation, these solitons exhibit interesting properties in terms of energy transmission. In this case, in fact, it is possible to deduce precise values of the transmission and reflection coefficients.

References

[1] V.B. Matveev, M.A. Salle. Darboux transformations and solitons. Springer-Verlag (1991).

[2] M. Jaworski. Breather-like solution of the Korteweg-de Vries equation. Physics Letters A, Vol. 104, 5, Elsevier(1984), 245-247.

Travelling waves and a fruitful 'time' reparametrization in relativistic electrodynamics

Gaetano Fiore
Dip. di Matematica e Applicazioni, Università di Napoli "Federico II",
& INFN, Sez. di Napoli, Complesso MSA, Via Cintia, 80126 Napoli, Italy

We simplify [1] the nonlinear equations of motion of charged particles in an external electromagnetic field that is the sum of a plane travelling wave $F_t^{\mu\nu}(ct{-}z)$ and a static part $F_s^{\mu\nu}(x,y,z)$: by adopting the light-like coordinate $\xi = ct{-}z$ instead of time t as an independent variable in the Action, Lagrangian and Hamiltonian, and deriving the new Euler-Lagrange and Hamilton equations accordingly, we make the unknown $z(t)$ disappear from the argument of $F_t^{\mu\nu}$. We study and solve first the single particle equations in few significant cases of extreme accelerations. In particular we obtain a rigorous formulation of a *Lawson-Woodward*-type (no-final-acceleration) theorem and a compact derivation of *cyclotron autoresonance*, beside new solutions in the presence of uniform $F_s^{\mu\nu}$. We then extend our method to plasmas in hydrodynamic conditions and apply it to plane problems: the system of partial differential equations may be partially solved and sometimes even completely reduced to a family of decoupled systems of ordinary ones; this occurs e.g. with the impact of the travelling wave on a vacuum-plasma interface (what may produce the *slingshot effect*).

Since Fourier analysis plays no role in our general framework, the method can be applied to all kind of travelling waves, ranging from almost monochromatic to socalled "impulses", which contain few, one or even no complete cycle.

References

[1] G. Fiore, Travelling waves and a fruitful 'time' reparametrization in relativistic electrodynamics. arXiv:1607.03482; and references therein.

On Algebro–Geometric Approach to Semi–Hamiltonian Systems of Hydrodynamic Type

E. V. Glukhov
Faculty of Mechanics and Mathematics, Moscow State University

An algebro–geometric approach to semi–Hamiltonian systems of hydrodynamic type is presented. Semi–Hamiltonian systems of hydrodynamic type compose a class of integrable diagonalizable hydrodynamic type systems. There is a correspondence between such systems and orthogonal curvilinear coordinate systems in spaces of diagonal curvature. The construction introduced in the work is based on generalized Krichever construction for orthogonal coordinate systems in flat spaces. Formulae for coefficients of semi–Hamiltonian systems of hydrodynamic type is obtained in terms of Baker–Akhiezer functions on a spectral curve. It is also proved that Baker–Akhiezer function values at each point of a spectral curve are first integrals of the constructed system.

The work is supported by Russian Science Foundation under grant 16-11-10260

References

[1] B. A. Dubrovin, S. P. Novikov, Hamiltonian formalism of one-dimensional systems of hydrodynamic type and the Bogolyubov-Whitham averaging method, Dokl. Akad. Nauk SSSR, 1983, v. 270, N 4, 781–785.

[2] S. Tsarev, The geometry of Hamiltonian systems of hydrodynamic type: The generalized hodograph method, Math. USSR-Izv. 37 (1991), 397–419.

[3] I. M. Krichever, Algebraic-geometric n-orthogonal curvilinear coordinate systems and solutions of the associativity equations, Funct. Anal. Appl., 31(1), 25–39 (1997)

[4] A. E. Mironov, I. A. Taimanov, Orthogonal curvilinear coordinate systems that correspond to singular spectral curves. (Russian) Tr. Mat. Inst. Steklova 255 (2006), Funkts. Prostran., Teor. Priblizh., Nelinein. Anal., 180–196; translation in Proc. Steklov Inst. Math. 2006, no. 4 (255), 169–184

[5] D. A. Berdinskii, I. P. Rybnikov, On orthogonal curvilinear coordinate systems in spaces of constant curvature. (Russian) Sibirsk. Mat. Zh. 52 (2011), no. 3, 502–511; translation in Sib. Math. J. 52 (2011), no. 3, 394–401

Basic deformed q-calculus and quantum mechanics

A. Lavagno
Department of Applied Science and Technology,
Politecnico di Torino, Torino, Italy and
INFN, Sezione di Torino, Italy

Starting on the basic deformed calculus and q-symmetric oscillator algebra [1, 2], we study the q-deformed adjoint and q-hermitian operator properties in a basic square-integrable space. In this framework, we introduce a generalized deformed Schrödinger equation which can be viewed as the quantum stochastic counterpart of a generalized classical kinetic equation. Such equation admits factorized time-space solutions and the free plane wave functions can be expressed in terms of the so-called basic-hypergeometric functions.

References

[1] L. Biedenharn, J. Phys. A 22 (1989) L873.

[2] R. J. Finkelstein, Int. J. Mod. Phys. A 13 (1998) 1795.

[3] H. Exton, *q-hypergeometric functions and applications*, (1983) Ellis Horwood Ltd, Publisher.

Localized-wave solutions for the nonlocal Davey-Stewartson I equation via Darboux transformation

Min Li[1], Tao Xu[2] and Yehui Huang[1]

1. Department of Mathematics and Physics, North China Electric Power University, Beijing 102206, China.

2. College of Science, China University of Petroleum, Beijing 102249, China.

Recently, much attention has been paid to the nonlocal integrable equations, e.g., the $\mathcal{PT}$-symmetric nonlocal nonlinear Schrödinger model [1,2,3]. In this poster, we study the nonlocal Davey-Stewartson (DS) I equation which was proposed by Fokas [4]. Based on the Darboux transformation, we derive several new types of localized-wave solutions from the zero seed solution, including the single-dromion solution, periodic-dromion solution and line-soliton solution with local oscillation. We give the nonsingular parametric conditions for the three types of localized-wave solutions, and discuss the propagation characteristics of those localized waves. It's interesting to find that there exists a breather-like wave in the center of the periodic dromion wave, while for the line soliton, an local oscillation wave arises in the center. In addition, we also examine the modulation instability for the nonlocal DS I equation.

References

[1] M. J. Ablowitz and Z. H. Musslimani, Phys. Rev. Lett. **110**, 064105 (2013).

[2] M. J. Ablowitz and Z. H. Musslimani, Phys. Rev. E **90**, 032912 (2014).

[3] M. Li and T. Xu, Phys. Rev. E **91**, 033202 (2015).

[4] A. S. Fokas, Nonlinearity **29**, 319 (2016).

An integrable self-adaptive moving mesh scheme for the modified short pulse equation

Ken-ichi Maruno
Department of Applied Mathematics, Waseda University
3-4-1 Okubo, Shinjuku-ku, Tokyo 169-8555 Japan

Self-adaptive moving mesh schemes have been recently developed for various nonlinear wave equations with solutions having singularities or multivaluedeness which are derived from the Wadati-Konno-Ichikawa (WKI) type linear system[2]. It is known that soliton equations in the WKI class are transformed into certain soliton equations which are derived from the AKNS type linear system via hodograph transformations. A key of construction of self-adaptive moving mesh schemes for nonlinear wave equations in the WKI class is discrete hodograph transformations and discrete conservation laws. An important property of self-adaptive moving mesh schemes is that mesh intervals are conserved densities for discrete conservation laws.

In this presentation, we propose an integrable self-adaptive moving mesh schemes for the modified short pulse (mSP) equation[3]

$$u_{xt} = u + \frac{1}{2} u \left(u^2\right)_{xx} . \tag{1}$$

It is known that the mSP equation has cusped soliton solutions. Note that the original short pulse equation $u_{xt} = u + \frac{1}{6}\left(u^3\right)_{xx}$ has loop soliton solutions and does not have cusped soliton solutions[1]. We construct an integrable self-adaptive moving mesh scheme for the mSP equation by using bilinear equations of the mSP equation and the hodograph transformation. Our integrable self-adaptive moving mesh scheme can be used for numerical simulations of cusped solitons of the mSP equation. We show some examples of numerical simulations and the accuracy of self-adaptive moving mesh scheme.

The contents in this talk are based on the joint work with Prof. Bao-Feng Feng, Prof. Yasuhiro Ohta and Mr. Shuntei Jo.

References

[1] T. Schäfer and C. E. Wayne, "Propagation of ultra-short optical pulses in cubic nonlinear media", Physica D **196** (2004) 90-105.

[2] B-F. Feng, K. Maruno, and Y. Ohta, "Self-adaptive moving mesh schemes for short pulse type equations and their Lax pairs" Pacific J. Math. for Industry **6** (2014) 1–14.

[3] S. Sakovich, "Transformation and integrability of a generalized short pulse equation" **39** (2016) 21–28.

Multi-Dimensional Conservation Laws for Integrable Systems

Z. V. Makridin[1,2], M. V. Pavlov[2]
Lavrentyev Institute of Hydrodynamics SB RAS,
Novosibirsk State University

We introduce and investigate a new phenomenon in the Theory of Integrable Systems — the concept of multi-dimensional conservation laws for two- and three-dimensional integrable systems.

Existence of infinitely many local two-dimensional conservation laws is a well-known property of two-dimensional integrable systems.

We show that pairs of commuting two-dimensional integrable systems possess infinitely many three-dimensional conservation laws.

Examples: the Benney hydrodynamic chain, the Korteweg de Vries equation.

Simultaneously three-dimensional integrable systems (like the Kadomtsev — Petviashvili equation) have infinitely many three-dimensional conservation laws.

The method is based on introducing of auxiliary quasi-local variables (moments). It allows us to construct infinitely many multi-dimensional conservation laws depending on an arbitrary number of independent variables, which is higher time variable for commuting flows of each integrable hierarchy.

We illustrate our approach considering the dispersionless limit of the Kadomtsev — Petviashvili equation and the Mikhalëv equation.

Applications in three-dimensional case: In the theory of shock waves and the Whitham averaging approach.

Third-order superintegrable systems with potentials satisfying only nonlinear equations

Antonella Marchesiello, Sarah Post, Libor Šnobl
Faculty of Information Technology,
Czech Technical University in Prague.
Department of Mathematics,
University of Hawai'i at Mānoa.
Faculty of Nuclear Sciences and Physical Engineering,
Czech Technical University in Prague.

The conditions for superintegrable systems in two-dimensional Euclidean space admitting separation of variables in an orthogonal coordinate system and a functionally independent third-order integral are studied. In particular, we consider scalar Hamiltonians on 2D Euclidean space defined by

$$H = \frac{1}{2}(p_1^2 + p_2^2) + V(x_1, x_2), \tag{1}$$

where either p_j is the conjugate momentum for classical mechanics or $p_j = -i\hbar\partial_{x_j}$ for a quantum mechanical operator. Furthermore, we suppose the Hamiltonian (1) admits an integral of the motion that is a third order polynomial in the momenta.

Each coordinate system for which the Helmholtz (zero-potential) equation admits separation of variables is determined by a second-order linear operator that commutes with the Laplacian. Up to conjugation by elements of the Euclidean group e_2, the second order operators are

$$p_1^2, \quad L_3^2, \quad p_1 L_3, \quad L_3^2 + c p_1^2, \tag{2}$$

$L_3 = x_1 p_2 - x_2 p_1$ being the generator of rotation. The first two, p_1^2 and L_3^2 are second-order invariants of the two, inequivalent maximal Lie subalgebras of e_2, namely the subalgebra generated by $\{p_1, p_2\}$ and $\{L_3\}$ respectively. Thus, these coordinates, Cartesian and polar respectively, are referred to as subgroup type coordinates.

The goal of this work is to review the previous results concerning systems that admit a third-order integral and are separable in Cartesian [1], polar [4] and parabolic coordinates [3] as well as to present new results for the elliptic case [2]. We shall show that "nonlinear" superintegrable potentials, i.e. potentials depending on solutions of non-linear ODEs including Painlevé transcendents and elliptic functions, are present only in the subgroup type coordinate. For the other two coordinate systems all potentials satisfy some set of linear ODEs. As a consequence, systems separating in parabolic or elliptic coordinates can have potentials with only non-movable singularities. The mechanism for this disparity is still not well understood and so, in the conclusions, we offer some conjectures and open problems suggested by these results.

References

[1] S. Gravel, P. Winternitz. Superintegrability with third-order integrals in quantum and classical mechanics, J. Math. Phys. 46 (2002), 5902–5912.

[2] A. Marchesiello, S. Post, L. Šnobl. Third-order superintegrable systems with potentials satisfying only nonlinear equations, J. Math. Phys. 56 (2015), 102104, 11 pp.

[3] I. Popper, S. Post, P. Winternitz. Third-order superintegrable systems separable in parabolic coordinates, J. Math. Phys. 53 (2012), 2105, 20 pp.

[4] F. Tremblay, P. Winternitz. Third order superintegrable systems separating in polar coordinates, J. Phys. A: Math. Theor. 43 (2010) p. 175206 (2010), 17 pp.

Skyrmion States in Chiral Liquid Crystals

L. Martina, G. De Matteis, V. Turco
Dip. Matematica e Fisica Università del Salento - Sez. INFN di Lecce
Dept. of Mathematics, Physics and Electrical Engineering,
Northumbria University, Newcastle Upon Tyne,
Scuola Superiore ISUFI, Università del Salento

Within the framework of Oseen-Frank theory, we analyze the static configurations for chiral liquid crystals. In particular, we find numerical solutions for localized axisymmetric states in confined chiral liquid crystals with weak homeotropic anchoring at the boundaries. These solutions describe the distortions of two-dimensional skyrmions, known as either *spherulites* or *cholesteric bubbles*, which have been observed experimentally in these systems. Relations with nonlinear integrable equations have been outlined and are used to study asymptotic behaviours of the solutions. By using analytical methods, we build approximated solutions of the equilibrium equations and we analyze the generation and stabilization of these states in relation to the material parameters, the external fields and the anchoring boundary conditions.

References

[1] A. O. Leonov, I. E. Dragunov, U. K. Rößler, and A. N. Bogdanov. Theory of skyrmion states in liquid crystals. Phys. Rev. E 90 (2014) 042502.

[2] A. Bogdanov and A. Hubert. Thermodynamically stable magnetic vortex states in magnetic crystals. Journal of Magnetism and Magnetic Materials 138 (1994) 255-269.

[3] Sajedeh Afghah and Jonathan V. Selinger. Theory of helicoids and skyrmions in confined cholesteric liquid crystals. cond-mat.soft (2017).

[4] Jun-ichi Fukuda, Slobodan Žumer. Quasi-two-dimensional Skyrmion lattices in a chiral nematic liquid crystal. Nature Communications 2 (2011) 246.

On the discrete nonlinear Schrödinger equation with $\mathcal{PT}$-symmetry

Amal J. Mohammed
Department of Mathematical Sciences, University of Essex
Wivenhoe Park, Colchester CO4 3SQ (UK)

17-6-2017

In this poster we will present the inverse scattering transform for a nonlocal discrete nonlinear Schrödinger equation (NLS) with $\mathcal{PT}$-symmetry [1, 2].

This includes: the eigenfunctions (Jost solutions) of the associated Lax pair, the scattering data and the fundamental analytic solutions (FAS). Furthermore, we will present the derivation of 1- and 2-soliton solutions for the nonlocal discrete NLS equation.

Finally, we will outline the completeness relation for the Jost solutions and squared solutions of the nonlocal discrete NLS equation. This will allow one to interpret the inverse scattering transform as a generalised Fourier transform. This makes possible introducing the associated generating (recursion) operator and to describe the corresponding hierarchy of integrable equations [2, 3].

References

[1] M. J. Ablowitz, and Z. H. Musslimani, *Integrable discrete PT symmetric model*, Phys. Rev. E **90** (2014), 032912.

[2] M. J. Ablowitz, and Z. H. Musslimani, *Inverse scattering transform for the integrable nonlocal nonlinear Schrödinger equation*, Nonlinearity **29** (2016), 915–946.

[3] V. S. Gerdjikov and A. Saxena, *Complete integrability of Nonlocal Nonlinear Schrödinger equation*, J. Math. Phys. **58** (2017) 013502 (33pp.).

On Multi-Component Nonlinear Schrödinger Equation with $\mathcal{PT}$-Symmetry

Junaid I. Mustafa
Department of Mathematical Sciences, University of Essex
Wivenhoe Park, Colchester CO4 3SQ (UK)

June 7, 2017

In this poster we will present the inverse scattering transform for multi-component generalisations of nonlinear Schrödinger (NLS) equation with $\mathcal{PT}$-symmetry related to symmetric spaces of **A.III**-type and **BD.I**-type [1, 2]. This includes the Manakov vector Schrödinger equation (related to **A.III**-symmetric spaces) and the multi-component NLS equations of Kullish-Sklyanin type (related to **BD.I**-symmetric spaces).

This includes: the spectral properties of the associated Lax operator, Jost function, the scattering matrix and the minimal set of scattering data, the fundamental analytic solutions. Furthermore, the 1- and 2-soliton solutions are found by using the Zakharov-Shabat dressing method. It is shown, that the multi-component NLS equations of these types allow both regular and singular soliton configurations.

Finally, we will outline the completeness relation for the Jost solutions and squared solutions of the nonlocal multi-component NLS equation. This will allow one to interpret the inverse scattering transform as a generalised Fourier transform. This makes possible introducing the associated generating (recursion) operator and to describe the corresponding hierarchy of integrable equations [3, 4].

References

[1] M. J. Ablowitz, and Z. H. Musslimani, *Integrable nonlocal nonlinear Schrödinger equation*, Phys. Rev. Lett. **110** (2013), 064105.

[2] A. P. Fordy and P. P. Kulish, *Nonlinear Schrödinger equations and simple Lie algebras*, Commun. Math. Phys. **89** (1983), 427–443.

[3] M. J. Ablowitz, and Z. H. Musslimani, *Inverse scattering transform for the integrable nonlocal nonlinear Schrödinger equation*, Nonlinearity **29** (2016), 915–946.

[4] V. S. Gerdjikov and A. Saxena, *Complete integrability of Nonlocal Nonlinear Schrödinger equation*, J. Math. Phys. **58** (2017) 013502 (33pp.).

Emergent Berezinskii-Kosterlitz-Thouless phase in low-dimensional ferroelectrics

Yousra Nahas[1], Sergei Prokhorenko[1], Igor Kornev[2], Laurent Bellaiche[1]
[1] University of Arkansas, Fayetteville, AR 72701, USA
[2] Laboratoire SPMS, Centrale Supelec, France

Low-dimensional systems with continuous symmetry and with short-range interactions are substantially affected by diverging fluctuations that lead to a lack of long-range ordering at any finite temperature [1, 2]. Particularly, the peculiarity and unique character of dimension two, embodied in the Berezinskii-Kosterlitz-Thouless (BKT) phenomenology and captured by the two-dimensional XY-model [3], resides in the topological nature of the phase transition it supports. The question of whether ferroelectrics in thin film morphology would allow for a subtle topological manifestation has so far remained eluded. Here we show, using first-principles-based simulations merging a an effective Hamiltonian scheme [4] with scaling, symmetry, and topological arguments, that an overlooked BKT phase sustained by quasi-continuous symmetry emerges between the ferroelectric phase and the paraelectric one of the prototype of ferroelectrics, namely $BaTiO_3$, being under tensile strain. We find that this intermediate phase supports quasi-long-range order reflected in the algebraic decay of the correlation function and sustained by the existence of neutral bound pairs of vortices and antivortices, in accordance with defining characteristics of a BKT phase. Its lower and upper critical temperatures, are associated with the condensation and unbinding of vortex-antivortex pairs, respectively. Not only do these results provide an extension of BKT physics to the field of ferroelectrics, but also unveil their non-trivial critical behavior in low dimensions.

References

[1] Mermin, N.D. and Wagner H. Absence of ferromagnetism or antiferromagnetism in one- or two-dimensional isotropic Heisenberg Models. Phys. Rev. Lett. 17, 1113 (1966).

[2] Hohenberg, P.C. Existence of long-range order in one and two dimensions. Phys. Rev. 158, 383 (1967).

[3] Berezinskii, V.L. Destruction of long range order in one dimensional and two-dimensional systems with a continuous symmetry group. Sov. Phys. JETP, 34, 610 (1972). Kosterlitz, J.M. and Thouless, D.J. Ordering, metastability and phase transitions in two-dimensional systems. J. Phys. C: Solid State Physics 6, 1181 (1973).

[4] Walizer, L., Lisenkov, S. and Bellaiche, L. Phys. Rev. B 73, 144105 (2006)

Integrable two-layer spin systems with self-consistent potential

G. Nugmanova, Zh. Sagidullayeva, R. Myrzakulov
L.N. Gumilyov Eurasian National University, Astana, Kazakhstan

Integrable generalization with self-consistent potential for the Heisenberg ferromagnetic equation reads as

$$iS_t + \frac{1}{2}[S, S_{xx}] + \frac{1}{\omega}[S, W] = 0, \qquad (1a)$$

$$iW_x + \omega[S, W] = 0, \qquad (1b)$$

where $\omega = const$, $S = \Sigma_{j=1}^{3} S_j(x, y, t)\sigma_j$ is a matrix analogue of the spin vector, W - potential with the matrix form $W = \Sigma_{j=1}^{3} W_j(x, y, t)\sigma_j$, and σ_j are Pauli matrices. In work [1] was shown that spin system with self-consistent potentials (1) is gauge equivalent to the Schrödinger-Maxwell-Bloch (SMB) equations [2]

$$iq_t + q_{xx} + 2\delta|q|^2 q - 2ip = 0,$$

$$p_x - 2i\omega p - 2\eta q = 0,$$

$$\eta_x + \delta(q^* p + p^* q) = 0,$$

where q, p are complex functions, η is a real function, ω, δ are real contants ($\delta = \pm 1$). The Darboux transformation for equation (1) was constructed and some of its exact solutions were found [3].

In this presentation we show which two-layer spin system is associated with the two-layer SMB equation [4].

References

[1] R. Myrzakulov, G. Mamyrbekova, G. Nugmanova, M. Lakshmanan. Integrable Motion of Curves in Self-Consistent Potentials: Relation to Spin Systems and Soliton Equations. Phys. Lett. A 378 (2014), 2118 –2123.

[2] R. Myrzakulov, G. Mamyrbekova, G. Nugmanova, M. Lakshmanan. Integrable (2+1)-dimensional Spin Models with Self-Consistent Potentials. Symmetry. 7 (2015), 1352-1375.

[3] Z. Yersultanova, M. Zhassybayeva, G. Mamyrbekova, G. Nugmanova, R. Myrzakulov. Darboux Transformation and Exact Solutions of the Integrable Heisenberg Ferromagnetic Equation with Self-Consistent Potential. Int. J. Geom. Meth. in Mod. Phys. 12 (2015), 1550134.

[4] Xiao-Yu Wu, Bo Tian, Hui-Ling Zhen, Wen-Rong Sun and Ya Sun. Solitons for the (2+1)-dimensional nonlinear Schrödinger-Maxwell-Bloch equations in an erbium-doped fibre. J. Mod. Opt. 2015, http://dx.doi.org/10.1080/09500340.2015.1086031.

Breaking integrability at the boundary: the inverse scattering method as a soliton detector

Robert Arthur[1], Patrick Dorey[1], Robert Parini[2]
[1] Department of Mathematical Sciences, Durham University, UK
[2] Department of Mathematics, University of York, UK

This poster, based on [1], explores the boundary scattering of sine-Gordon solitons with a non-integrable Robin boundary,

$$u_x(t,0) + 2ku(t,0) = 0 \tag{1}$$

where k is a real parameter. The soliton content of the field produced by the collision is analysed using a numerical implementation of the direct scattering problem associated with the inverse scattering method.

Depending on the initial soliton velocity and boundary parameter k the collision produces various combinations of solitons, anti-solitons and breathers (bound states of solitons and anti-solitons). As $k \to \infty$ or 0 the boundary becomes integrable and the transition between these two extremes is described. A highlight of this investigation is the discovery of an intricate resonance structure arising from the creation of an intermediate breather whose subsequent recollision with the boundary is highly dependent on the breather phase.

References

[1] R. Arthur, P. Dorey, R. Parini. Breaking integrability at the boundary: the sine-gordon model with robin boundary conditions. *Journal of Physics A*, 49(16):165205, 2016 (arXiv:1509.08448)

On Hamiltonian geometry of the associativity equations

O.I. Mokhov, N.A. Pavlenko
Lomonosov Moscow State University
Lomonosov Moscow State University

In the case of three primary fields, the associativity equations or the Witten–Dijkgraaf–Verlinde–Verlinde (WDVV) equations of the two-dimensional topological quantum field theory can be represented as integrable nondiagonalizable systems of hydrodynamic type (O.I. Mokhov, [1]). After that the question about the Hamiltonian nature of such hydrodynamic type systems arose. The Hamiltonian geometry of these systems essentially depends on the metric of the associativity equations (O.I. Mokhov and E.V. Ferapontov, [2]). There are examples of the WDVV equations which are equivalent to the hydrodynamic type systems with local homogeneous first-order Dubrovin–Novikov type Hamiltonian structures, and those which are equivalent to the hydrodynamic type systems without such structures.

O.I. Mokhov and the author have obtained the classification of existence of a local first-order Hamiltonian structure for the hydrodynamic type systems which are equivalent to the WDVV equations in the case of three primary fields. The results of O.I. Bogoyavlenskij and A.P. Reynolds [3] for the three-component nondiagonalizable hydrodynamic type systems are essentially used for the solution of this problem. The results of classification will be presented.

The work is supported by Russian Science Foundation under grant 16-11-10260.

References

[1] O.I. Mokhov. Symplectic and Poisson structures on loop spaces of smooth manifolds, and integrable systems. Russian Mathematical Surveys. 53(3) (1998), 515–622.

[2] E.V. Ferapontov, O.I. Mokhov. The associativity equations in the two-dimensional topological field theory as integrable Hamiltonian nondiagonalizable systems of hydrodynamic type. Functional Anal. Appl. 30(3) (1996), 195–203. arXiv: hep-th/9505180.

[3] O.I. Bogoyavlenskij, A.P. Reynolds. Criteria for Existence of a Hamiltonian Structure. Regular and Chaotic Dynamics. 15(4–5) (2010), 431–439.

Fluctuations and topological defects in proper ferroelectrics

S. Prokhorenko, Y. Nahas, L. Bellaiche
Physics Department and Institute for Nanoscience and Engineering, University of Arkansas

Topologically nontrivial dipolar patterns like vortices, hedgehogs (monopoles) and skyrmions are not commonly expected to appear in bulk ferroelectric materials. Indeed, unlike ferroelectric nanostructures or relaxors, bulk ferroelectrics exhibit neither depolarizing nor random local fields that can render topological defects energetically favorable. Furthermore, polar bulks also appear to fall short of alternative, "topological", defect stabilization mechanisms as a result of inherent discrete symmetries of these systems.

In this study, we combine homotopy theory and effective Hamiltonian approach to explore stability of topological defects in bulk BaTiO3. Our results show that, against theoretical expectations, this proper ferroelectric material can exhibit stable topological point defects in its tetragonal polar phase and stable topological line defects in its orthorhombic polar phase. The stability of such defects originates from a novel mechanism of topological protection related to finite-temperature fluctuations of local dipoles. Large-scale effective Hamiltonian Monte Carlo simulations are then conducted to confirm these theoretical predictions. The results of our work, hence, reveal a novel mechanism of topological protection that can be realized in proper ferroelectrics and provide a theoretical framework for investigations of topological defects in systems with finite underlying symmetries.

References

[1] . Prokhorenko, Y. Nahas, L. Beallaiche. Fluctuations and Topological Defects in Proper Ferroelectric Crystals. Phys. Rev. Lett. 118 (2017), 147601.

Conservation and Balance Laws by means of Mixed Method

M. Ruggieri, M. P. Speciale
Faculty of Engineering and Architecture,
Kore University of Enna, Italy,
email: marianna.ruggieri@unikore.it

Department of Mathematics and Computer Science,
Physical Sciences and Earth Sciences, University of Messina,
University of Messina, Italy,
email: mpspeciale@unime.it

Recently the authors have introduced a mixed approach to construct some conservation laws of partial differential equations [1], the method merges the Ibragimov method and the one by Anco and Bluman [2]-[5].

In this note, our aim is to use the mixed approach to determine a link between balance and conservation laws for a system of $\overline{m}$ partial differential equations of order k,

$$F_{\overline{\alpha}}(\mathbf{x}, \mathbf{u}, \mathbf{u}_{(1)}, \mathbf{u}_{(2)}, \ldots, \mathbf{u}_{(k)}) = 0, \qquad \overline{\alpha} = 1, \ldots, \overline{m}, \tag{1}$$

with m dependent variables $u = (u^1, \ldots, u^m)$ and n independent variables $x = (x^1, \ldots, x^n)$, starting from conservation laws.

In particular, since many physical problems are mathematically modeled by nonautonomous and/or nonhomogeneous systems of PDE and the possibility of reducing the system into autonomous and/or homogeneous form is related to the Lie symmetry properties of the model under investigation [6], our purpose is to link the conserved quantities of an autonomous and/or a homogeneous system to the conserved or non conserved quantities of the nonautonomous and/or nonhomogeneous system obtained by Lie invertible transformation.

References

[1] M. Ruggieri, M. P. Speciale, Conservation laws for a model derived from two layer-fluids, Journal of Physics: Conference Series 482, 012037, 10 (2014).

[2] S. C. Anco, G. Bluman, Direct construction method for conservation laws of partial differential equations I: Examples of conservation law classifications, Euro. J. Appl. Math. 13, 545-566 (2002).

[3] S. C. Anco, G. Bluman, Direct construction method for conservation laws of partial differential equations II: General treatment, Euro. J. Appl. Math. 13, 567-585 (2002).

[4] N. H. Ibragimov, A new conservation theorem, J. Math. Anal. Appl. 333, 311–328 (2007).

[5] N. H. Ibragimov, Construction of Conservation Laws Using Symmetries, Springer (2014).

[6] F. Oliveri, General dynamical systems described by first order quasilinear PDEs reducible to homogeneous and autonomous form, International Journal of Non-Linear Mechanics 47, 5360 (2012).

Exact solutions of integrable spin system

Zh. Sagidullayeva, G. Nugmanova,
L.N. Gumilyov Eurasian National University, Astana, Kazakhstan

Integrable nonlinear differential equations admit soliton or soliton-like solutions. The research of solitons and related solutions has become one of the active fields of research in the field of physics and mathematics. Among integrable systems, the Heisenberg ferromagnet equation plays an important role in physics and mathematics.

In this paper, we consider an integrable hierarchy Heisenberg ferromagnet equation in the following form [1]

$$S_t - S_{xxx} - \frac{3}{4}tr(S_x^2)S_x - \frac{3}{4}[tr(S_x^2)]_x S = 0, \tag{1}$$

where $S = \Sigma_{j=1}^3 S_j(x,t)\sigma_j$ is a matrix analogue of the spin vector $\mathbf{S}$, and

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

are Pauli matrices. The components of the matrix function S satisfy the boundary condition $S(x,t)|_{x\longrightarrow\pm\infty} = (0,0,\pm1)$.

We construct a bilinear form for equation (1) and using Hirota's direct method find one-soliton solution. Further, using obtained solutions, we can recursively construct two-, three-, and n-soliton solutions that have specific applications in the natural sciences.

References

[1] G. Nugmanova, A. Azimkhanova Integrable hierarchies of Heisenberg ferromagnet equation, Journal of Physics: Conference Series 738 (2016) 012132.

MODELLING SOLUTIONS TO THE KdV-BURGERS EQUATION IN THE CASE OF NONHOMOGENEOUS DISSIPATIVE MEDIA

Alexey Samokhin
Moscow State Technical University of Civil Aviation

The behavior of the soliton type solutions to the KdV - Burgers equation is studied numerically in the case of nonhomogeneous dissipative media. A number of publications dealt with a similar problems recently, [1, 2]

The aim of the presented research is to study the behavior of the soliton that, while moving in nondissipative medium encounters a barrier (finite or infinite) with finite constant dissipation. The modelling included the case of a finite dissipative layer similar to a wave passing through the air-glass-air as well as a wave passing from a nondissipative layer into a dissipative one (similar to the passage of light from air to water).

The continuation of the author's publications [3, 4] is presented. New results include a numerical model of the wave's behavior for different types of the media non-homogeneity. The dissipation predictably results in reducing the soliton amplitude, but some new effects occur in the case of finite piecewise constant barrier on the soliton path: after the wave leaves the dissipative barrier it retains, on the whole, a soliton form yet a reflection wave arises as small and quasi-harmonic oscillations. These oscillations and spread as the soliton is moving through the barrier. The reflection oscillations are faster than the soliton. The modelling used the Maple software *PDETools* packet.

References

[1] A.P. Chugainova, V.A. Shargatov, Stability of the breaks structure described by the generalized Kortweg-de Vries-Burgers equation, Computational Math and Math Phys. 56:2 (2016), 259–274.

[2] R.L. Pego, P. Smereka, M.I. Weinstein. Oscillatory instability of traveling waves for a KdV-Burgers equation // Physica D. 1993. V. 67. P. 45-65.

[3] Samokhin A.V., On nonlinear superposition of the KdV-Burgers shock waves and the behavior of solitons in a layered medium.// Journal of Differential Geometry and its Applications. In press: available online 9 March 2017. 10p. http://dx.doi.org/10.1016/j.difgeo.2017.03.001

[4] Samokhin A.V., The Burgers equation with periodic boundary conditions on an interval // Theor. Math. Phys, **188**(3), pp. 1371–1376.

MODELLING SOLUTIONS TO THE KdV-BURGERS EQUATION IN THE CASE OF NONHOMOGENEOUS DISSIPATIVE MEDIA

Alexey Samokhin
Moscow State Technical University of Civil Aviation

The behavior of the soliton type solutions to the KdV - Burgers equation is studied numerically in the case of nonhomogeneous dissipative media. A number of publications dealt with a similar problems recently, [1, 2]

The aim of the presented research is to study the behavior of the soliton that, while moving in nondissipative medium encounters a barrier (finite or infinite) with finite constant dissipation. The modelling included the case of a finite dissipative layer similar to a wave passing through the air-glass-air as well as a wave passing from a nondissipative layer into a dissipative one (similar to the passage of light from air to water).

The continuation of the author's publications [3, 4] is presented. New results include a numerical model of the wave's behavior for different types of the media non-homogeneity. The dissipation predictably results in reducing the soliton amplitude, but some new effects occur in the case of finite piecewise constant barrier on the soliton path: after the wave leaves the dissipative barrier it retains, on the whole, a soliton form yet a reflection wave arises as small and quasi-harmonic oscillations. These oscillations and spread as the soliton is moving through the barrier. The reflection oscillations are faster than the soliton. The modelling used the Maple software *PDETools* packet.

References

[1] A.P. Chugainova, V.A. Shargatov, Stability of the breaks structure described by the generalized Kortweg-de Vries-Burgers equation, Computational Math and Math Phys. 56:2 (2016), 259–274.

[2] R.L. Pego, P. Smereka, M.I. Weinstein. Oscillatory instability of traveling waves for a KdV-Burgers equation // Physica D. 1993. V. 67. P. 45-65.

[3] Samokhin A.V., On nonlinear superposition of the KdV-Burgers shock waves and the behavior of solitons in a layered medium.// Journal of Differential Geometry and its Applications. In press: available online 9 March 2017. 10p. http://dx.doi.org/10.1016/j.difgeo.2017.03.001

[4] Samokhin A.V., The Burgers equation with periodic boundary conditions on an interval // Theor. Math. Phys, **188**(3), pp. 1371–1376.

Integrable Liénard–type equations and nonlocal transformations

Dmitry I. Sinelshchikov
National Research Nuclear University MEPhI

We consider the following family of ordinary differential equations

$$y_{zz} + f(y)y_z^2 + g(y)y_z + h(y) = 0,$$

where f, g and h are arbitrary functions. This family of equations contains both the classical Liénard equation (the case of $f(y) = 0$) and the quadratic Liénard equation (the case of $g(y) = 0$) and is often called the Liénard–type equation. This equation is widely used for the description of various processes and phenomena in physics, biology and mechanic (see, e.g. [1, 2]). For instance, traveling wave reductions of reaction-diffusion systems, the Rayleigh equation for spherical bubbles dynamics and the Van–der–Pol equation all belong to the family of Liénard–type equations.

While various attempts to find general analytical solutions of the Liénard–type equations have been made, there is still no complete answer to the question of integrability of this equation. In this talk, we discuss a recent approach (see, [3, 4, 5]) for finding integrable families of the Liénard–type equations. The main idea of this approach is to find connections, which are given by the generalized Sundman transformations, between the Liénard–type equations and the Painlevé–Gambier equations. As a result of application of this approach we find new integrable families of the Liénard–type equations. Note that under integrability we understand a possibility to find the general closed form solution of a representative of this family of equations.

First we consider the case of the quadratic Liénard equations, that is the case of $g(y) = 0$. It can be shown that the family of quadratic Liénard equations is closed with respect to the generalized Sundman transformations. This fact allows us to construct the general solutions of the quadratic Liénard equations for arbitrary functions $g(y)$ and $h(y)$ by transforming them into integrable subcases of this family of equations. We show that as such integrable subcases of the quadratic Liénard equations it is efficient to use equations of this type, which are solvable in term of the elliptic functions. We demonstrate applications of this approach by constructing the general closed form solutions of the Rayleigh equation for spherical bubble dynamics, traveling wave reductions of the K(m,n) equations and some other nonlinear differential equations.

Then, we consider the general case of the Liénard–type equations, assuming that $g(y) \neq 0$. Studying connections between the family of Liénard–type equations and the Painlevé–Gambier equations of classes I, II and III we obtain several new integrability conditions (i.e. correlations on functions f, g and h, which ensure integrability) for this family of equations. We demonstrate effectiveness of this approach by constructing several new integrable Liénard–type equations and their general solutions. We also demonstrate that some of the previously known integrability conditions for the Liénard–type equations follows from the linearizability of the corresponding equations via the generalized Sundman transformations. Finally, we demonstrate that this approach can be used for finding the Lagrangians and Jacobi multipliers for the Liénard–type equations. We find new Lagrangians and Jacobi last multipliers for some of the Painlevé–Gambier equations and extend this results to the Liénard–type equations with the help of the generalized Sundman transformations. In the same way we can also find new first integrals for the Liénard–type equations.

References

[1] A.A. Andronov, A.A. Vitt, S.E. Khaikin, Theory of Oscillators, Dover Publications, New York, 2011.

[2] V.F. Zaitsev, A.D. Polyanin, Handbook of Exact Solutions for Ordinary Differential Equations, Chapman and Hall/CRC, Boca Raton, 2002.

[3] N.A. Kudryashov, D.I. Sinelshchikov, On the criteria for integrability of the Liénard equation, Appl. Math. Lett. 57 (2016), 114–120.

[4] N.A. Kudryashov, D.I. Sinelshchikov, On the integrability conditions for a family of Liénard-type equations, Regul. Chaotic Dyn. 21 (2016), 548–555

[5] N.A. Kudryashov, D.I. Sinelshchikov, On connections of the Liénard equation with some equations of Painlevé–Gambier type, J. Math. Anal. Appl. 449 (2017), 1570–1580.

Elliptic diffeomorphisms of symplectic 4-manifolds

Vsevolod Shevchishin, Gleb Smirnov
Higher School of Economics (Moscow, Russia)
International School for Advanced Studies (Trieste, Italy)

The talk is devoted to symplectomorphism groups of 4-manifolds, topological methods in the algebraic geometry of rational and ruled surfaces, and applications of the pseudoholomorphic technique to symplectic topology.

In his PhD thesis, Paul Seidel found examples of symplectomorphisms which are smoothly isotopic to the identity, but not isotopic to the identity within the symplectomorphism group. These symplectomorphisms are known as Dehn twists, and they exist for those symplectic 4-manifolds which admit an embedded Lagrangian 2-sphere. Later Seidel proved that Dehn twists are of infinite order in the symplectic mapping class groups of certain symplectic K3 surfaces.

In spite of this deep result of Seidel, and other results that followed it, we still have no general way to construct non-isotopic symplectomorphisms for 4-manifolds. For instance, there are 4-manifolds which do not contain Lagrangian spheres, yet it is believed that they have a non-trivial symplectic mapping class group.

In this talk we will introduce and study a new type of symplectomorphisms for symplectic 4-manifolds and discuss some examples of symplectic 4-manifolds for which these new twists are infinite order elements in the symplectic mapping class groups.

References

[1] V. Shevchishin, G. Smirnov. Elliptic diffeomorphisms of symplectic 4-manifolds, **in preparation**

Helical solitons in the string model of classical particles

Nawin Raj, Yury Stepanyants
School of Agricultural, Computational and Environmental Sciences, University of Southern Queensland, Toowoomba, QLD, 4350, Australia.
Email of corresponding author: Yury.Stepanyants@usq.edu.au

As has been shown in the paper [4], flexural transverse long waves in an anharmonic chain of atoms can be described by the nonlinear vector equation

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} - c^2 \frac{\partial^2 \mathbf{u}}{\partial x^2} - a \frac{\partial^2 \left(|\mathbf{u}|^2 \mathbf{u}\right)}{\partial x^2} - b_1 \frac{\partial^4 \mathbf{u}}{\partial x^4} - b_2 \frac{\partial^6 \mathbf{u}}{\partial x^6} = 0, \tag{1}$$

where $\mathbf{u} = \partial \overrightarrow{\xi} / \partial x$, and $\overrightarrow{\xi}$ is the chain displacement in the direction perpendicular to x-axis; c, a, b_1, and b_2 are real constant coefficients. Depending on the value of these coefficients, equation (1) can be reduced to several different evolution equations; among them is the vector mKdV equation earlier derived in the context of chain particle model in Refs. [1, 2]:

$$\frac{\partial \mathbf{u}}{\partial t} + \alpha |\mathbf{u}|^2 \frac{\partial \mathbf{u}}{\partial x} + \beta \frac{\partial^3 \mathbf{u}}{\partial x^2} = -\alpha_1 \mathbf{u} \frac{\partial |\mathbf{u}|^2}{\partial x}. \tag{2}$$

Here $\alpha_1 = \alpha$, but it is convenient to use different notations for the coefficients, which allows us to consider in parallel another model with $\alpha_1 = 0$.

Equation (2) was derived for the first time as earlier as in 1975 for the lower hybrid plasma waves – see Ref. [3] and references therein. As was shown in Ref. [3], the equation is non-integrable in general, except the particular case when $\alpha_1 = 0$.

In the papers cited above solitary and periodic stationary solutions were found within the framework of equation (2). Some of them represent helical periodic waves, others represent plane solitary waves which can propagate along the chain at different angles with respect to each other. Each solitary wave performs a chain displacement in a certain plane only. The interactions of solitary waves are non-elastic in general, except the case when they are in the same plane; in such case the basic equation (2) reduces to the completely integrable scalar mKdV equation.

In this work we present the results of numerical study of interactions between plane solitary waves of different polarization, i.e. laying in different planes. We also consider a family of helical solitary waves and study the interaction between them and between plane and helical solitary waves. In the case of interactions between helical solitary waves we show that the result depends upon the direction and strength of helicity. Two different cases were studied in details, (i) when equation (2) is completely integrable ($\alpha_1 = 0$), and (ii) when it is non-integrable ($\alpha_1 = \alpha$).

The problem studied has numerous physical applications in plasmas, solid state physics, physics of alpha-spiral molecules, etc.

It is still a challenge to construct by means of the inverse scattering transform an exact analytical solution describing the interaction of helical solitons, at least in the integrable case and compare the results obtained with the numerical solutions of non-integrable equation. Another challenge is the finding of breather solutions, which apparently should exist within the framework of equation (2). This work is currently underway.

References

[1] M. Destrade, G. Saccomandi. Nonlinear transverse waves in deformed dispersive solids. Wave Motion 45 (2008), 325–336.

[2] O.B. Gorbacheva, L.A. Ostrovsky. Nonlinear vector waves in a mechanical model of a molecular chain. Physica D 8 (1983), 223–228.

[3] C.F. Karney, A. Sen, F.Y. Chu. Nonlinear evolution of lower hybrid waves. Phys. Fluids 22 (1979), 940–952.

[4] S.P. Nikitenkova, N. Raj, Y.A. Stepanyants. Nonlinear vector waves of a flexural mode in a chain model of atomic particles. Comm. Nonlin. Sci. Num. Simulation 20 (2015), 731–742.

Bifurcations in molecular vibrational modes under symmetry breaking isotopic substitutions: semi-classical and quantum analyses

Vladimir Tyuterev[1], Frédéric Mauguière[2], Roman Kochanov[3,4]
[1]GSMA, UMR 7331, CNRS, University of Reims, France
[2]School of Mathematics, University of Bristol, UK
[3]QUAMER, Tomsk State University, Russia
[4]Harvard University, Cambridge MA, USA

A complex shape of molecular potential energy surfaces near the dissociation threshold gives rise to strong coupling of quantum states of the nuclear motion resulting to drastic changes in the vibrational modes. In this work, we study the corresponding non-linear phenomena using two complementary approaches. The first one aims at understanding the quasi-classical dynamics in terms of bifurcation phenomena occurring for the periodic orbits in the phase space [1]. These bifurcations generate new types of vibrational modes emanating from the fundamental ones. We consider their stability using the monodromy theory. The study is carried out both in full and in reduced phase spaces corresponding to the polyad models involving the study of relative equilibria. On the other hand, we investigate the corresponding transformations of quantum wavefunction and their nodal lines at high energy. The particular focus is the investigation of the consequences of breaking the molecular symmetry by isotopic substitution in the classical and quantum dynamics. Such studies are theoretical prerequisites for interpretation of laser experiments on multi-photon molecular excitation [2] and on high-sensitivity spectroscopy near the dissociation limit for hydrogen bound OH $\Rightarrow$ OD substitutions. It is also mandatory for the understanding of the metastables states and the dynamics of molecular formation and fragmentation that is a long-standing issue for the ozone molecule [3], [4], [5].

We acknowledge the support from LEFE-Chat INSU CNRS program of France, and Mendeleev funding grant of Tomsk State University, Russia.

References

[1] F. Mauguiere, Vl. Tyuterev, S. Farantos. Bifurcation effects and patterns in the vibrational excited states of isotopically substituted water. Chem. Phys. Lett. 494 (2010), 163–169.

[2] M. Grechko, O. Boyarkin, T. Rizzo. State–selective spectroscopy of water up to its first dissociation limit. J. Chem. Phys. 131 (2009), 221105.

[3] R. A. Marcus. Theory of mass–independent fractionation of isotopes, phase space accessibility, and a role of isotopic symmetry. PNAS, 110 (2013), 17703–17707.

[4] Vl. Tyuterev, R. Kochanov, A. Campargue, et al. Does the "Reef Structure" at the Ozone Transition State towards the Dissociation Exist? New Insight from Calculations and Ultrasensitive Spectroscopy Experiments. Phys. Rev. Lett. 113 (2014), 143002.

[5] D. Lapierre, A. Alijah, R. Kochanov, V. Kokoouline, Vl. Tyuterev. Lifetimes and wave functions of ozone metastable vibrational states near the dissociation limit in a full-symmetry approach. Phys. Rev. A. 94 (2016), 042514.

Bi-Hamiltonian structures of KdV type

P. Lorenzoni[1], A. Savoldi[2], R. Vitolo[3]
[1]Department of Mathematics,
University of Milano Bicocca, Milano, Italy
[2]Department of Mathematical Sciences,
Loughborough University, Loughborough, UK
[3]Department of Mathematics and Physics "E. De Giorgi",
University of Salento and Sezione INFN, Lecce, Italy

Combining an old idea of Olver and Rosenau with the classification of second and third order homogeneous Hamiltonian operators (Ferapontov, Pavlov, V. 2014-2016) we classify compatible trios of two-component homogeneous Hamiltonian operators. The trios yield pairs of compatible bi-Hamiltonian operators whose structure is a direct generalization of the bi-Hamiltonian pair of the KdV equation. The bi-Hamiltonian pairs give rise to multi-parametric families of bi-Hamiltonian systems. We recover known examples and we find apparently new integrable systems. (Joint work with P. Lorenzoni, A. Savoldi.)

Preprint available at `https://arxiv.org/abs/1607.07020`

Systems of conservation laws with third-order Hamiltonian structures

EV Ferapontov[1], MV Pavlov[2], RF Vitolo[3]
[1]Department of Mathematical Sciences,
Loughborough University, Loughborough, UK
[2]Department of Mechanics and Mathematics,
Novosibirsk State University, Russia
[3]Department of Mathematics and Physics "E. De Giorgi",
University of Salento and Sezione INFN, Lecce, Italy

We investigate n-component systems of conservation laws that possess third-order Hamiltonian structures of differential-geometric type. Examples include equations of associativity of two-dimensional topological field theory (WDVV equations) and various equations of Monge-Ampère type. The classification of such systems is reduced to the projective classification of linear congruences of lines in P^{n+2} satisfying additional geometric constraints.

Preprint available at `https://arxiv.org/abs/1703.06173`

Fast Nonlinear Fourier Transforms

Sander Wahls
Delft Center for Systems and Control
TU Delft, The Netherlands

It was more than 20 years ago, when Hasegawa and Nyu proposed to apply the scattering transform of Zakharov and Shabat for the solution of the nonlinear Schrödinger equation to fiber-optic communcations [1, 2]. Their proposal did not receive much attention until a few years ago, when the idea resurfaced in the context of digital coherent transmission systems [3]. Efficient algorithms for the numerical computation of the forward and inverse scattering transform, which are also known as forward and inverse nonlinear Fourier transforms in this area, are a key element for the practical implementation for these schemes. In the last few years, algorithms for the forward and inverse transform were developed with complexities close to that of the celebrated fast Fourier transform (FFT) algorithm [4, 5, 6, 7]. The goal of this talk is to review these developments and to present the key ideas behind the algorithms.

References

[1] A. Hasegawa, T. Nyu. Eigenvalue communication. J. Lightwave Technol., vol. 11, pp, 395–399, 1993.

[2] V. E. Zakharov, A. B. Shabat. Exact theory of 2-dimensional self-focusing and one-dimensional self-modulation of waves in nonlinear media. Sov. Phys. J. Exp. Theor. Phys., vol. 34, pp. 62–69, 1972.

[3] S. K. Turitsyn, J. E. Prilepsky, S. T. Le, S. Wahls, L. L. Frumin, M. Kamalian, S. A. Derevyanko. Nonlinear Fourier transform for optical data processing and transmission: Advances and perspectives. Optica, vol. 4, no. 3, pp. 307–322, Mar. 2017.

[4] S. Wahls, H. V. Poor. Introducing the fast nonlinear Fourier transform. Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP), Vancouver, Canada, May 2013.

[5] S. Wahls, H. V. Poor. Fast Numerical Nonlinear Fourier Transforms. IEEE Transactions on Information Theory, vol. 61, no. 12, pp. 6957–6974, Dec. 2015.

[6] S. Wahls, H. V. Poor. Fast Inverse Nonlinear Fourier Transform For Generating Multi-Solitons In Optical Fiber. Proc. IEEE International Symposium on Information Theory (ISIT), Hong Kong, China, Jun. 2015.

[7] V. Vaibhav, S. Wahls. Introducing the Fast Inverse NFT. Proc. Optical Fiber Communication Conference (OFC), Los Angeles, CA, Mar. 2017.

Symbolic Dynamics and Topological Analysis of Diamagnetic Kepler Problem

Zuo-Bing Wu[1,2]
[1]State Key Laboratory of Nonlinear Mechanics,
Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190, China
[2]School of Engineering Science,
University of Chinese Academy of Sciences, Beijing 100049, China

The diamagnetic Kepler problem (DKP), i.e., a hydrogen atom moving in a uniform magnetic field, is a simple physical system for studying the correspondence relation between classical chaos and quantum behaviors. In the semiclassical quantization of a classically chaotic system, quantum properties are related to unstable periodic orbits. In this paper, firstly, symbolic dynamics of the DKP is established based on the lift of the annulus map, which is obtained from a Poincare section associated with the axes. The correspondence between the coding derived from this axis Poincare section is compared with the coding based on bounces. Symmetry is used to reduce the symbolic dynamics. By means of symbolic dynamics the admissibility of periodic orbits is analyzed, and the symmetry of orbits discussed. Then, a method to determine the admissibility of symbolic sequences and to find the unstable periodic orbits corresponding to allowed symbolic sequences for the DKP is proposed by using the ordering of stable and unstable manifolds. By investigating the unstable periodic orbits up to length 6, a one to one correspondence between the unstable periodic orbits and their corresponding symbolic sequences is shown under the system symmetry decomposition. Furthermore, a method to construct topological template in terms of symbolic dynamics for the DKP is proposed. To confirm the topological template, rotation numbers of invariant manifolds around unstable periodic orbits in a phase space are taken as an object of comparison. The rotation numbers are determined from the definition and connected with symbolic sequences encoding the periodic orbits in a reduced Poincare section. Only symbolic codes with inverse ordering in the forward mapping can contribute to the rotation of invariant manifolds around the periodic orbits. By using symbolic ordering, the reduced Poincare section is constricted along stable manifolds and a topological template, which preserves the ordering of forward sequences and can be used to extract the rotation numbers, is established. The rotation numbers computed from the topological template are the same as those computed from their original definition. This work was supported by the National Natural Science Foundation of China through the Grants Nos. 11172310 and 11472284.

References

[1] M. C. Gutzwiller. Chaos in classical and quantum mechanics. Springer-Verlag, New York, 1992.

[2] Z.-B. Wu, W.-M. Zheng. Symbolic dynamics of the diamagnetic kepler problem without involving bounced. Phys. Script. 59 (1999), 266–273.

[3] Z.-B. Wu, J.-Y. Zeng. A mehtod to find unstable periodic orbits for the diamagnetic kepler problem. Phys. Script. 61 (2000), 406–411.

[4] Z.-B. Wu. Rotation numbers of invariant manifolds around unstable periodic orbits for the diamagnetic kepler problem. Fract. 16 (2008), 11–23.

Soliton solutions of (1+1)-dimensional nonlocal nonlinear Schrodinger and Maxwell-Bloch equations

K. Yesmakhanova[1], G.Shaikhova[2], G. Bekova[2]

[1]Department of Higher Mathematics,
Eurasian National University, Astana, Kazakhstan
[2]Department of General and Theoretical Physics,
Eurasian National University, Astana, Kazakhstan

In this work, we present the (1+1)-dimensional nonlocal nonlinear Schrodinger and Maxwell-Bloch (SMB) equations:

$$iq_t(x,t) + q_{xx}(x,t) + 2q(x,t)q^*(-x,t)q(x,t) - 2p(x,t) = 0, \quad (1)$$

$$p_x(x,t) = 2[q(x,t)\eta(x,t) - i\omega p(x,t)], \quad (2)$$

$$\eta_x(x,t) = q(x,t)p^*(-x,t) - p(x,t)q^*(-x,t), \quad (3)$$

where $q,\ q^*,\ p,\ p^*$ are complex functions, η is real function and ω is constant. Subscripts x, t denote partial derivatives with respect to the variables.

If $p = 0, \eta = 0$, then the equations (1)-(3) reduce to nonlocal nonlinear Schrödinger equation of the type

$$iq_t(x,t) + q_{xx}(x,t) + 2q(x,t)q^*(-x,t)q(x,t) = 0. \quad (4)$$

The equation (4) is integrable [1, 2, 3]. Using the Lax pair of the equations (1)-(3), we construct the one-fold Darboux transformation and obtain the determinant representation of the N-fold Darboux transformation. Soliton solutions of the nonlocal nonlinear SMB equations are obtained by using the determinant representation of the Darboux transformation.

References

[1] M.Li-Yuan, Zh.Zuo-Nong. Nonlocal nonlinear Schrödinger equation and its discrete version: Soliton solutions and gauge equivalence. Journal of Mathematical Physics. 57, 083507 (2016).

[2] M.J. Ablowitz , Z.H. Musslimani. Integrable Nonlocal Nonlinear Schrödinger Equation. Physical Review Letters, 110, 064105 (2013).

[3] T. A. Gadzhimuradov , A. M. Agalarov. Towards a gauge-equivalent magnetic structure of the nonlocal nonlinear Schrödinger equation. Physical Review A, 93, 062124 (2016).

Lagrangian dynamics by nonlocal constants of motion

Gianluca Gorni, Gaetano Zampieri
Università di Udine
gianluca.gorni@uniud.it
Università di Verona
gaetano.zampieri@univr.it

Consider the Lagrange equation

$$\frac{d}{dt}\partial_{\dot{q}}L\big(t,q(t),\dot{q}(t)\big)-\partial_{q}L\big(t,q(t),\dot{q}(t)\big)=Q\big(t,q(t),\dot{q}(t)\big) \tag{1}$$

for smooth $L(t,q,\dot{q})\in R$, $Q(t,q,\dot{q})\in R^n$, with $t\in R$, $q,\dot{q}\in R^n$.

We show some applications of a simple technique to find constants of motion, along solutions of (1), of the nonlocal form

$$N\big(t,q(t),\dot{q}(t)\big)+\int_{t_0}^{t}M\big(s,q(s),\dot{q}(s)\big)ds\,. \tag{2}$$

The first example taken from [1] shows a genuine first integral (i.e. $M\equiv 0$) for $L=\frac{1}{2}|\dot{q}|^2-U(q)$ with U homogeneous of degree -2, in particular Calogero's potential, and $Q\equiv 0$.

Next, from [2], we find nonlocal constants of motion which give global existence and estimates for the solutions of the dissipative equation $\ddot{q}=-k\dot{q}-\partial_q U(q)$, when $k>0$ and $U:R^n\to R$ is bounded from below.

The third example, taken from [3], is a nonlocal constant of motion for the dissipative Maxwell-Bloch system for lasers in Lagrangian formulation $L=\frac{1}{2}\big(\|\dot{q}\|^2+(q_1^2+q_2^2)(\dot{q}_3-ab)\big)$, $Q=-\big((a+b)\dot{q}_1,(a+b)\dot{q}_2,a\big(q_1^2+q_2^2\big)+c(\dot{q}_3-k)\big)$, which leads to a nonstandard separation of variables whenever $c=2a$.

References

[1] G. Gorni, G. Zampieri. Revisiting Noether's theorem on constants of motion. *Journal of Nonlinear Mathematical Physics* 21 (2014), 43–73.

[2] G. Gorni, G. Zampieri. Nonlocal variational constants of motion in dissipative dynamics. Submitted by Jean Mawhin, *Differential and Integral Equations* 30 (2017), 631–640.

[3] G. Gorni, G. Zampieri. Nonlocal and nonvariational extensions of Killing-type equations. To appear in *Discrete and Continuous Dynamical Systems-Series S*.

Exact solution for the stationary shape of a conducting liquid jet deformed by a transverse electric field

Volkov N.B., Zubarev N.M., Zubareva O.V.
Institute of Electrophysics, UB RAS, Ekaterinburg, Russia

The electrostatic problem of finding possible equilibrium configurations of an uncharged cylindrical jet of a conducting fluid in a transverse electric field is considered. A cross-section of the jet placed between two flat electrodes is deformed by the electrostatic forces (the capillary forces play a stabilizing role). The main difficulty in finding solutions to the problem consists in the necessity to calculate the electric field distribution around the jet with unknown boundary, which is determined by the essentially nonlinear condition of balance between electrostatic and capillary forces. Note that this problem is mathematically equivalent to the problem concerning the shape of a two-dimensional bubble deformed by the steady flow of an inviscid incompressible fluid in a flat channel.

In the present work, a one-parametric family of the exact solutions of the problem is obtained using the conformal mapping technique [1]. The problem reduces to the successive solving of nonlinear ordinary differential equations if we assume that the relation between the absolute value of the electric field strength and its slope angle is invariant under the deformation of the jet surface (similar assumption was applied in Ref. [2] for finding equilibrium configurations of a periodic system of liquid metal columns in an external high-frequency magnetic field). It is interesting that the only previously known zero-parametric solution of the problem (it corresponds to the limiting case of infinite interelectrode distance [3]) does not belong to the obtained family of solutions.

The work was supported by the Russian Foundation for Basic Research (Project Nos. 17-08-00430 and 16-08-00228) and by the Presidium of the Ural Branch of the Russian Academy of Sciences (Project No. 15-8-2-8).

References

[1] N.B. Volkov, N.M. Zubarev, O.V. Zubareva. Exact solutions to the problem on the shape of an uncharged conducting liquid jet in a transverse electric field. JETP 122 (2016), 950-955

[2] J.A. Shercliff. Magnetic shaping of molten-metal columns. Proc. R. Soc. Lond. A 375 (1981), 455-473.

[3] E.B. McLeod. The explicit solution of a free boundary problem involving surface tention. J. Rat. Mech. Anal. 4 (1955), 557-567.

Geometry of rupture formed under the action of a tangential magnetic field in a conducting fluid layer

Zubarev N.M., Zubareva O.V.
Institute of Electrophysics, UB RAS, Ekaterinburg, Russia

A tangential magnetic field has a stabilizing influence on the surface of a liquid possessing magnetic properties. For relatively small deformations of the flat surface of a fluid, the magnetic forces, as well as the capillary ones, tend to return the free surface to its original state. In the situation where the amplitude of the surface deformation exceeds the depth of the liquid layer, the geometry of the system radically changes: the rupture of the layer can appear (see, for instance, experimental work [1], where the ruptures of a horizontal layer of ferrofluid were investigated).

In the present work we consider the simplest case where the magnetic field does not penetrate into the fluid, and the force lines of the field are tangential to the liquid boundary (this corresponds to the high-frequency magnetic field). Using the conformal mapping technique (the domain above the liquid and the substrate is mapped onto the half-plane), we have found exact solutions describing the formation of the rupture in a horizontal fluid layer for the plane symmetry of the problem [2]. For these solutions, capillary and magnetic forces are mutually compensated on the free surface of a liquid. The dependence of the width of the rupture on the parameters of the problem (namely, on the magnetic field value and the surface tension coefficient) has been established.

The work was supported by the Russian Foundation for Basic Research (Project Nos. 17-08-00430 and 16-08-00228) and by the Presidium of the Ural Branch of the Russian Academy of Sciences (Project No. 15-8-2-8).

References

[1] C.A. Bushueva, K.G. Kostarev. Behavior of a ferrofluid layer with stable surface rupture subjected to a tangential magnetic field. Fluid Dyn. 46 (2011), 707-711.

[2] N.M. Zubarev, O.V. Zubareva. Formation of rupture in a conducting fluid layer under the action of an oscillating tangential magnetic field. Journal of Magnetism and Magnetic Materials. 431 (2017), 226-228.

Talks

KP theory, total positivity and rational degenerations of $\mathtt{M}$–curves

Simonetta Abenda, Petr G. Grinevich
University of Bologna (Italy)
Landau Institute of Theoretical Physics (Russian Federation)

A relevant finite dimensional reduction of the Sato Grassmannian is parametrized by soliton data $(K, [A])$ with $K = \{k_1 < ... < k_n\}$ and $[A] \in Gr^{\text{TNN}}(k, n)$. To $(K, [A])$ there correspond regular bounded $(n-k, k)$–line soliton KP solutions $u(x, y, t)$ whose asymptotics behavior and tropicalization have been recently characterized in a series of papers by Kodama-Chakravarty[4] and Kodama-Williams[6, 7].

The reality condition for regular finite–gap solutions to the KP-II equation[8, 5] singles out genus g regular M–curves, with maximum number $g+1$ of real components (ovals) and Krichever divisors in the so–called finite ovals. The soliton solutions parametrized by $(K, [A])$ are also associated to rational degenerations of such M–curves.

In this talk I shall explain how to associate a rational degeneration of an M–curve of genus equal to the dimension of the Gelfand-Serganova matroid stratum to which $[A]$ belongs and prove the existence and uniqueness of a Baker–Akhiezer function with pole divisor in the position prescribed by the reality condition [2, 3].

I shall also explain how to reconstruct soliton data $(K, [A])$ when the curve is the rational degeneration of a hyperelliptic M–curve and the effective divisor satisfies the reality condition[1]. Indeed, such KP soliton family may be explicitly characterized and is naturally linked to the finite Toda system. In particular, I shall explain the relation to the spectral problem for the finite Toda system[9].

References

[1] Abenda S. On a family of KP multi-line solitons associated to rational degenerations of real hyperelliptic curves and to the finite non-periodic Toda hierarchy. arXiv:1605.00995.

[2] Abenda S., Grinevich P.G. Rational degeneration of M-curves, totally positive Grassmannians and KP-solitons. arXiv:1506.00563.

[3] Abenda S., Grinevich P.G. Le–diagrams, KP theory and rational degenerations of $\mathtt{M}$–curves, in press (2017)

[4] Chakravarty S., Kodama Yu. Soliton solutions of the KP equation and application to shallow water waves. Stud. Appl. Math. **123** (2009) 83–151.

[5] Dubrovin, B. A., Natanzon S. M., Real theta-function solutions of the Kadomtsev-Petviashvili equation, Izv. Akad. Nauk SSSR Ser. Mat., 52:2 (1988), 267–286

[6] Kodama, Yuji; Williams, Lauren The Deodhar decomposition of the Grassmannian and the regularity of KP solitons, Adv. Math. 244 (2013), 979–1032

[7] Kodama, Yuji; Williams, Lauren KP solitons and total positivity for the Grassmannian, Invent. Math. 198 (2014), no. 3, 637–699.

[8] Krichever, I. M. An algebraic-geometric construction of the Zakharov-Shabat equations and their periodic solutions. (Russian) Dokl. Akad. Nauk SSSR 227 (1976), no. 2, 291–294.

[9] Krichever, I.; Vaninsky, K. L. The periodic and open Toda lattice. Mirror symmetry, IV (Montreal, QC, 2000), AMS/IP Stud. Adv. Math., 33, Amer. Math. Soc., Providence, RI, (2002),139–158.

Solitons and Integrable Nonlocal Nonlinear Equations

M.J. Ablowitz
University of Colorado at Boulder (USA)

Solitons and the Inverse Scattering Transform (IST) are well known in the Math/Physics community. A surprisingly large number of recently found 'simple' nonlocal integrable equations and solutions will be discussed.

Solving the generalized Jacobi inversion problem by means of degenerate multivariative sigma-functions

Julia Bernatska, Dmitry Leykin
National University of Kyiv-Mohyla Academy
NASU Institute of Magnetism

We present the technique of multivariative sigma functions [1], in terms of which Abelian functions on Jacobians of hyperelliptic and nonhyperelliptic curves are constructed. The Abelian functions appear as finite-gap quasi-periodic solutions for integrable systems.

A commonly encountered situation, which attracts particular interest, concerns the case when the dimension of a finite-gap system is greater than the genus of its spectral curve. In this case Abelian functions have to be defined on a generalized Jacobian. We propose an effective way of constructing such functions by means of degenerate multivariative sigma functions. As an example we consider the case of genus 2 degenerate sigma-function [2].

References

[1] Buchstaber V. M., and Leykin D. V. Heat Equations in a Nonholonomic Frame, Functional Analysis and Its Applications, **38**(2), 88–101 (2004)

[2] Bernatska J., Leykin D. On degenerate sigma-functions of genus two, arXiv:1509.01490 [math-ph]

On a discretization of confocal quadrics: Geometric parametrizations, integrable systems and incircular nets

Alexander Bobenko
Technische Universität Berlin

Confocal quadrics lie at the heart of the system of confocal coordinates (also called elliptic coordinates). We suggest a geometric discretization which leads to factorisable discrete nets with a novel discrete analog of the orthogonality property and to an integrable discretization of the Euler-Poisson-Darboux equation. The coordinate functions of discrete confocal quadrics are computed explicitly. We demonstrate that special discrete confocal conics lead to incircular nets introduced in [3].

The talk is based on joint papers with W. Schief, Yu. Suris and J. Techter [1], [2].

References

[1] A.I. Bobenko, W.K. Schief, Yu.B. Suris, J. Techter, On a discretization of confocal quadrics. I. An integrable systems approach, Journal of Integrable Systems (2016) Volume 1:1, 1-34, arXiv:1511.01777

[2] A.I. Bobenko, W.K. Schief, Yu.B. Suris, J. Techter, On a discretization of confocal quadrics. II. A geometric approach to general parametrizations, Preprint (2017)

[3] A. Akopyan, A.I. Bobenko, Incircular nets and confocal conics, Trans. AMS (to appear), arXiv:1602.04637

Spectral compression by self-phase modulation in optical fibres

Sonia Boscolo, Christophe Finot
Aston Institute of Photonic Technologies,
School of Engineering and Applied Science, Aston University,
Birmingham B4 7ET, United Kingdom
Laboratoire Interdisciplinaire Carnot de Bourgogne,
UMR 6303 CNRS - Université de Bourgogne-Franche-Comté,
21078 Dijon Cedex, France

Self-phase modulation (SPM) in optical fibre is ordinarily associated with spectral broadening of an ultra-short optical pulse. However, for appropriate initial conditions of the input pulse, SPM can result in significant spectral compression. Indeed, SPM causes spectral compression or broadening depending on the initial frequency modulation (chirp) of the pulse electric field. Specifically, a pulse with a negative chirp, such as that imparted by an anomalously dispersive element, is compressed by the effects of SPM [1]. This method of spectral compression has been reported for various parameters and system configurations.

In this contribution, we review our recent results and advances in the area. Firstly, we provide insight into the influence of the temporal intensity profile of the initial pulse on the spectral dynamics of the nonlinear propagation in fibre. In particular, we emphasise that spectral narrowing does not occur for any initial pulse shape, and that there are significant differences between the propagation of parabolic, Gaussian, super-Gaussian or triangular pulses, which we elucidate with the aid of a time-frequency analysis [2]. The main limitation of SPM-driven spectral compression in the nonlinearity-dominant regime of propagation is the presence of residual side lobes in the compressed spectrum that result from the mismatch between the initial linear chirp of the pulse and the SPM-induced nonlinear chirp. Hence we discuss several strategies to overcome this limitation. We demonstrate that the use of pre-shaped input pulses with a parabolic waveform can achieve spectral compression to the Fourier transform limit owing to the fact that for such pulses the cancellation of the linear and nonlinear phases can be made complete [3]. We show that the intensity level of the side lobes in the compressed spectrum of pulses with standard temporal intensity profiles (such as Gaussian or hyperbolic secant shapes) can be efficiently reduced by an additional sinusoidal modulation of the temporal phase applied to the pulse [4]. Another strategy to enhance the quality of the compressed pulse spectrum is to select a dispersive nonlinear regime of propagation in which the combined action of normal group-velocity dispersion and SPM results in a deformation of the temporal profile of the pulse tending to acquire a rectangular shape while nearly complete compensation of the pulse chirp occurs [5]. Lastly, we unveil the rather good stability of the spectral compression process against amplitude fluctuations and optical signal-to-noise degradation of the seed pulses. We therefore evaluate the capability of the process of being used in the context of optical regeneration of intensity-modulated signals by describing an optical power limiting scheme that combines nonlinear spectral compression with centred optical bandpass filtering [6].

References

[1] M. Oberthaler, R.A. Höpfel. Spectral narrowing of ultrashort laser pulses by self-phase modulation in optical fibers. Appl. Phys. Lett. 63 (1993), 1017–1019.

[2] S. Boscolo, F. Chaussard, E. Andresen, H. Rigneault, C. Finot. Impact of initial pulse shape on the nonlinear spectral compression in optical fibre. (2017), Submitted.

[3] E.R. Andresen, C. Finot, D. Oron, H. Rigneault. Spectral analog of the Gouy phase shift. Phys. Rev. Lett. 110 (2013), 143902.

[4] S. Boscolo, L.Kh. Mouradian, C. Finot. Enhanced nonlinear spectral compression in fiber by external sinusoidal phase modulation. J. Opt. 18 (2016), 105504 (7pp).

[5] C. Finot, S. Boscolo. Design rules for nonlinear spectral compression in optical fibers. J. Opt. Soc. Amer. B 33 (2016), 760–767.

[6] S. Boscolo, J. Fatome, C. Finot. Impact of amplitude jitter and signal-to-noise ratio on the nonlinear spectral compression in optical fibres. Opt. Commun. 389 (2017), 197–202.

Zeros of polynomials and solvable nonlinear evolution equations

Francesco Calogero
Dipartimento di Fisica, Università di Roma "La Sapienza"
and Istituto Nazionale di Fisica Nucleare, Sezione di Roma

Recent findings concerning the relations between the coefficients and the zeros of time-dependent monic polynomials will be reported. They underline a differential algorithm to compute all the zeros of a generic polynomial and allow the identification of novel classes of dynamical systems solvable by algebraic operations, including hierarchies of such Newtonian ("accelerations equal forces") problems describing an arbitrary number of nonlinearly interacting point-particles moving in the complex plane. And the related notion of generations of (monic) polynomials will be introduced and discussed. Part of this work has been done with Oksana Bihun and with Mario Bruschi.

References

[1] F. Calogero, *New solvable variants of the goldfish many-body problem*, Studies Appl. Math. **137** (1), 123-139 (2016); `DOI:10.1111/sapm.12096`.

[2] O. Bihun and F. Calogero, *A new solvable many-body problem of goldfish type*, J. Nonlinear Math. Phys. **23**, 28-46 (2016).

[3] O. Bihun and F. Calogero, *Novel solvable many-body problems*, J. Nonlinear Math. Phys. **23**, 190-212 (2016).

[4] O. Bihun and F. Calogero, *Generations of monic polynomials such that the coefficients of the polynomials of the next generation coincide with the zeros of polynomial of the current generation, and new solvable many-body problems*, Lett. Math. Phys. **106** (7), 1011-1031 (2016).

[5] F. Calogero, *A solvable N-body problem of goldfish type featuring N^2 arbitrary coupling constants*, J. Nonlinear Math. Phys. **23**, 300-305 (2016).

[6] F. Calogero, *Three new classes of solvable N-body problems of goldfish type with many arbitrary coupling constants*, Symmetry **8**, 53 (2016).

[7] M. Bruschi and F. Calogero, *A convenient expression of the time-derivative $z_n^{(k)}(t)$, of arbitrary order k of the zero $z_n(t)$ of a time-dependent polynomial $p_N(z;t)$ of arbitrary degree N in z, and solvable dynamical systems*, J. Nonlinear Math. Phys. **23**, 474-485 (2016).

[8] F. Calogero, *Novel isochronous N-body problems featuring N arbitrary rational coupling constants*, J. Math. Phys. **57**, 072901 (2016); http://dx.doi.org/10.1063/1.4954851.

[9] F. Calogero, *Yet another class of new solvable N-body problems of goldfish type*, Qualit. Theory Dyn. Syst. (printed online: 2016.10.04); DOI: 10.1007/s12346-016-0215-y.

[10] F. Calogero, *New solvable dynamical systems*, J. Nonlinear Math. Phys. **23**, 486-493 (2016).

[11] F. Calogero, *Integrable Hamiltonian N-body problems in the plane featuring N arbitrary functions*, J. Nonlinear Math. Phys. **24** (1), 1-6 (2017).

[12] F. Calogero, *New C-integrable and S-integrable systems of nonlinear partial differential equation*, J. Nonlinear Math. Phys. **24** (1), 142-148 (2017).

[13] F. Calogero, *Nonlinear differential algorithm to compute all the zeros of a generic polynomial*, J. Math. Phys. **57**, 083508 (4 pages) (2016); http://dx.doi.org/10.1063/1.4960821). arXiv:1607.05081v1 [math.CA].

[14] F. Calogero, *Comment on "Nonlinear differential algorithm to compute all the zeros of a generic polynomial" [J. Math. Phys. **57**, 083508 (2016)]*, J. Math. Phys. 57, 104101 (4 pages) (2016).

[15] O. Bihun and F. Calogero, *Generations of solvable discrete-time dynamical systems*, J. Math. Phys. (submitted to, 2016.06.23).

[16] F. Calogero, Zeros of polynomials and solvable nonlinear evolution equations, (short book: about 250 pages), to be published.

Abelian and non-Abelian Bäcklund Charts

Sandra Carillo
Dip. "Scienze di Base e Applicate per l'Ingegneria",
Sapienza - University of Rome,
16, Via A. Scarpa, 00161 Rome, Italy
&
I.N.F.N. - Sez. Roma1, Gr. IV
Mathematical Methods in NonLinear Physics, Rome, Italy

Results concerning nonlinear evolution equations, mainly in the non-Abelian setting are considered. Specifically, an overview which takes its origin in the late 80s [2, 1] in the commutative setting, find their non-Abelian generalization in recent studies [3, 4, 5, 6, 7, 8]. The latest achievements are presented and briefly discussed.

References

[1] B. Fuchssteiner and S. Carillo, Soliton structure versus singularity analysis: Third order completely integrable nonlinear equations in 1+1-dimensions, *Phys. A* **154** (1989), 467-510.

[2] C. Rogers and S. Carillo, On reciprocal properties of the Caudrey-Dodd-Gibbon and Kaup-Kupershmidt hierarchies, *Physica Scripta* **36** (1987), 865-869.

[3] S. Carillo and C. Schiebold, Non-commutative KdV and mKdV hierarchies via recursion methods, *J. Math. Phys.* **50** (2009), 073510.

[4] S. Carillo and C. Schiebold, Matrix Korteweg-de Vries and modified Korteweg-de Vries hierarchies: Non-commutative soliton solutions, *J. Math. Phys.* **52** (2011), 053507.

[5] S. Carillo, M. Lo Schiavo and C. Schiebold, Bäcklund Transformations and Non Abelian Nonlinear Evolution Equations: a novel Bäcklund chart, *SIGMA*, 12 (2016), 087, 17 pages, doi:10.3842/SIGMA.2016.087.

[6] S. Carillo, M. Lo Schiavo and C. Schiebold, Recursion Operators admitted by non-Abelian Burgers equations: Some Remarks, (2016), 12 pages, arXiv:1606.07270v1.

[7] S. Carillo,
A novel KdV-type equation ?, arXiv:1702.06874, submitted feb. 2017.

[8] S. Carillo, M. Lo Schiavo, E. Porten, C. Schiebold,
A novel noncommutative KdV-type equation, its recursion operator, and solitons, arXiv:1704.03208, submitted apr. 2017.

Normal forms of dispersive scalar Poisson brackets for two-dimensional systems

Guido Carlet, Matteo Casati, Sergey Shadrin
KdV Institut voor Wiskunde, Universiteit van Amsterdam
Dept. of Mathematical Sciences, Loughborough University
KdV Institut voor Wiskunde, Universiteit van Amsterdam

This contribution presents the classification of dispersive deformations of the two-dimensional scalar Dubrovin–Novikov (DN) Poisson brackets [2]. DN brackets of this class are Poisson structures on the space of local functionals, where the densities are functions from a 2–dimensional to a 1–dimensional manifold. They were introduced as a generalisation of the Poisson brackets of hydrodynamic type, as suitable structure to define $2+1$ dimensional Hamiltonian PDEs [3].

In analogy with the Poisson geometry of finite dimensional manifolds, we can regard DN brackets as being defined by a Poisson bivector on an infinite dimensional manifold; moreover, we can use such a bivector to define the Lichnerowicz cochain complex of (local) p-vectors. For the case of one independent variable, Ezra Getzler proved that all the positive cohomology groups of the complex vanish [4], regardless of the number of components. In particular, that means that all the dispersive deformations of the brackets are trivial, i.e. they are generated by a Miura transformation of the undeformed bracket.

In a previous paper [1] we have proved that the cohomology groups for $D \geq 2$ independent variables are in general not trivial, and we explicitly computed the dimension of all the cohomology groups for $D = 2$. In particular, both the cohomology groups H^2 (describing nontrivial infinitesimal deformations of the DN bracket) and H^3 (related to the obstructions in extending the infinitesimal deformations to finite ones) are not vanishing and grow bigger with the growth of the differential order of the deformations considered.

Let $H^p_d(\mathcal{F})$ be the homogeneous, d-th order component of the p-th cohomology group for the local p-vectors. The main result of the previous paper [1] is that, in the twodimensional case,

$$H^p_d(\mathcal{F}) \cong \frac{\Theta^p_d}{\partial\Theta^p_{d-1}} \oplus \frac{\Theta^{p+1}_d}{\partial\Theta^{p+1}_{d-1}} := \Xi^p_d \oplus \Xi^{p+1}_d$$

where Θ^p_d is the component of of bidegree (p, d) of the polynomial ring $\mathbf{R}[\{\theta^s, s \in \mathbf{Z}_{\geq 0}\}]$, with θ's Grassmann variables, and $\partial\theta^s = \theta^{s+1}$.

In this contribution, we show how to explicitly write representatives of the cohomology classes in $H^2_d(\mathcal{F})$ and prove that all the ones generated by elements of Ξ^3_d cannot be extended to finite deformations.

Recalling that, in some coordinate system, the scalar two-dimensional DN bracket has the form

$$\{u(x^1, x^2), u(x^1, x^2)\}^0 = \delta(x^1 - y^1)\delta'(x^2 - y^2), \tag{1}$$

this leads us to conclude that the normal form of the dispersive Poisson bracket with leading term (1) is, under Miura transformations of the second kind, given by

$$\{u(x^1, x^2), u(x^1, x^2)\} = \delta(x^1 - y^1)\delta'(x^2 - y^2) + \\ + \sum_{k\geq 1} c_k \delta^{(2k+1)}(x^1 - y^1)\delta(x^2 - y^2) \tag{2}$$

for a sequence of constants $(c_1, c_2, \ldots)$.

References

[1] G. Carlet, M. Casati, S. Shadrin. Poisson cohomology of scalar multidimensional Dubrovin–Novikov brackets. J. Geom. Phys. 114 (2017), 404–419

[2] G. Carlet, M. Casati, S. Shadrin, Normal forms of dispersive scalar Poisson brackets with two independent variables, *in preparation*

[3] B.A. Dubrovin, S.P. Novikov. Poisson brackets of hydrodynamic type. Doklad. Akad. Nauk USSR 279 (1984), 294–297.

[4] E. Getzler. A Darboux theorem for Hamiltonian operators in the formal calculus of variations. Duke Math. J. 111 (2002), 535-560.

Dunkl–Cherednik operators and quantum Lax pairs

Oleg Chalykh
University of Leeds

A quantum Lax pair for the Calogero–Moser system was first proposed in [1], cf. [2, 3]. Later it was generalized to arbitrary root systems in [4, 5], based on earlier results in the classical case [6, 7, 8]. It was noted in those works that the quantum Lax matrix had a close resemblance to a Dunkl operator, but a precise relationship was not estiblished. More recently, there have also been works [9, 10, 11], pursuing a link between Dunkl operators and quantum Lax operators, but it remained more of an empirical observation.

We will explain a direct conceptual link between Dunkl operators and quantum Lax pairs. In fact, a Lax pair L, A can be associated to any of the commuting quantum Calogero–Moser Hamiltonians, so we obtain a family of compatible quantum Lax pairs. This approach also works for more general Calogero–Moser systems associated with complex reflection groups. Moreover, it can be used in the elliptic case to construct previously unknown quantum Lax pairs.

Replacing Dunkl operators by their q-analogues, known as Cherednik operators, leads to quantum Lax pairs for the relativistic Calogero–Moser system given by the trigonometric Macdonald–Ruijsenaars operators (Koornwinder–van Diejen operators in the BC_n-case). This gives a uniform construction of both classical and quantum Lax pairs for the systems of Ruijsenaars–Schenider type for all root systems, including the Koornwinder–van Diejen system with five coupling parameters. In the latter case, we combine our approach with an idea from [11] to calculate a Lax pair for the Koornwinder–van Diejen system, which remained elusive until now: the best previous result in that direction was a construction of a classical Lax pair [12] for a two-parametric subfamily.

References

[1] H. Ujino, K. Hikami, M. Wadati. Integrability of the quantum Calogero-Moser model. J. Phys. Soc. Japan 61 (1992), 3425.

[2] B. S. Shastri, B. Sutherland. Super Lax pairs and infinite symmetries in the $1/r^2$ system. Phys. Rev. Lett. 70 (1993), 4029.

[3] D. Bernard, M. Gaudin, F. D. M. Haldane, V. Pasquier. Yang-Baxter equation in long-range interacting systems. J. Phys. A: Math. Gen. 26 (1993), 5219.

[4] A. J. Bordner, N. S. Manton, R. Sasaki. Calogero-Moser Models V: Supersymmetry, and quantum Lax pair. Prog. Theor. Phys. 103 (2000), 463–487.

[5] S. P. Khastgir, A. J. Pocklington, R. Sasaki. Quantum Calogero-Moser models: integrability for all root systems. J. Phys. A: Math. Gen. 33 (2000), 9033–9064.

[6] I. M. Krichever. Elliptic solutions of the Kadomtsev-Petviashvili equation and integrable systems of particles. Funct. Anal. Appl. 14 (1980), 282–289.

[7] E. D'Hoker, D. H. Phong. Calogero-Moser Lax pairs with spectral parameter for general Lie algebras. Nucl. Phys. B 530 (1998), 537–610.

[8] A. J. Bordner, E. Corrigan, R. Sasaki. Generalised Calogero-Moser models and universal Lax pair operators. Prog. Theor. Phys. 102 (1999), 499–529.

[9] A. N. Sergeev, A. P. Veselov. Dunkl operators at infinity and Calogero–Moser systems. IMRN 21 (2015), 10959-10986.

[10] M. L. Nazarov, E. K. Sklyanin. Lax operator for Macdonald symmetric functions. Lett. Math. Phys. 105(7) (2015), 901–916.

[11] M. L. Nazarov, E. K. Sklyanin. Cherednik operators and Ruijsenaars-Schneider model at infinity. Preprint, March 2017. Available at: arXiv:1703.02794 [nlin.SI].

[12] B. G. Pusztai, T. F. Gorbe. Lax representation of the hyperbolic van Diejen dynamics with two coupling parameters. Preprint, March 2016. Available at: arXiv:1603.06710 [math-ph].

Stability of shear shallow water flows with free surface

A.A. Chesnokov[1], G.A. El[2], S.L. Gavrilyuk[3], M.V. Pavlov[1]
[1]Novosibirsk State University
[2]Loughborough University
[3]Aix-Marseille University

Stability of shear flows for the full Euler equations is a fundamental problem of Fluid Mechanics (see e.g. Drazin, 2002). The classical stability and instability criteria formulated in terms of growth of linear perturbations (Rayleigh, Fjortoft) are usually obtained for flows between rigid walls. Some recent works use the generalized notion of stability as the well-posedness of time evolution, i.e. hyperbolicity (see Chumakova, Manzaque, Milewski et al., 2009), but they also consider either flows under closed lid or use periodic boundary conditions in the vertical direction which greatly simplifies the analysis. However, the presence of free surface can obviously change the flow stability criteria and, to our knowledge, stability of shallow water shear flows with a free surface has not been studied before. A generalized theory of characteristics and the notion of hyperbolicity for integrodifferential equations of the long wave theory was introduced by Teshukov [1]. He developed new mathematical tool for the qualitative study of integro-differential hyperbolic equations. We apply this approach in the present work.

Stability of inviscid shear shallow water flows with free surface is studied in the framework of the Benney equations. This is done by investigating the generalized hyperbolicity of the integrodifferential Benney system of equations. It is shown that all shear flows having monotonic convex velocity profiles are stable. The hydrodynamic approximations of the model corresponding to the classes of flows with piecewise linear continuous and discontinuous velocity profiles are derived and studied. It is shown that these approximations possess Hamiltonian structure and a complete system of Riemann invariants, which are found in an explicit form. Sufficient conditions for hyperbolicity of the governing equations for such multilayer flows are formulated. The generalization of the above results to the case of stratified fluid is less obvious, however, it is established that vorticity has a stabilizing effect (see [2] for details).

References

[1] V.M. Teshukov. Hyperbolicity of long-wave equations. Dokl. Akad. Nauk. 284 (1985), 555–559.

[2] A.A. Chesnokov, G.A. El, S.L. Gavrilyuk, M.V. Pavlov. Stability of shear shallow water flows with free surface. arxiv.org/abs/1610.04331

Painlevé VI surfaces*

Robert Conte
1. Centre de mathématiques et de leurs applications,
École normale supérieure de Cachan, CNRS, Université Paris-Saclay, France.
2. Department of mathematics, The University of Hong Kong,
Pokfulam road, Hong Kong.
Robert.Conte@cea.fr

The surfaces built by Bonnet in 1867 [2] are the first occurrence ever of the sixth Painlevé equation $\mathrm{P_{VI}}$ [1]. We extrapolate their moving frame to a new [3], second order, isomonodromic matrix Lax pair for $\mathrm{P_{VI}}$, which is in some sense its natural Lax pair in CP^1. Its elements depend rationally on the dependent variable and quadratically on the monodromy exponents θ_j, while the one of Jimbo and Miwa [4] is meromorphic in θ_∞. Moreover, this extrapolation lifts Bonnet surfaces to generalized surfaces which depend on two more degrees of freedom.

References

[1] A.I. Bobenko, U. Eitner and A.V. Kitaev, Surfaces with harmonic inverse mean curvature and Painlevé equations, Geometriae dedicata **68** (1997) 187–227.

[2] O. Bonnet, Mémoire sur la théorie des surfaces applicables sur une surface donnée. Deuxième partie : Détermination de toutes les surfaces applicables sur une surface donnée, J. École polytechnique **42** (1867) 1–151.
http://gallica.bnf.fr/ark:/12148/bpt6k433698b/f5.image

[3] R. Conte, Surfaces de Bonnet et équations de Painlevé,
C.R. Math. Acad. Sci. Paris **355** (2017) 40–44.
http://arxiv.org/abs/1607.01222v2 [math-ph]

[4] M. Jimbo and T. Miwa, Monodromy preserving deformations of linear ordinary differential equations with rational coefficients. II, Physica D **2** (1981) 407–448.

*PMNP 2017, Gallipoli, 17–24 June 2017. 20 March 2017

Geometric Deautonomization and Discrete Painlevé Equations

Adrian Stefan Carstea[§], Anton Dzhamay[¶], and Tomoyuki Takenawa[$]
[§] National Inst. of Physic and Nuclear Engineering, Bucharest, Romania
[¶] University of Northern Colorado, Greeley, CO, USA
[$] Tokyo University of Marine Science and Technology, Tokyo, Japan

It is known that classical differential Painlevé equations have autonomous limits, and the resulting equations can be solved in terms of elliptic functions. A similar statement holds in the discrete case as well. Conversely, starting with an autonomous integrable two-dimensional mapping, such as the QRT or the Quispel-Roberts-Thompson mapping, one can apply the so-called *deautonomization procedure* to construct examples of discrete Painlevé equations. In this was many examples were originally obtained by B. Grammaticos, A. Ramani, and their collaborators using the *singularity confinement* principle, as explained in the survey [1]. In that way one can obtain discrete Painlevé equations of *different* types starting from the *same* autonomous mapping. In [2] we explain this phenomenon using the the geometric theory of discrete Painlevé equations proposed by H. Sakai, [3]. In this talk we explain the main ideas behind our approach.

In a nutshell, we observe that a QRT mapping is naturally defined on a rational elliptic surface X and it induces a linear map on the Picard lattice $\mathrm{Pic}(X)$ of the surface. A deautonomization procedure results in a discrete Painlevé equation that has the same action on $\mathrm{Pic}(X)$, but the parameters describing the surface now change under the mapping. Knowing only the action on $\mathrm{Pic}(X)$ is not enough information to reconstruct the mapping. We also need to choose a fiber that corresponds to the *fixed anti-canonical divisor* in the discrete Painlevé setting. In addition to a generic smooth fiber, rational elliptic surfaces have singular fibers; choosing fibers of different types then results in different deautonomizations, which also explains how the same autonomous system can be an autonomous limit of different non-autonomous systems. From this data (i.e., the choice of an elliptic fiber and the linear action on $\mathrm{Pic}(X)$) we are then able to explicitly write down discrete Painlevé equation using the *rational mapping factorization* technique.

In this way we construct different examples of discrete Painlevé equations. Of particular interest are constrained examples that have special symmetry groups, such as $D_6^{(1)}$, i.e., the groups that do not appear explicitly in Sakai's classification scheme (w.r.t. the symmetry type).

References

[1] Grammaticos, B. Ramani, A., Discrete Painlevé equations: a review, in *Discrete integrable systems*, LNP **644** (Springer, Berlin, 2004) 245–321

[2] Adrian Stefan Carstea, Anton Dzhamay, and Tomoyuki Takenawa, *Fiber-dependent deautonomization of integrable 2D mappings and discrete Painlevé equations*,`arXiv:1702.04907 [math.DS]`

[3] Hidetaka Sakai, *Rational surfaces associated with affine root systems and geometry of the Painlevé equations*, Comm. Math. Phys. **220** (2001), no. 1, 165–229.

Lump solitons in a higher-order nonlinear equation in $2 + 1$ dimensions

Pilar G. Estévez
Professor of Theoretical Physics. Universidad de salamanca SPAIN

We propose and examine an integrable system of nonlinear equations that generalizes the nonlinear Schrödinger equation to $2 + 1$ dimensions. This integrable system of equations is a promising starting point to elaborate more accurate models in nonlinear optics and molecular systems within the continuum limit. The Lax pair for the system is derived after applying the singular manifold method. We also present an iterative procedure to construct the solutions from a seed solution. Solutions with one, two and three lump solitons are thoroughly discussed.

References

[1] A. S. Davydov and N. I. Kislukha, Phys. stat. sol. B 59, (1973), 465

[2] M. Daniel and K. Deepamala, Physica A 221, (1995), 241255

[3] M. Lakshmanan, K. Porsezian, and M. Daniel, Phys. Lett. A 133 (1988), 483

[4] A. Ankiewicz, Y. Wang, S. Wabnitz, and N. Akhmediev, Phys. Rev. E 89, (2014), 012907

[5] P. G. Estevez and G. A. Hernaez, J. Nonlinear Math. Phys. 8, (2001), 106

[6] P. G. Estevez J. Prada, and J. Villarroel, J. Physics A: Math. Theor. 40, (2007), 7213

[7] J. S. He, H. R. Zhang, L. H. Wang, K. Porsezian, and A. S. Fokas, Phys. Rev. E 87, (2013), 052914

1

Integrability in AGT duality

Matteo Beccaria, Alberto Fachechi, Guido Macorini, Luigi Martina
Dipartimento di Matematica e Fisica Ennio De Giorgi,
Università del Salento & INFN, Via Arnesano, 73100 Lecce, Italy

In modern theoretical physics, a prominent role is covered by the concepts of integrability and conformal invariance. On one side, integrable systems appear in many settings, ranging from classical mechanics to string theory, and present a very particular behaviour due to the large extent of underlying symmetries. On the other side, field theories based on conformal invariance are an ubiquitous topic, emerging in the physics of critical phenomena and in string theory. Due to their large amount of symmetries, they are ideal subjects where to study its consequences in the light of integrability. The two-dimensional case is an extreme example of this idea, since the conformal algebra is infinite-dimensional. As a consequence, 2d CFTs present a suggestive integrable structure whose investigation has been carried out some time ago in the celebrated works by Bazhanov, Lukyanov and Zamolodchikov. More recently, 2d CFTs earned a renewed interest due to the so-called AGT duality [1]. This correspondence relates 2d Liouville conformal field theories on suitable Riemann surfaces and Ω-deformed Seiberg-Witten gauge theories. Therefore, the integrable structure of the 2d CFT would be inherited by the corresponding supersymmetric Yang-Mills theory. In fact, the algebra of integrals of motion of CFTs is mapped to the chiral ring of its AGT-dual. Moreover, the correspondence matches the conformal blocks of the CFT with a suitable operator insertion and the instanton partition function of the supersymmetric gauge theory dual.
We present some recent results in the context of AGT duality. We studied [2] the chiral ring of the $\mathcal{N} = 2$ pure and $\mathcal{N} = 2^*$ Seiberg-Witten theories and how it gets deformed once that the Ω-background is switched on, interpreting the resulting structure on the CFT side by means of AGT duality. Moreover, we explored [3] the $SU(2)$ $N_f = 4$ Ω-deformed Seiberg-Witten theory for fixed values of the matter masses and one of the deformation parameters and required a simple polar structure for the Nekrasov formulae. In this way, we were able to determine in closed form the exact partition function of the gauge theory, therefore giving non-trivial predictions on the conformal block on the 4-punctured sphere with fixed central charge and conformal dimensions for external operators. The results are in agreement with the Zamolodchikov recursion relation.

References

[1] L. F. Alday, D. Gaiotto and Y. Tachikawa, "Liouville Correlation Functions from Four-dimensional Gauge Theories," Lett. Math. Phys. **91**, 167 (2010) doi:10.1007/s11005-010-0369-5 [arXiv:0906.3219 [hep-th]].

[2] M. Beccaria, A. Fachechi and G. Macorini, "Chiral trace relations in Ω-deformed $\mathcal{N} = 2$ theories," arXiv:1702.01254 [hep-th].

[3] M. Beccaria, A. Fachechi, G. Macorini and L. Martina, "Exact partition functions for deformed $\mathcal{N} = 2$ theories with $N_f = 4$ flavours," JHEP **1612** (2016) 029 doi:10.1007/JHEP12(2016)029 [arXiv:1609.01189 [hep-th]].

Integrable systems in 4D associated with sixfolds in $\mathbf{Gr}(4,6)$

B Doubrov[1], E Ferapontov[2], B Kruglikov[3], V Novikov[2]

[1] Department of Mathematical Physics
Faculty of Applied Mathematics,
Belarussian State University,
Nezavisimosti av. 4, 220030 Minsk, Belarus,

[2] Department of Mathematical Sciences
Loughborough University
Loughborough, Leicestershire LE11 3TU, UK

[3]Department of Mathematics and Statistics
Faculty of Science and Technology
UiT the Arctic University of Norway
Tromsø 90-37, Norway

Let $\mathbf{Gr}(d,n)$ be the Grassmannian of d-dimensional linear subspaces of an n-dimensional vector space V. A submanifold $X \subset \mathbf{Gr}(d,n)$ gives rise to a differential system $\Sigma(X)$ that governs d-dimensional submanifolds of V whose Gaussian image is contained in X. We investigate a special case of this construction where X is a sixfold in $\mathbf{Gr}(4,6)$. The corresponding system $\Sigma(X)$ reduces to a pair of first-order PDEs for 2 functions of 4 independent variables. Equations of this type arise in self-dual Ricci-flat geometry. Our main result is a complete description of integrable systems $\Sigma(X)$. These naturally fall into two subclasses.

- Systems of Monge-Ampère type. The corresponding sixfolds X are codimension 2 linear sections of the Plücker embedding $\mathbf{Gr}(4,6) \hookrightarrow P^{14}$.
- General linearly degenerate systems. The corresponding sixfolds X are images of quadratic maps $P^6 \to \mathbf{Gr}(4,6)$ given by the classical construction of Chasles.

We prove that integrability is equivalent to the requirement that the characteristic variety of system $\Sigma(X)$ gives rise to a conformal structure which is self-dual on every solution.

References

[1] B. Doubrov, E.V. Ferapontov, B. Kruglikov, V.S. Novikov, On the integrability in Grassmann geometries: integrable systems associated with fourfolds in Gr(3, 5), arXiv:1503.02274.

Revisiting the classics:Fourier,Laplace and Riemann

Athanassios F. Fokas
DAMTP, University of Cambridge, UK

The quest for extending the inverse scattering transform to the analysis of boundary value problems has led to the emergence of the unified transform (also referred to as the Fokas method). It will be shown that this transform, in addition to providing explicit asymptotics for nonlinear problems like the NLS on the half-line, it also yields unexpected results for such classical problems as the heat equation on the interval, first investigated by Fourier, as well as for the classical elliptic PDEs including the Laplace equation. An interesting connection of this new method with the Riemann hypothesis provided the motivation for a novel approach to the asymptotics of the Riemann zeta function, which has led to the proof of the Lindelof hypothesis for a close variant of the Riemann zeta function.

Multiparametric families of solutions to the KPI equation, their Fredholm, wronskians and degenerate determinant representations and multi-rogue waves.

Pierre Gaillard
Institut Mathématique de Bourgogne,
Université de Bourgogne, Dijon, 21000, France

We construct solutions to the Kadomtsev-Petviashvili equation (KPI) in terms of Fredholm determinants. We deduce solutions written as a quotient of wronskians of order $2N$. These solutions called solutions of order N depend on $2N-1$ parameters.
They can also be written as a quotient of two polynomials of degree $2N(N+1)$ in x, y and t depending on $2N-2$ parameters. The maximum of the modulus of these solutions at order N is equal to $2(2N+1)^2$.
We explicitly construct the expressions until the order 6 and we study the patterns of their modulus in the plane (x, y) and their evolution according to time and parameters.

References

[1] P. Gaillard, Families of quasi-rational solutions of the NLS equation and multi-rogue waves, J. Phys. A : Meth. Theor., V. **44**, 1-15, 2011

[2] P. Gaillard, Wronskian representation of solutions of the NLS equation and higher Peregrine breathers, J. Math. Sciences : Adv. Appl., V. **13**, N. 2, 71-153, 2012

[3] P. Gaillard, Degenerate determinant representation of solution of the NLS equation, higher Peregrine breathers and multi-rogue waves, Jour. Of Math. Phys., V. **54**, 013504-1-32, 2013

[4] P. Gaillard, Other 2N-2 parameters solutions to the NLS equation and 2N+1 highest amplitude of the modulus of the N-th order AP breather, J. Phys. A: Math. Theor., V. **48**, 145203-1-23, 2015

[5] P. Gaillard, Multi-parametric deformations of the Peregrine breather of order N solutions to the NLS equation and multi-rogue waves, Adv. Res., V. **4**, 346-364, 2015

[6] P. Gaillard, Towards a classification of the quasi rational solutions to the NLS equation, Theor. And Math. Phys., V. **189**, 1440-1449, 2016

Conservation laws, symmetries, and exact solutions of generalized KP and Boussinesq equations in two dimensions.

S.C. Anco and M.L. Gandarias
Brock University,Ontario Canada
Cádiz University, Cádiz Spain

Nonlinear generalizations of integrable equations in one-dimension, such as the KdV and Bousinesq equations with p-power nonlinearities, arise in many physical applications and are interesting in analysis due to critical behaviour. In this talk, we study analogous nonlinear generalizations of the integrable KP equation and the 2D Boussinesq equation. We give a complete classification of low-order conservation laws and Lie symmetries for these two-dimensional equations with p-power nonlinearities. We also consider exact solutions obtained by symmetry reduction.

References

[1] S. C. Anco and G. Bluman, Direct construction method for conservation laws of partial differential equations Part II: General treatment, Euro. J. Appl. Math. 41 (2002), 567–585.

[2] S.C. Anco Generalization of Noether's theorem in modern form to non-variational partial differential equations. To appear in Fields Institute Communications: Recent progress and Modern Challenges in Applied Mathematics, Modeling and Computational Science; arXiv: mathph/1605.08734 (2016).

[3] R. Naz, Z. Ali, and I. Naeem, Reductions and New Exact Solutions of ZK, Gardner KP, and Modified KP Equations via Generalized Double Reduction Theorem. Abstract and Applied Analysis (2013), 340564–340575.

Conservation laws, symmetries, and exact solutions of generalized KP and Boussinesq equations in two dimensions.

S.C. Anco and M.L. Gandarias
Brock University,Ontario Canada
Cadiz University, Cadiz Spain

Nonlinear generalizations of integrable equations in one-dimension, such as the KdV and Bousinesq equations with p-power nonlinearities, arise in many physical applications and are interesting in analysis due to critical behaviour. In this talk, we study analogous nonlinear generalizations of the integrable KP equation and the 2D Boussinesq equation. We give a complete classification of low-order conservation laws and Lie symmetries for these two-dimensional equations with p-power nonlinearities. We also consider exact solutions obtained by symmetry reduction.

References

[1] S. C. Anco and G. Bluman, Direct construction method for conservation laws of partial differential equations Part II: General treatment, Euro. J. Appl. Math. 41 (2002), 567–585.

[2] S.C. Anco Generalization of Noether's theorem in modern form to non-variational partial differential equations. To appear in Fields Institute Communications: Recent progress and Modern Challenges in Applied Mathematics, Modeling and Computational Science; arXiv: mathph/1605.08734 (2016).

[3] R. Naz, Z. Ali, and I. Naeem, Reductions and New Exact Solutions of ZK, Gardner KP, and Modified KP Equations via Generalized Double Reduction Theorem. Abstract and Applied Analysis (2013), 340564–340575.

Symmetric spaces and new 2-component integrable NLS equations

V. S. Gerdjikov
Institute of Mathematics and Informatics,
Bulgarian Academy of Sciences, 1113 Sofia, Bulgaria.

The fundamental properties of the multi-component nonlinear Schrödinger (NLS) type models related to the BD.I symmetric spaces are analyzed. Their Lax operator in the case $SO(9)/S(O(6)\otimes O(3))$ take the form:

$$L\psi \equiv \left(i\frac{\partial}{\partial x} + Q(x,t) - \lambda J \right) \psi(x,t,\lambda) = 0,$$

$$Q(x,t) = \begin{pmatrix} 0 & \boldsymbol{q}^T & 0 \\ \boldsymbol{p} & 0 & s_0\boldsymbol{q} \\ 0 & \boldsymbol{p}^T s_0 & 0 \end{pmatrix}, \quad J = \begin{pmatrix} \mathbb{1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\mathbb{1} \end{pmatrix}. \quad s_0 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

The matrices $Q(x,t)$ and J are 3×3 block matrices, each block is also 3×3. New types of $\mathbb{Z}_6\times\mathbb{Z}_2$-reductions of these systems are constructed. After the reduction $Q(x,t)$ depends on only two complex functions as follows:

$$\boldsymbol{q}(x,t) = \frac{1}{\sqrt{6}} \begin{pmatrix} v_1 & \sqrt{2}v_2 & -v_1 \\ -v_1 & -\sqrt{2}v_2 & v_1 \\ v_1 & \sqrt{2}v_2 & -v_1 \end{pmatrix}, \qquad \boldsymbol{p} = \boldsymbol{q}^\dagger.$$

As a result we obtain a new 2-component NLS type system with Hamiltonian

$$H = \left|\frac{\partial v_1}{\partial x}\right|^2 + \left|\frac{\partial v_2}{\partial x}\right|^2 - \frac{1}{2}\left(|v_1|^2 + |v_2|^2\right)^2 + \frac{1}{2}\left(v_1 v_2^* - v_1^* v_2\right)^2,$$

The spectral properties of the reduced Lax operator L and the fundamental properties of the relevant class of nonlinear evolution equations are described. Special attention is paid to the recursion operators and the bi-Hamiltonian properties of the relevant NLEE.

Other examples of new 2-component NLS equations related to other types of symmetric spaces are presented in [1].

References

[1] V.S. Gerdjikov, A. A. Stefanov. New types of two component NLS-type equations. Pliska Studia Mathematica **26**, 53–66 (2016).

Integrable nonlocal multi-component equations with $\mathcal{PT}$ and $\mathcal{CPT}$ symmetries

G. G. Grahovski [(1)], V. S. Gerdjikov [(2)], R. I. Ivanov [(3)]
[(1)] Department of Mathematical Sciences,
University of Essex, UK
[(2)] Institute of Mathematics and Informatics,
Bulgarian Academy of Sciences, Sofia, Bulgaria
[(3)] School of Mathematical Sciences,
Dublin Institute of Technology, Ireland

We will present extensions of N-wave and derivative NLS types of equations with $\mathcal{PT}$ and $\mathcal{CPT}$-symmetries [1]. The types of (nonlocal) reductions leading to integrable equations invariant with respect to $\mathcal{C}$- (charge conjugation), $\mathcal{P}$- (spatial reflection) and $\mathcal{T}$- (time reversal) symmetries are described. The corresponding constraints on the fundamental analytic solutions and the scattering data are derived.

Based on examples of 3-wave (related to the algebra $sl(3,\mathbb{C})$) and 4-wave (related to the algebra $so(5,\mathbb{C})$) systems, the properties of different types of 1- and 2-soliton solutions are discussed. It is shown that the $\mathcal{PT}$ symmetric 3-wave equations may have regular multi-soliton solutions for some specific choices of their parameters [1].

Furthermore, we will present multi-component generalizations of derivative nonlinear Schrodinger (DNLS) type of related to **A.III** symmetric spaces and having with $\mathcal{CPT}$-symmetry [2]. This includes equations of Kaup-Newell (KN) and Gerdjikov-Ivanov (GI) types.

References

[1] VS Gerdjikov, GG Grahovski, RI Ivanov, *On the N-wave Equations with $\mathcal{PT}$-symmetry*, Theor. Math. Phys. **188** (2016), 1305 – 1321 [E-print: `arXiv:1601.01929`].

[2] VS Gerdjikov, GG Grahovski, RI Ivanov, *On integrable wave interactions and Lax pairs on symmetric spaces*, Wave Motion (to appear) [E-print: `arXiv:1607.06940`].

Exceptional Hermite polynomials and rational solutions of the Painlevé IV chain

David Gomez-Ullate[a], Yves Grandati[b] and Robert Milson[c]

The recently discovered exceptional orthogonal polynomials (X-OP) appear to be intimately related to the structure of closed form exactly solvable quantum potentials.

For instance, the X-Hermite polynomials are (up to a gauge factor) the eigenstates of the rational extensions of the harmonic oscillator obtained via some specific dressing chains.

Considering chains which are cyclic, we propose a new scheme which allows to generate in a direct way the polynomials associated to the rational solutions of the Painlevé IV equation and higher order analogues from the X-Hermite family.

On a direct algoritm for constructing recursion operators and Lax pairs for integrable models

A.R.Khakimova, Presenting I.T.Habibullin
Bashkir State University
Institute of Mathematics, Ufa Scientific Center, RAS

In the talk the method for constructing the recursion operators and the Lax pairs for integrable discrete and continuous models will be discussed proposed in [1], [2]. Let us explain the essence of the algorithm, for the certainty take the class of the equations

$$u_t = f(u, u_1, u_2, ..., u_k), \quad \text{where} \quad u_j = D_x^j u. \tag{1}$$

Here D_x stands for the operator of the total differentiation with respect to x. At the first stage we construct a surface defined by the equation

$$H(U, U_1, \dots, U_m; u, u_1, \dots, u_{m_1}) = 0 \tag{2}$$

consistent with the linearization of (1):

$$U_t = \left(\frac{\partial f}{\partial u} + \frac{\partial f}{\partial u_1} D_x + \frac{\partial f}{\partial u_2} D_x^2 + ... + \frac{\partial f}{\partial u_k} D_x^k\right) U \tag{3}$$

for all values of the dynamical variables of the equation (1): u, u_1, u_2, ... We concentrate on the following important cases:

1. H is a linear form $H = \sum_{j=0}^{k} \alpha_j(u, u_1, \dots)U_j$;
2. H is a quadratic form $H = \sum_{i,j=0}^{k-1} \alpha_{ij}(u, u_1, \dots)U_i U_j$.

In the first case equation (2) is easily rewritten as $RU = \lambda U$, where R is the recursion operator for the equation (1) (see [1]). In the second case a couple of the equations (2)-(3) constitutes a nonlinear Lax pair for (1). By a simple point transformation the nonlinear pair is reduced to a true (not fake!) linear Lax pair for (1)) (see [2]). Efficiency of the algorithm is approved by numerous examples.

References

[1] I T Habibullin, A R Khakimova and M N Poptsova 2016 On a method for constructing the Lax pairs for nonlinear integrable equations, J. Phys. A: Math. Theor. 49 035202 (35pp).

[2] I T Habibullin, A R Khakimova 2017 On a method for constructing the Lax pairs for integrable models via quadratic ansatz, arXiv:1702.04533 [nlin.SI].

Noncommutative Solitons

Masashi Hamanaka
Nagoya University

For the last several years, extension of integrable systems and soliton theories to non-commutative (NC) spacetimes has been studied intensively and various kind of integrability aspects have been revealed (see e.g. [3, 4]). This is partially motivated by recent developments of noncommutative gauge theories on D-branes, where the noncommutative extension corresponds to existence of background flux.

In this conference, I would like to discuss exact soliton solutions to the noncommutative integrable (KP and its generalizations) hierarchy. The solutions are represented in terms of quasideterminants in a compact form. Quasideterminants, defined by Gelfand and Retakh in 1991, are noncommutative generalization of determinants in a sense (For a survey, see e.g. [1].) They play crucial roles in noncommutative solitons: the derivations and proofs are surplisingly simplified much more than commutative ones. This might suggest quasideterminant formulations would be more essential. Relation to the Yang-Mills theory and twistor theory is also found ([4] and references therein). Scattering process and conserved quantities are also discussed ([2, 3] and references therein).

References

[1] I. Gelfand, S. Gelfand, V. Retakh and R. L. Wilson, "Quasideterminants," Adv. Math. **193**, 56 (2005).

[2] M. Hamanaka, "Notes on exact multi-soliton solutions of noncommutative integrable hierarchies," JHEP **0702**, 094 (2007).

[3] M. Hamanaka, "Noncommutative Integrable Systems and Quasideterminants," AIP Conf. Proc. **1212**, 122 (2010).

[4] M. Hamanaka, "Noncommutative Solitons and Quasideterminants," Phys. Scripta **89**, 038006 (2014).

Fluid flow in a plane channel with deformable walls

Alexander Khe[1,2], Alexander Chesnokov[2,1], Valery Liapidevskii[1,2]
[1]Lavrentyev Institute of Hydrodynamics, Novosibirsk, Russia
[2]Novosibirsk State University, Novosibirsk, Russia

This work presents a mathematical model governing a flow in a channel with collapsible walls. The model describes both stationary and non-stationary (oscillatory) modes. The governing system is obtained under a two-layer flow assumption, used to describe pseudoshocks or regions of continuous transition from supercritical to subcritical flows.

A study of a fluid flow in a channel with a deformable wall is of high interest not only from the point of view of many applications, but also in terms of construction and investigation of mathematical models describing the mutual motion of the fluid and the channel wall. One of the major problems in this theory is a problem of description of a pulse wave propagation. To study these wave processes, one-dimensional models are widely use, which are similar in structure to the shallow water equations and equations of one-dimensional gas dynamics. The closure relation ("equation of state") in this case is a relation between the pressure and cross-section area of the channel lumen ("tube law"). In the case of axially symmetric channel of a circular cross section (a tube with an elastic wall) with positive transmural pressure (pressure difference in the fluid and in the environmental medium), one deals with a medium with a convex equation of state, which sufficiently well describes the propagation of the pulsation waves [1, 2]. However, in the case of a relatively small discharge when the transmural pressure is negative, the tube starts to shrink non-axisymmetrically which leads to new phenomena, such as for example sudden collapse of the tube and the development of large-amplitude self-excited oscillations. For negative transmural pressures, the equation of state looses its convexity, and in the shallow water theory, formation of the hydraulic jumps corresponding to rarefaction shock waves is possible [3, 4, 5].

An attempt to describe the flow instability faces two major challenges: the need for taking into account energy losses due to the development of a turbulent boundary layer and setup of the boundary conditions on the boundaries of flexible and rigid tubes [1]. These problems can be solved within the framework of a two-layer flow scheme. An approach developed in [6, 7] for description of a pseudoshock or a region of continuous transition from supercritical (supersonic) to subcritical (subsonic) flow in incompressible fluid and barotropic gas through the development of a turbulent boundary layer is used.

To illustrate the ability of the model to describe various flow regimes, we performed a number of computer simulations. The results of the calculations show that in the region of convexity of the equations of state, smooth pseudoshock is formed, which can be either stationary or moving upstream depending on the incoming flow parameters and the boundary conditions. At the same time if the initial cross-section area is below the inflection point of the tube law then even in the absence of flow perturbations and steady-state boundary conditions, the flow becomes substantially non-stationary and exhibits quasi-periodic waves of large amplitude.

The proposed mathematical model is able to describe such flow regimes due to introduction of energy loss in boundary layer, without which no oscillatory solutions exist [1]. The mathematical model can be used to simulate flows in Starling resistor, which is a basic experimental setup for many physiological flows such as cardiovascular system, pulmonary system and others.

Acknowledgements. This work was supported by the Russian Foundation for Basic Research (project 16-01-00156) and the Support Program for Leading Scientific Schools (grant NSh-8146.2016.1).

References

[1] M. Heil, O. E. Jensen. Flows in deformable tubes and channels: Theoretical models and biological applications. In: P. W. Carpenter, T. J. Pedley (eds.). *Flow past highly compliant boundaries and in collapsible tubes*, Fluid Mechanics and Its Applications. Vol. 72. Springer, 2003. Pp. 15–49.

[2] L. Formaggia, D. Lamponi, A. Quarteroni. One-dimensional models for blood flow in arteries. *Journal of Engineering Mathematics*, 47:251–276, 2003.

[3] A. H. Shapiro. Steady flow in collapsible tubes. *Journal of Biomechanical Engineering*, 99(3):126–147, 1977.

[4] C. Cancelli, T. J. Pedley. A separated-flow model for collapsible-tube oscillations. *Journal of Fluid Mechanics*, 157:375–404, 1985.

[5] B. S. Brook, S. A. E. G. Falle, T. J. Pedley. Numerical solutions for unsteady gravity-driven flows in collapsible tubes: evolution and roll-wave instability of a steady state. *Journal of Fluid Mechanics*, 396:223–256, 1999.

[6] V. Yu. Liapidevskii, A. A. Chesnokov. Mixing layer under a free surface. *Journal of Applied Mechanics and Technical Physics*, 55(2):299–310, 2014.

[7] I. I. Lipatov, V. Yu. Liapidevskii, A. A. Chesnokov. An unsteady pseudoshock model for barotropic gas flow. *Doklady Physics*, 61(2):82–86, 2016.

Regular triangulations of point sets and solitons in two dimensions II

Yuji Kodama
Department of Mathematics, The Ohio State University

This is a continuation of my talk in the previous PMNP 2015: I will report on an interesting connection between triangulations of a point set on the plane and soliton graphs in two-dimension. A soliton graph of two-dimensional integrable system is defined as a tropical (or ultra-discrete) limit of the contour plot of the soliton solution. Then the dual graph of the soliton graph gives a triangulation of a point set determined by the spectral parameters of the solution. Each soliton graph is generated by a point of the multi-time space of the hierarchy of the soliton equation. In particular, I will discuss the case of the KP solitons associated with the totally positive Grassmannian $\mathrm{Gr}(N, M)_{>0}$.

References

[1] Y. Kodama, L. K. Williams. KP solitons and total positivity for the Grassmannian, Invent. Math. 198 (2014), 637–699.

[2] Y. Kodama, L. K. Williams. The Deodhar decomposition of the Grassmannian and the regularity of KP solitons. Adv. Math. 244 (2013) 979–1032.

[3] J. Huang. Classification of soliton graphs on totally non-negative Grassmannians. PhD thesis, The Ohio State University (2015).

Nonlocal symmetries of integrable linearly degenerate equations: a comparative study

H. Baran, I. Krasil′shchik, O. Morozov, P. Vojčák
Mathematical Institute, Silesian University in Opava
Independent University of Moscow
& V. A. Trapeznikov Institute of Control Sciences
of Russian Academy of Sciences
65 Profsoyuznaya street, Moscow 117997, Russia
AGH University of Science and Technology, Kraków
Mathematical Institute, Silesian University in Opava

We continue here the study of Lax integrable equations. We consider four three-dimensional equations: (1) the rdDym equation $u_{ty} = u_x u_{xy} - u_y u_{xx}$, (2) the 3D Pavlov equation $u_{yy} = u_{tx} + u_y u_{xx} - u_x u_{xy}$; (3) the universal hierarchy equation $u_{yy} = u_t u_{xy} - u_y u_{tx}$, and (4) the modified Veronese web equation $u_{ty} = u_t u_{xy} - u_y u_{tx}$. For each equation, using the know Lax pairs and expanding the latter in formal series in spectral parameter, we construct two infinite-dimensional differential coverings and give a full description of nonlocal symmetry algebras associated to these coverings. For all the for pairs of coverings, the obtained Lie algebras of symmetries manifest similar (but not the same) structures: the are (semi) direct sums of the Witt algebra, the algebra of vector fields on the line, and loop algebras; all of them contain a component of finite grading. We also discuss actions of recursion operators on shadows of nonlocal symmetries.

References

[1] H. Baran, I.S. Krasil′shchik, O.I. Morozov, P. Vojčák, Nonlocal symmetries of Lax integrable equations: a comparative study, `arXiv:1611.04938v2 [nlin.SI]`

Spectral theory of periodic 2D Shrodinger operators integrable on a “singular” energy level

I. Krichever
Columbia University, New York, NY (USA)

An algebraic-geometrical construction of “finite-gap” 2D Shrodinger operators integrable on one energy level was proposed by Novikov and Veselov in 1980-s. In the talk we present further development of this theory related to a study of special energy levels for which the corresponding spectral curve has to be singular. The study is motivated by recent interest to complex saddle points of 2D sigma-models

Higher-order partial differential equations for description of the Fermi-Pasta-Ulam and the Kontorova-Frenkel models

Nikolay A. Kudryashov
National Research Nuclear University MEPHI

We consider the following dynamical system:

$$m\,\frac{d^2y_i}{dt} = F_{i+1,i} - F_{i,i-1} - f_0\,\sin\left(\frac{2\,\pi\,y_i}{a}\right), \qquad (i = 1, \dots, N), \tag{1}$$

where y_i measures the displacement of the i-th mass from equilibrium in time t, the force $F_{i+1,i}$ describes the nonlinear interaction between atpms dislocations in the crystal lattice in case of dislocations

$$F_{i+1,i} = \gamma\,(y_{i+1} - y_i) + \alpha\,(y_{i+1} - y_i)^2 + \beta\,(y_{i+1} - y_i)^3, \tag{2}$$

and f_0, a, γ, α, β are constant parameters of system (1).

The system of equations (1) is the generalization of some well-known dynamical systems. At $\alpha = 0$ and $\beta = 0$ the system of equations (1) is the mathematical model introduced by Frenkel and Kontorova for the description of dislocations in the rigid body [1]. In this model it was suggested that the influence of atoms in the crystal is taken into account by term $f_0\,\sin\frac{2\,\pi\,y_i}{a}$ but the atoms in case of dislocations interact by means of linear low. Assuming that $N \to \infty$ and $h \to 0$ where h is the distance between atoms, we can get the Sine-Gordon equation.

In case of $f = 0$ and $\beta = 0$ system of equations (1) is the well-known Fermi-Pasta-Ulam model [2] which was studied many times. It is known that the Fermi-Pasta-Ulam model is transformed at $N \to \infty$ and $h \to 0$ to the Korteweg-de Vries equation [3].

The main result of work [3] was the introduction of solitons as solutions of the Koryeweg-de Vries equation. It was shown in 1967 that the Cauchy problem for this equation can be solved by the Inverse Scattering transform [4].

Assuming $f_0 = 0$, $\alpha \neq 0$ and $\beta \neq 0$ at $N \to \infty$ and $h \to 0$ one can find the modified Korteweg-de Vries equation for the description of nonlinear waves.

In papers [5, 6] the author took into account high order terms in the Taylor series for the description of nonlinear waves in the Fermi-Pasta-Ulam and the Kontorova-Frenkel models assuming that $\alpha \neq 0$ and $\beta \neq 0$ and did not obtain nonlinear integrable differential equations in mass chain. Here we assume that the interaction between dislocations in crystal is described by means of nonlinear low at $\alpha \neq 0$ and $\beta \neq 0$ and consider the other equations. The aim of this talk is to present the nonlinear partial differential equations corresponding to dynamical system (1) and to discuss the properties of these equations.

References

[1] T.A. Kontorova, Ya. I. Frenkel, On theory of plastic deformation, JETP **89** (1938) 1340, 1349 (in Russian)

[2] E. Fermi, J.R. Pasta, S.Ulam, Studies of nonlinear problems, Report LA-1940, 1955. Los Alamos: Los Alamos Scientific Laboratory

[3] N.J. Zabusky, M.D. Kruskal, Interactions of solitons in a collisionless plasma and the recurrence of initial states, Physical Review Letters, **15** (1965) 240.

[4] C.S. Gardner, J.M. Greene, M.D. Kruskal, R.M. Miura, Method for solving the Korteweg de Vries equation. J. Math. Phys. **19** (1967) 1095

[5] N.A. Kudryashov, Refinment of the Korteweg-de Vries equation from the Fermi-Pasta-Ulam. Phys. Lett. A **379** (2015) 2610

[6] N.A. Kudryashov, Analytical properties of nonlinear dislocaton equation, Appl. Math. Lett. **69** (2017) 29

Soliton hierarchy with self-consistent sources: inverse scattering method and Darboux transformation

Runliang LIN[†], Xiaojun Liu[‡], Yunbo Zeng[†]
[†]Department of Mathematical Sciences, Tsinghua University, Beijing 100084, P.R.China
[‡]Department of Applied Mathematics, China Agricultural University, Beijing 100083, P.R. China

In this contribution, we will briefly review the inverse scattering method and the soliton solutions for the KdV hierarchy with self-consistent sources. Then we will show how to generate the KP hierarchy with self-consistent sources (KPHSCSs), which includes two types of KP equation with sources (KPES). The Wronskian solutions for the KPHSCSs can be obtained by the generalized dressing approach. We will also show that the bilinear identities and the tau-function for the KPHSCSs (and also for the B-type KPHSCSs) can be derived by introducing an auxiliary parameter, whose flow corresponds to the so-called squared eigenfunction symmetry of the KP hierarchy. The bilinear identities can generate all the Hirota's bilinear equations for the zero-curvature forms of the KPHSCSs, and the Hirota's bilinear equations obtained here for the KPES are in a simpler form by comparing with the results by X.B. Hu and H.Y. Wang.

References

[1] Runliang Lin, T.C. Cao, X.J. Liu, Y.B. Zeng, Bilinear identities for an extended B-type Kadomtsev-Petviashvili hierarchy, Theor. Math. Phys., 186 (2016) 307-319.

[2] A. Doliwa, Runliang Lin, Discrete KP equation with self-consistent sources, Phys. Lett. A, 378 (2014) 1925-1931.

[3] Runliang Lin, X.J. Liu and Y.B. Zeng, Binlinear identities and Hirota's bilinear forms for an extended Kadomtsev-Petviashvili hierarchy, J. Nonl. Math. Phys., 20 (2013) 214-228.

[4] Runliang Lin, H. Peng and M. Mañas, The q-deformed mKP hierarchy with self-consistent sources, Wronskian solutions and solitons, J. Phys. A: Math. Theor., 43 (2010) 434022 (15pp).

[5] Runliang Lin, Yunbo Zeng and Wen-Xiu Ma, Solving the KdV hierarchy with self-consistent sources by inverse scattering method, Physica A, 291 (2001) 287-298.

Integrable potentials by Darboux Transformations in rings and quantum/classical problems

Sergey Leble, Sergey Leble,
Immanuel Kant Baltic Federal University,
236004, Nevskogo 14, Kaliningrad, Russia

We consider two problems of an operator factorization in associative rings. One relates to the polynomial differential operators in differential rings and other to operators as polynomials of an automorphism in rings with automorphism [1]. The solution of the operators left/right division problems [2] leads to time-dependent Darboux transformations (DT) definition [1] on base of an intertwine relations and correspondent Miura maps and generalized Burgers equations. The generalized binary Bell polynomials are introduced as convenient terms for explicit expressions of the transformations. The generalized Burgers equation turns to be linearizable. Its stationary version is identified as (generalized) Miura relation [3]. Between the operators there are some of interest in classical or quantum mechanics. Its structure introduces a notion of potentials as the operators coefficients. Such potentials can be originated from some solvable one by the Darboux Transformations. In such case we name it integrable because we obtain its form and, simultaneously, its eigenfunctions. It means, that we derive a chain of solvable quantum problems with time-dependent potentials.

Trying to incorporate the mentioned algebraic properties at minimal operator level we consider Zakharov-Shabat - like problem . The subclasses that allow a DT symmetry (covariance) are considered from a point of view of dressing chain equations. Classic DT with a kind of Miura link is considered. Examples of sl(2,C) (NS equation) and quadratic creation-annihilation opertor algebras are considered.

References

[1] Matveev, V.B. (1998) Darboux transformations in associative rings and functional-difference equations, in J. Harnad and A. Kasman (eds.), The Bispectral Problem, AMS series CRM Proceedings and Lecture Notes, Montreal, 14, pp. 211-226.

[2] S. Leble, A. Zaitsev Division of differential operators, intertwine relations and Darboux transformations Rep. Math. Phys. 40 (2000) 165-174

[3] Evgeny V. Doktorov, Sergey B. Leble Dressing Method in Mathematical Physics (Springer-Verlag, 2007)

On the Hirota's tau function

Franco Magri
Dipartimento di Matematica, Universita' di Milano Bicocca

April 5, 2017

The purpose of the talk is to present a simple recursive scheme allowing to construct hierarchies of integrable systems of hydrodynamic type possessing a tau function, in the sense of Hirota . The construction makes use of the concept of Lenard's complex on a Haantjes manifold. If time shall permit, the new procedure will be compared to the bihamiltonian scheme, and the differences pointed out.

References

[1] F.Magri. Haantjes manifolds and Veselov systems.Theoretical and Mathematical Physics. 189 (2016), 1486–1499.

Maximally superintegrable classical and quantum Stäckel systems

Krzysztof Marciniak
Department of Science and Technology
Campus Norrköping, Linköping University
601-74 Norrköping, Sweden
krzma@itn.liu.se

This talk is based on a joint work with M. Blaszak, published in Ref. 3 below.

Stäckel separable systems are most conveniently introduced by the separation relations

$$\sigma(\lambda_i) +_{j=1}^{n} h_j \lambda_i^{\gamma_j} = \frac{1}{2} f(\lambda_i)\mu_i^2, \quad i = 1, \ldots, n, \quad \gamma_i \in \mathbf{N} \tag{1}$$

(where σ and f are arbitrary functions of one variable) on a $2n$-dimensional manifold $M = T^*Q$ equipped with the Poisson tensor $\Pi =_{i=1}^{n} \frac{\partial}{\partial\lambda_i} \wedge \frac{\partial}{\partial\mu_i}$. Solving (1) with respect to h_j we obtain n functions $h_j = h_j(\lambda, \mu)$ on M of the form $h_j = \frac{1}{2}\mu^T A_j(\lambda)\mu + U_j(\lambda)$ which are called (classical) Stäckel hamiltonians. In this talk I discuss maximal superintegrability of first classical and then quantum Stäckel systems, the latter in the framework of so called minimal quantization (see Ref 2 below). I prove a sufficient condition for a flat or constant curvature Stäckel system to be maximally superintegrable. Further, I prove a sufficient condition for a Stäckel transform to preserve maximal superintegrability and apply this condition to a broad class of Stäckel systems, which yields new maximally superintegrable systems as conformal deformations of the original systems. Finally, I demonstrate how to perform the procedure of minimal quantization to considered systems in order to produce quantum superintegrable and quantum separable systems.

References

[1] Błaszak, M. and Marciniak, K. (2012), *On Reciprocal Equivalence of Stäckel Systems*. Studies in Applied Mathematics. doi: 10.1111/j.1467-9590.2011.00544.x

[2] Błaszak M., Marciniak K. and Domanski Z., *Separable quantizations of Stäckel systems*, Annals of Physics **371** (2016) pp. 460-471, doi:10.1016/j.aop.2016.06.007

[3] Błaszak M. and Marciniak K., *Classical and quantum superintegrability of Stäckel systems*, Symmetry, Integrability and Geometry: Methods and Applications **13** (2017) 008 doi.org/10.3842/SIGMA.2017.008

LD Faddeev, long range scattering and some unsolved problems in the IST method.

VB Matveev

L.D. Faddeev, contributed in the long range scattering theory in several ways. First, together with P.P. Kulish, he developed a new approach to regularise infrared divergences in QED [1]. His earlier work [2] was devoted to removal of the ultra-violet divergences in QFT.

In my talk, together with some personal recollections, I will explain how the works [1, 2] both stimulated important new developments in quantum long range scattering theory and I will also comment on the related unsolved problems in the IST method.

References

[1] L.D. Faddeev, P.P.Kulish. Asymptotic conditions and infrared divergences in quantum electrodynamics. Th.Math. Phys.4 (1970), 745–757.

[2] L.D. Faddeev. On the separation of self-action and scattering effects in perturbation theory. Soviet Physics Doklady 8 (1964), 881- 883

The space of symmetric squares of hyperelliptic curves: infnite-dimensional Lie algebras and polynomial integrable dynamical systems on $\mathbb{C}^4$

V.M. Buchstaber and A.V. Mikhailov

We construct Lie algebras of vector fields on universal bundles of symmetric squares of hyperelliptic curves of genus $g = 1, 2, \ldots$. For each of these Lie algebras, the Lie subalgebra of vertical fields has two commuting generators, while the generators of the Lie subalgebra of projectable vector fields determines the canonical representation of the Lie subalgebra with generators L_{2q}, $q = -1, 0, 1, 2, \ldots$, of the Witt algebra. The vertical vector fields yield two commuting integrable polynomial dynamical systems on $\mathbb{C}^4$, while the projectable fields provide us with the Lie algebra of derivations of the solutions with respect to the curve parameters. The method can be extended to higher symmetric powers and more general algebraic curves.

Pseudo-Wronskians of Hermite polynomials

David Gomez-Ullate, Yves Grandati, Robert Milson
U. Complutense
U. Lorraine
Dalhousie U.

In this talk we consider a certain class of identities involving determinants of Hermite polynomials. The prototypical example of such a determinant is the Wronskian of a finite set of Hermite polynomials. Remarkably, all such Wronskians may be re-expressed as an infinity of other determinants of a certain structure, determinants that we refer to as *pseudo-Wronskians*. The theory can be easily understood in terms of Maya diagrams and partitions and allows for the construction of the optimal determinantal representation of Hermite wronskians. We will then apply these results to give novel descriptions of rational solutions of the Painleve IV equation.

Dressing networks: towards an integrability approach to collective and complex phenomena

Antonio Moro

A large variety of real world systems can be naturally modelled by networks, i.e. graphs whose nodes represent the components of a system linked (interacting) according to specific statistical rules. A network is realised by a graph typically constituted by a large number of nodes/links. Fluid and magnetic models in Physics are just two among many classical examples of systems which can be modelled by simple or complex networks. In particular "extreme" conditions (thermodynamic regime), networks, just like fluids and magnets, exhibit a critical collective behaviour intended as a drastic change of state due to a continuous change of the model parameters. Using an approach to thermodynamics, recently introduced to describe a general class of van der Waals type models and magnetic systems in mean field approximation, we analyse the integrable structure of corresponding networks and use the theory of integrable conservation laws combined with a suitable "dressing" procedure to calculate order parameters outside and inside the critical region.

Extensions of integrable systems via deformations of binary Darboux transformations

Folkert Müller-Hoissen
Max Planck Institute for Dynamics and Self-Organization,
37077 Göttingen, Germany

Joinly with O. Chvartatskyi and A. Dimakis, we explored in [1] a method of deformation of the "potential" that plays a crucial role in binary Darboux transformations. Such a deformation either leads to an extension of an integrable system by so-called "self-consistent sources", or to the kind of systems that are known to arise via a squared eigenfunction symmetry reduction from a higher-dimensional system (like KP). The essence of the method can be expressed concisely in the framework of bidifferential calculus. This then allows a straight application to a variety of continuous and discrete integrable systems [1, 2]. In this talk, we present an introduction to the method and an overview of corresponding results.

References

[1] O. Chvartatskyi, A. Dimakis, F. Müller-Hoissen. Self-consistent sources for integrable equations via deformations of binary Darboux transformations. Lett. Math. Phys. 106 (2016), 1139–1179.

[2] F. Müller-Hoissen, O. Chvartatskyi, K. Toda. Generalized Volterra lattices: Binary Darboux transformations and self-consistent sources. J. Geom. Phys. 113 (2017) 226–238.

Two Interacting Surfaces and Curves Corresponding to Periodic Solutions of Manakov System

Akbota Myrzakul, Ratbay Myrzakulov
Eurasian National University, Astana, Kazakhstan

In this talk, we want to consider the following Manakov system [1]

$$iq_{1t} + q_{1xx} + 2(|q_1|^2 + |q_2|^2)q_1 = 0, \tag{1a}$$

$$iq_{2t} + q_{2xx} + 2(|q_1|^2 + |q_2|^2)q_2 = 0, \tag{1b}$$

where q_j are complex functions. The gauge and geometrical equivalent counterpart of the Manakov system (1) is given by [2]

$$iA_t + \frac{1}{2}[A, A_{xx}] + iu_1 A_x + v_1[\sigma_3, A] = 0, \tag{2a}$$

$$iB_t + \frac{1}{2}[B, B_{xx}] + iu_2 B_x + v_2[\sigma_3, B] = 0, \tag{2b}$$

where u_j and v_j are some real functions (potentials) and

$$A = \begin{pmatrix} A_3 & A^- \\ A^+ & -A_3 \end{pmatrix}, \quad B = \begin{pmatrix} B_3 & B^- \\ B^+ & -B_3 \end{pmatrix}, \quad A^2 = B^2 = I. \tag{3}$$

It is the 2-layer M-LIII equation. It is well known that the 2-layer M-LIII equation and the Manakov system are integrable by IST. The Darboux transformation (DT) for the simple periodic "seed" solution of the Manakov system (1) is presented. Using this DT, the exact solutions of the Manakov system is considered. Next, using the Sym-Tafel formula, the two interacting surfaces and curves related with the solutions of the Manakov system were constructed.

References

[1] S.V. Manakov. On the theory of two-dimensional stationary self-focusing of electromagnetic waves. Sov. Phys. JETP, **38**, 248 (1974).

[2] Akbota Myrzakul, Ratbay Myrzakulov. Integrable motion of two interacting curves, spin systems and the Manakov system. International Journal of Geometric Methods in Modern Physics (accepted), (2017). [arXiv:1606.06598]

Solutions of the KP-hierarchy corresponding to degenerate trigonal curves

Atsushi Nakayashiki
Department of Mathematics, Tsuda University

We study solutions of the KP-hierarchy corresponding to some trigonal curves, called (3,s) curves, and their degenerations. It is well known that solutions corresponding to rational degeneration of hyperelliptic curves with only ordinary double points are soliton solutions. We study what happens if we consider trigonal curves. We determine the limits of theta function solutions when the (3,s) curve degenerate to singular rational curves with only ordinary triple points. They are not solitons but are some intermidiate solutions between solitons and rational solutions. We extensively use Sato Grassmannian since it directly connects solutions of the KP-hierarchy to the defining equations of algebraic curves.

References

Quantum variational principle and quantum multiforms

S.D. King, F.W. Nijhoff
School of Mathematics, University of Leeds

A modern notion of integrability is that of multidimensional consistency (MDC) which classically implies the coexistence of commuting dynamical flows in several (possibly infinitely many) variables imposable on one and the same dependent variable. Recently a novel variational principle was introduced to capture this phenomenon within a Lagrangian framework, namely that of Lagrangian multiforms [1, 2]. We will review some of the classical results and propose a quantum analogue of this theory in terms of quantum mechanical propagators.

References

[1] S.B. Lobb and F.W. Nijhoff, *Langrangian multiforms and multidimensional consistency*, J. Phys. A: Math & Theor **42** (2009) 454013.

[2] S.B. Lobb and F.W. Nijhoff, *A variational principles for discrete integrable systems*, arXiv: 1312.1440v2, (submitted to SIGMA).

[3] S. King and F.W. Nijhoff, *Quantum variational principle and quantum multiform structure: the case of quadratic Lagrangians*, arXiv: 1702.08709, (submitted to Annals of Physics).

Towards global classification of integrable $2+1$-dimensional equations

V.S. Novikov

Department of Mathematical Sciences,
Loughborough University, LE11 3TU, UK

Following the global (at all orders) classification of $1+1$-dimensional scalar polynomial homogeneous equations [1] we extend this result to $2+1$-dimensions and classify scalar polynomial homogeneous equations with degenerate dispersion laws [2]. We first classify the rational degenerate dispersion laws and show that the admissible cases are the dispersion relations of KP, BKP/CKP and Veselov-Novikov type only. In the framework of the Perturbative Symmetry Approach [3] we then classify the homogeneous polynomial equations with these 3 classes of dispersion laws at all orders.

References

[1] Jan A Sanders, Jing Ping Wang, On the integrability of homogeneous scalar evolution equations, Journal of Differential Equations 147, 2, (1998) 410-434.

[2] AM Balk, EV Ferapontov, Invariants of wave systems and web geometry, AMS Translations, 1-30, (1998).

[3] Alexander V Mikhailov, Vladimir S Novikov, Perturbative Symmetry Approach, J. Phys. A: Math. Theor., 35, 22, 4775 (2002).

Superintegrability and linearity

M.C. Nucci,
University of Perugia & INFN-Perugia, Italy

In 2011, a two-dimensional superintegrable system such that the corresponding Hamilton-Jacobi equation does not admit separation of variables in any coordinates was studied in [1]. In [2] it was found that its Lagrangian equations can be transformed into a linear third-order equation by applying the reduction method [3].

In [4], a conjecture was made, namely that all classical superintegrable systems in two-dimensional space are linearizable. Indeed, known examples in two-dimensional Euclidean space were shown to yield linear equations, e.g. the Tremblay-Turbiner-Winternitz system [5].

In [6] and [7] the hidden linearity of superintegrable systems in two-dimensional non-Euclidean space, e.g. Darboux spaces [8], [9], was determined.

We will show that many other maximally superintegrable systems, e.g. Evans' systems [10], can be transformed into linear equations by applying the reduction method in order to search for hidden Lie symmetries.

References

[1] S. Post and P. Winternitz. A nonseparable quantum superintegrable system in 2D real Euclidean space. J. Phys. A: Math. Theor. 44 (2011) 162001.

[2] M.C. Nucci and S. Post. Lie symmetries and superintegrability. J. Phys. A: Math. Theor. 45 (2012) 482001.

[3] M.C. Nucci. The complete Kepler group can be derived by Lie group analysis. J. Math. Phys. 37 (1996) 1772–1775.

[4] G. Gubbiotti and M.C. Nucci. Are all classical superintegrable systems in two-dimensional space linearizable? J. Math. Phys. 58 (2017) 012902.

[5] F. Tremblay, A.V. Turbiner, and P. Winternitz. An infinite family of solvable and integrable quantum systems on a plane. J. Phys. A: Math. Theor. 42 (2009) 242001.

[6] M.C. Nucci. Ubiquitous symmetries. Theor. Math. Phys. 188 (2016) 1361–1370.

[7] G. Gubbiotti and M.C. Nucci. Superintegrable systems in non-Euclidean plane: hidden symmetries leading to linearity. (2017) in preparation.

[8] E.G. Kalnins, J.M. Kress, P. Winternitz. Superintegrability in a two dimensional space of nonconstant curvature. J. Math. Phys. 43 (2002) 970–983.

[9] E.G. Kalnins, J.M. Kress, W. Miller, P. Winternitz. Superintegrable systems in Darboux spaces. J. Math. Phys. 44 (2003) 5811–5848.

[10] N.W. Evans. Superintegrability in classical mechanics. Phys. Rev. A 41 (1990) 5666–5676.

Hydrodynamics of the FPU problem and its integrable aspects

M. Gallone[a], A. Ponno[b], B. Rink[c]

[a] SISSA, Trieste
[b] Università di Padova
[c] Vrije Universiteit, Amsterdam

It is shown that the the resonant normal form equations of motion of generic Fermi-Pasta-Ulam systems (α-model class) are integrable to the second perturbative order. In particular, the normal form Hamiltonian of the system consists, up to a remainder, of the first two integrals of the KdV-hierarchy [1],[2]. Such a second order tangency to an integrable model explains why the FPU systems display a rather slow approach to the statistical equilibrium. The result holds in the limit of large size, at finite small energy per particle, and implies a lower bound to the relaxation time that is compatible with the one numerically observed [3]. Non-generic models (β-model class, and so on) are also treated. A conjecture on the break down of the integrability of the normal form at a finite order is discussed.

References

[1] M. Gallone, A. Ponno, B. Rink, *Hydrodynamics of the FPU problem and its integrable aspects*; preprint, 2017.

[2] M. Gallone, A. Ponno, B. Rink, *An integrable normal form for the Fermi-Pasta-Ulam problem*; preprint, 2017.

[3] G. Benettin and A. Ponno, *Time-Scales to Equipartition in the Fermi-Pasta-Ulam Problem: Finite-Size Effects and Thermodynamic Limit*, J. Stat. Phys. **144** (2011), 793-812.

D. J. Benney

New Three-Dimensional Integrable Discrete Systems and Their Dispersionless Limits

M.V. Pavlov,
Department of Mechanics and Mathematics,
Novosibirsk State University,
Pirogova street 2, 630090, Novosibirsk, Russia

In this talk we utilise a simple mechanism, which allows to construct three-dimensional discrete integrable systems. One of these systems is a generalisation of the Vector (Manakov) Nonlinear Schroedinger Equation. Separately we consider a new multi-component reduction of the Benney system describing one-dimensional propagation of long waves on surface of finite depth fluid. We show that this reduction is a continuous limit (a continuous limit for discrete systems is an analogue of dispersionless limit for dispersive systems) of the new discrete vector system.

References

, *Some properties of long nonlinear waves, Stud. Appl. Math. 52 (1973), pp. 45–50.*

, Symmetry constraints for dispersionless integrable equations and systems of hydrodynamic type, Phys. Lett. A 330 (2004), pp. 448–459.

, *Stability of shear shallow water flows with free surface to appear in SIAM (2017) https://arxiv.org/abs/1610.04331.*

, Benney equations and quasi-classical approximation in the method of inverse problem, Funk. Anal. Prilozh. 14 (1980), pp. 15–24.

Continuous symmetries of the Hirota difference equations

A.K.Pogrebkov
Steklov Mathematical Institute of Russian Academy of Science
and National Research University "Higher School of Economics"
Moscow, Russia

Continuous symmetries of the Hirota difference equation, commuting with shifts of independent variables, are derived by means of the dressing procedure. Action of these symmetries on the dependent variables of the equation is presented. Commutativity of these symmetries enables interpretation of their parameters as “times” of the nonlinear integrable partial differential-difference and differential equations. Examples of equations resulting in such procedure and their Lax pairs are given. Besides these ordinary symmetries, the additional ones are introduced and their action on the Scattering data is presented.

New findings on Perlick system I: from the algebra of symmetries to the geometry of the orbits

O. Ragnisco
Instituto Nazionale di Fisica Nucleare, Sezione di Roma 3
Via della Vasca Navale 84, 00146 Roma, Italy

We investigate the main algebraic properties of the maximally superintegrable system known as "Perlick system type I", by using a classical variant of the so called factorization method. As it is expected for maximally superintegrable systems, the algebraic structure sheds light also on the geometric features of the trajectories. The crucial role played by the rational parameter β will be seen "in action".

References

[1] Ş. Kuru, J. Negro and O. Ragnisco, *The Perlick system type I: from the algebra of symmetries to the geometry of the trajectories*, arXiv:1705.04618v1

Solitons and spin waves in the spiral structure of magnets

Kiselev V.V., Raskovalov A.A.
Institute of Metal Physics UB RUS, Russia
Institute of Metal Physics UB RUS, Russia

Solitons and waves on the spatially inhomogeneous ground state background are still little known. For the first time, the solitons against the nonlinear cnoidal wave background were described in [1] by the inverse scattering method in the framework of KdV - model. We present the modification of the inverse scattering problem to integrate the sine-Gordon equation and to study the solitons and waves for magnets with a spiral (stripe domain) structure [2]. The U-V-pair rationally depends on the spectral parameter. However, the problem of analytical functions conjugation, which is the basic of the method, is formulated not on the complex plane, but on the Riemann surface, which is topologically equivalent to a torus. The Riemann surface is naturally related with the periodic spiral structure.

To investigate the nonlinear dynamics of spiral structure, we use the technique of «dressing» the partial solution of sine-Gordon equation by means of Riemann problem on a torus. The «dressing» technique allows us to make complete analysis of solitons and waves in the spiral structure at localized initial conditions and boundary conditions at infinity. Due to the periodic background, the solutions of the Riemann problem become quasi-periodic («Bloch») functions. This is the difference between our problem and the problem to integrate the elliptic sine-Gordon equation [3], [4].

We have studied the soliton and multi-soliton solutions against the spiral structure background in detail [5]–[8]. It has been shown, that the formation and motion of solitons are accompanied by the local translations of the spiral structure and by the oscillations of its domain walls, which manifest themselves as «precursors» and «tails» of the solitons. The large-time behavior of the weak-nonlinear dispersive wave field, generated by an initial localized perturbation of the structure has been investigated. The ways of observing and exciting the solitons in the spiral structure of magnets are discussed.

This work was carried out within the state assignment of FASO of Russia (theme «Quantum», no. 01201463332) and is supported in part by the project of UB RUS no. 15-8-2-7.

References

[1] E.A. Kuznetsov, A.V. Mikhailov. Stability of stationary waves in nonlinear weakly dispersive media. Sov. Phys. – JETP. 40, No 5 (1974), 855-859.

[2] V.V. Kiselev, A.A. Raskovalov. Nonlinear dynamics of a quasi-one-dimensional helicoidal structure. Theor. Math. Phys. 173, No 2 (2012), 1565-1586.

[3] A.B. Borisov, V.V. Kiseleiv. Vortex dipoles on a solution lattice background: Solution of the boundary-value problem by the inverse spectral transform. Physica D. 111 (1998), 96-128.

[4] V.V. Kiseleiv. 2D-vortices in incommensurate (stripe-domain) magnetic structures. Phys. Met. Mellogr. 95. Suppl. 1 (2003), 28-34.

[5] V.V. Kiselev, A.A. Raskovalov. Nonlinear dynamics of breathers in the spiral structures of magnets. JETP. 122, No 6 (2016), 1094-1103.

[6] V.V. Kiselev, A.A. Raskovalov. Solitons of electric polarization in multiferroics. Physics of the Solid State. 58, No 3 (2016), 497-502.

[7] V.V. Kiselev, A.A. Raskovalov. Twokink excitation in a spiral magnetic structure. Low Temperature Physics. 42, No 1 (2016), 50-56.

[8] V.V. Kiselev, A.A. Raskovalov. Solitons and nonlinear waves in the spiral magnetic structures. Chaos, Solitons and Fractals. 84 (2016), 88-103.

Ermakov-Painlevé II Reduction in Cold Plasma Physics. Application of a Bäcklund Transformation

Colin Rogers
School of Mathematics and Statistics,
The University of New South Wales,
Sydney, NSW 2052, Australia

A class of symmetry transformations of a type originally introduced in a nonlinear optics context is used here to isolate an integrable Ermakov-Painlevé II reduction of a resonant NLS equation which encapsulates a nonlinear system in cold plasma physics descriptive of the uni-axial propagation of magneto-acoustic waves. A Bäcklund transformation is used in the iterative generation of novel classes of solutions to the cold plasma system which involve either Yablonski-Vorob'ev polynomials or classical Airy functions.

Sub-symmetries and their applications

V. Rosenhaus, and Ravi Shankar
California State University, Chico
University of Washington

We introduce a generalization of symmetries of systems of differential equations. An infinitesimal transformation is a sub-symmetry if it leaves a subsystem invariant on the solution set of the entire system. We show that sub-symmetries can be very helpful in the decoupling of a system of equations. For a large class of differential systems, we demonstrate a natural association between sub-symmetries and local conservation laws.

We prove an analogue of the first Noether Theorem for sub-symmetries, and demonstrate its applications. We also discuss the role of sub-symmetries in the deformation of known conservation laws of a system into other (often, new) conservation laws, and demonstrate that in this regard, a sub-symmetry can be a considerably more powerful tool than a regular symmetry, [1], [2].

References

[1] V. Rosenhaus, Ravi Shankar. Sub-symmetries I. Main properties and applications.

https://www.researchgate.net/publication/313892895_Sub-symmetries_I_Main_properties_and_applications.

[2] V. Rosenhaus, Ravi Shankar. Sub-symmetries II. Sub-symmetries and Conservation Laws.

https://www.researchgate.net/publication/313892898_Sub-symmetries_II_Sub-symmetries_and_Conservation_Laws.

School of Mathematics, University of Leeds, UK siruleeds@gmail.com

Sine-Gordon field theory vs. relativistic Calogero-Moser N-particle sytems: An overview

Simon Ruijsenaars

June 7, 2017

Abstract

The sine-Gordon equation $\phi_{xx} - \phi_{tt} = \sin(\phi)$ is one of the earliest and most important examples of an integrable 2D field theory solved in great detail via the IST. The quantum counterpart of the IST revealed that the classical soliton, antisoliton and breather solutions turn into solitonic fermions, antifermions and their bound states, with particle number preservation under scattering and factorized amplitudes governed by the Yang-Baxter equations. In this talk we survey the relation between the relativistic sine-Gordon model and (a special case of) the hyperbolic relativistic integrable N-particle systems of Calogero-Moser type. More specifically, we review the intimate link between the classical version of the latter and the particle-like sine-Gordon solutions, and present compelling evidence that this soliton-particle correspondence turns into a physical equivalence on the quantum level, in the sense that the same scattering amplitudes and bound state energies arise in the quantum field-theoretic and N-particle models.

Analytic description of the exact rogue wave recurrence in the periodic NLS Cauchy problem

P. G. Grinevich, P. M. Santini
Landau Institute of Theoretical Physics, Chernogolovka, Russia
Department of Physics, University of Roma "La Sapienza", Roma, Italy

Rogue Waves (RWs) are transient waves appearing, apparently from nowhere, in several physical contexts, like in water waves, nonlinear optics, Bose-Einstein condensates, liquid Helium, etc.. [1]. It is by now well understood that the basic physical mechanism for the appearance of RWs is the modulation instability of the amplitude of quasi monochromatic waves [2, 3], and the simplest nonlinear model for the description of such phenomenon is the integrable [4] self-focusing Nonlinear Schrödinger Equation (sfNLS) $iu_t + u_{xx} + 2|u|^2u = 0$ and its unstable background, described by the simple exact solution $u_0(t) = e^{2it}$. We have recently solved the Cauchy problem of the sfNLS equation for generic, x-periodic, $O(\epsilon)$ initial perturbations of the background, in the case of N unstable modes, with $1/N \gg \epsilon$. The solution of the problem, given in terms of elementary functions in different time intervals, describes the exact recurrence of an infinite sequence of rogue waves, and the n^{th} rogue wave of the sequence is described, in the finite time interval in which it appears, by the N breather solution of Akhmediev type, whose paremeters are expressed in terms of the initial data via elementary functions. Explicit details are given for $N = 1, 2$. The above results have been obtained in [5] and in [6] through two different approaches: the finite gap method and, respectively, matched asymptotic expansions techniques.

References

[1] M. Onorato, S. Residori, U. Bortolozzo, A. Montina, F.T. Arecchi, Rogue waves and their generating mechanisms in different physical contexts, Physics Reports 528 (2013) 47-89.

[2] T. B. Benjamin, J. E. Feir, The disintegration of wave trains on deep water. Part I. Theory, Journal of Fluid Mechanics 27 (1967) 417-430.

[3] V. E. Zakharov, Stability of period waves of finite amplitude on surface of a deep fluid, Journal of Applied Mechanics and Technical Physics 9 (1968) 190-194.

[4] V. E. Zakharov and A. B. Shabat, Exact theory of two-dimensional delf-focusing and one-dimensional self-modulation of waves in nonlinear media, Sov. Phys. JETP 34, 1, 62-69, 1972.

[5] P. G. Grinevich and P. M. Santini, Analytic description of the exact recurrence of rogue waves in the periodic NLS setting, using the finite gap method. Preprint (2017).

[6] P. G. Grinevich and P. M. Santini, Analytic description of the exact recurrence of rogue waves in the periodic NLS setting, using matched asymptotic expansions. Preprint (2017).

Integrable discretisation of hodograph-type systems, hyperelliptic integrals and Whitham equations

B.G. Konopelchenko, W.K. Schief
Department of Mathematics and Physics "Ennio de Giorgi",
University of Salento and sezione INFN, Lecce, 73100, Italy
School of Mathematics and Statistics, The University of
New South Wales, Sydney, NSW 2052, Australia

Based on the well-established theory of discrete conjugate nets in discrete differential geometry, we propose and examine discrete analogues of important objects and notions in the theory of semi-Hamiltonian systems of hydrodynamic type. In particular, we present discrete counterparts of (generalised) hodograph equations, hyperelliptic integrals and associated cycles, characteristic speeds of Whitham type and (implicitly) the corresponding Whitham equations. By construction, the intimate relationship with integrable system theory is maintained in the discrete setting.

Some Unfamiliar Aspects of Bäcklund Transformations

Alexander G. Rasin
Department of Mathematics,
Ariel University, Ariel 40700, Israel
Jeremy Schiff
Department of Mathematics,
Bar-Ilan University, Ramat Gan, 52900, Israel

This presentation gives an overview of our recent work on Bäcklund transformations (BTs), including the papers [1, 2] and some more recent results. The BTs for the Camassa-Holm (CH) [1] and Harry Dym (HD) equations are examples of BTs that act on both the dependent *and* the independent variables of an equation. Nevertheless, there exist algebraic superposition principles for a pair of BTs, and the application of these gives a simple construction of numerous interesting solutions, such as cuspon-soliton superpositions of CH. For the Boussinesq equation [2], as well as other equations associated with 3×3 matrix Lax pairs, there does not exist a pure-algebraic superposition principle for a pair of BTs, but remarkably there does for three. In addition to standard soliton solutions, the Boussinesq equation has "merging soliton" solutions, in which a pair of solitary waves merge into a single one. We present various (nonsingular and singular) superpositions of these; such solutions have been discovered several times over the years, for example in [3].

We elucidate how for many integrable equations the existence of an infinite number of conservation laws, an infinite hierarchy of local symmetries and so-called nonlocal symmetries can all be derived directly from a BT (and its algebraic superposition principle).

References

[1] Alexander Rasin and Jeremy Schiff, *Bäcklund Transformations for the Camassa Holm Equation*, J. Nonlinear Science, **27** 45-69 (2017).

[2] Alexander Rasin and Jeremy Schiff, *Bäcklund Transformations for the Boussinesq Equation and Merging Solitons*, arXiv:1701.00243.

[3] L.V.Bogdanov and V.E.Zakharov, *The Boussinesq equation revisited*, Physica D 165 137-162 (2002).

Hamiltonian aspects of the Classical Inverse Scattering Method in the work of L.D.Faddeev and his laboratory

Michael Semenov-Tian-Shansky
Steklov Mathematical Institute, St.Petersburg, Russia and Université de Bourgogne, Dijon, Fr

Professor Faddeev who passed away in last February was at the origin of many analytic tools and concepts that have shaped the Classical Inverse Scattering Method. I shall discuss his contribution to the understanding of the Hamiltonian aspects of this method, in connection with the main analytic tools such as the Riemann problem and with the underlying geometry of the phase spaces of non-linear integrable systems. The development of the Quantum Inverse Scattering Method shed a new light on this subject and brought about new important ideas into the Poisson and symplectic geometry.

L.D. Faddev's work on quantum integrability

E. Sklyanin
University of York, UK

A review of L.D. Faddeev's works on quantum integrability, quantum inverse scattering method and quantum groups, with personal recollections.

Separation of variables in anisotropic models and non-skew-symmetric elliptic r-matrix

T. Skrypnyk
Universita degli Studi di Milano-Bicocca,
via Roberto Cozzi, 53, 20125, Milano, Italy

Using the classical r-matrix approach [1] we resolve a problem of separation of variables for all the classical integrable hamiltonian systems possessing Lax matrices that satisfy linear Poisson brackets with non-skew-symmetric, non-dynamical elliptic $so(3)\otimes so(3)$-valued classical r-matrix. Using the corresponding Lax matrices we present a general form of the "separating functions" $B(u)$ and $A(u)$ that generate the coordinates and the momenta of separation for the associated models [2]. We consider several examples and perform the separation of variables for the classical anisotropic Euler's top, Steklov-Lyapunov model of the motion of anisotropic rigid body in the liquid [3]-[4], two-spin generalized Gaudin model [4] and "spin" generalization of Steklov-Lyapunov model [2].

References

[1] Sklyanin E. *Progress of Theoretical Physics*, Supplement No.**118**, 35 (1995).

[2] Skrypnyk, T. *Lett Math Phys* (2016). doi:10.1007/s11005-016-0920-0

[3] Steklov V. On the motion of a rigid body in a fluid, Kharkov, (1903).

[4] Lyapunov A. *Soob. Khark. Matem. Ob.*,ser. 2,**vol.** 4,no. 1-2, pp.81-88(1893).

[4] Skrypnyk T. *J. Geom. Phys.* **57** , no.1, 53-67 (2006).

Superintegrable 3D systems in a magnetic field corresponding to Cartesian separation of variables

Antonella Marchesiello†and Libor Šnobl‡,
†Faculty of Information Technology,
Czech Technical University in Prague
‡Faculty of Nuclear Sciences and Physical Engineering,
Czech Technical University in Prague

We consider three dimensional superintegrable systems, i.e. Hamiltonian systems that have more integrals of motion than degrees of freedom, in a magnetic field. Such systems are described by

$$H = \frac{1}{2}(\vec{p} + \vec{A})^2 + V(\vec{x}) \tag{1}$$

where $\vec{p}$ is the momentum, $\vec{A}$ is the vector potential and V is the effective potential. The magnetic field $\vec{B} = \mathrm{rot}\vec{A}$ is assumed to be nonvanishing. Superintegrability of such systems was studied in two spatial dimensions in [1]. In three dimensions the conditions for integrals of motion, assumed to be up to second order in the momenta. i.e. of the form

$$X = \sum_{j=1}^{3} h_j(\vec{x}) p_j^A p_j^A + \sum_{j,k,l=1}^{3} \frac{1}{2}|\epsilon_{jkl}| n_j(\vec{x}) p_k^A p_l^A + \sum_{j=1}^{3} s_j(\vec{x}) p_j^A + m(\vec{x}), \tag{2}$$

with $\vec{p}^A = \vec{p} + \vec{A}$, were studied in [2] where also several classes of these systems were investigated.

In [3] the question of integrability of systems with magnetic field in three dimensions was studied for the class of systems which separate in Cartesian coordinates in the limit when the magnetic field vanishes, i.e. they possess two second order integrals of motion of the "Cartesian type". For all systems constructed there we look for additional integrals up to second order in momenta which make these systems minimally or maximally superintegrable and study their trajectories. We find three classes of minimally superintegrable systems which lead to four maximally superintegrable subclasses. Thus, despite the fact that in the analogous case in two dimensions the only possibility for quadratic superintegrability is for the magnetic field $\vec{B}$ and the effective potential V both constant, in three dimensions the results are much richer.

We explicitly demonstrate that the structure of the leading order terms of the Cartesian type integrals needs to be considered in a more general form than for the case without magnetic field.

References

[1] J. Bérubé, P. Winternitz. Integrable and superintegrable quantum systems in a magnetic field. J. Math. Phys. 45 (2004), no. 5, 1959–1973.

[2] A. Marchesiello, L. Šnobl, and P. Winternitz. Three-dimensional superintegrable systems in a static electromagnetic field. J. Phys. 48 (2015), no. 39, 395206, 24pp.

[3] A. Zhalij. Quantum integrable systems in three-dimensional magnetic fields: the Cartesian case. J. Phys. Conf. Ser. 621 (2015) 012019, 8pp.

Quantum spectral curve of melting crystal model and its 4D limit

Kanehisa Takasaki, Kinki University

The melting crystal model is a statistical model of 3D Young diagrams. Its partition function can be reduced to the sum

$$Z = \sum_{\lambda\in\mathcal{P}} s_\lambda(q^{-\rho})^2 Q^{|\lambda|}, \quad s_\lambda(q^{-\rho}) = \frac{q^{-\kappa(\lambda)/4}}{\prod_{(i,j)\in\lambda}(q^{-h(i,j)/2} - q^{h(i,j)/2})},$$

over the set $\mathcal{P}$ of ordinary partitions. $s_\lambda(q^{-\rho})$ is a kind of principal specialization of the infinite-variate Schur function s_λ. This sum can be identified with the instanton partition function of 5D $\mathcal{N} = 1$ supersymmetric $U(1)$ Yang-Mills theory on $\mathbf{R}^4 \times S^1$. Its deformation

$$Z(\boldsymbol{t}) = \sum_{\lambda\in\mathcal{P}} s_\lambda(q^{-\rho})^2 Q^{|\lambda|} e^{\phi(\boldsymbol{t},\lambda)}, \quad \phi(\boldsymbol{t},\lambda) = \sum_{k=1}^{\infty} t_k \phi_k(\lambda),$$

by a set of external potentials $\phi_k(\lambda)$ is known to be a tau function of the KP hierarchy [1]. An associated quantum spectral curve is obtained by the method of q-difference Kac-Schwarz operators [2]. In the 4D limit where the radius R of S^1 tends to 0, this quantum curve turns into the quantum spectral curve

$$\left(e^{\hbar d/dx} + e^{-\hbar d/dx} - x\right)\Psi(x) = 0$$

of Gromov-Witten theory on $\mathbf{CP}^1$ derived by Dunin-Barkowski,Mulase, Norbury, Popolitov and Shadrin [3]. We can thus reproduce the results of Dunin-Barkowski et al. from a different approach.

Moreover, the partition function $Z(\boldsymbol{t})$ itself has a natural limit to a generating function $Z_{\mathrm{4D}}(\boldsymbol{T})$ of the Gromov-Witten invariants. We can derive Fay-type bilinear identities for $Z_{\mathrm{4D}}(\boldsymbol{T})$ from those of $Z(\boldsymbol{t})$, which imply that $Z_{\mathrm{4D}}(\boldsymbol{T})$, too, is a tau function of the KP hierarchy.

References

[1] T. Nakatsu and K. Takasaki, Melting crystal, quantum torus and Toda hierarchy, Comm. Math. Phys. **285** (2009), 445–468, arXiv:0710.5339 [hep-th].

[2] K. Takasaki and T. Nakatsu, q-difference Kac-Schwarz operators in topological string theory, SIGMA **13** (2017), 009, 28 pages, arXiv:1609.00882 [Math-ph].

[3] P. Dunin-Barkowski, M. Motohico Mulase, P. Norbury, A. Popolitov and S. Shadrin, Quantum spectral curve for the Gromov-Witten theory of the complex projective line, arXiv:1312.5336 [math-ph].

Melting crystal model and its 4D limit

Kanehisa Takasaki, Kinki University

The melting crystal model is a statistical model of 3D Young diagrams. Its partition function can be reduced to the sum

$$Z = \sum_{\lambda \in \mathcal{P}} s_\lambda(q^{-\rho})^2 Q^{|\lambda|}, \quad s_\lambda(q^{-\rho}) = \frac{q^{-\kappa(\lambda)/4}}{\prod_{(i,j)\in\lambda}(q^{-h(i,j)/2} - q^{h(i,j)/2})},$$

over the set $\mathcal{P}$ of ordinary partitions. $s_\lambda(q^{-\rho})$ is a kind of principal specialization of the infinite-variate Schur function s_λ. This sum can be identified with the instanton partition function of 5D $\mathcal{N} = 1$ supersymmetric $U(1)$ Yang-Mills theory on $\mathbf{R}^4 \times S^1$. Its deformation

$$Z(\boldsymbol{t}) = \sum_{\lambda \in \mathcal{P}} s_\lambda(q^{-\rho})^2 Q^{|\lambda|} e^{\phi(\boldsymbol{t},\lambda)}, \quad \phi(\boldsymbol{t},\lambda) = \sum_{k=1}^{\infty} t_k \phi_k(\lambda),$$

by a set of external potentials $\phi_k(\lambda)$ with coupling constants $\mathbf{t} = (t_1, t_2, \ldots)$ is known to be a tau function of the KP hierarchy [1]. An associated quantum spectral curve is obtained by the method of q-difference Kac-Schwarz operators [2].

In the 4D limit where the radius R of S^1 tends to 0, this quantum curve turns into the quantum spectral curve

$$\left(e^{\hbar d/dx} + e^{-\hbar d/dx} - x\right)\Psi(x) = 0$$

of Gromov-Witten theory on $\mathbf{CP}^1$ derived by Dunin-Barkowski,Mulase, Norbury, Popolitov and Shadrin [3]. We can thus reproduce the results of Dunin-Barkowski et al. from a different approach. Moreover, the partition function $Z(\boldsymbol{t})$ itself has a natural limit to a generating function $Z_{\mathrm{4D}}(\boldsymbol{T})$, $\mathbf{T} = (T_1, t_2, \ldots)$, of the Gromov-Witten invariants of $\mathbf{CP}^1$. We can derive Fay-type bilinear equations for $Z_{\mathrm{4D}}(\boldsymbol{T})$ from those of $Z(\boldsymbol{t})$. This implies that $Z_{\mathrm{4D}}(\boldsymbol{T})$, too, is a tau function of the KP hierarchy.

References

[1] T. Nakatsu and K. Takasaki, Melting crystal, quantum torus and Toda hierarchy, Comm. Math. Phys. **285** (2009), 445–468, arXiv:0710.5339 [hep-th].

[2] K. Takasaki and T. Nakatsu, q-difference Kac-Schwarz operators in topological string theory, SIGMA **13** (2017), 009, 28 pages, arXiv:1609.00882 [Math-ph].

[3] P. Dunin-Barkowski, M. Motohico Mulase, P. Norbury, A. Popolitov and S. Shadrin, Quantum spectral curve for the Gromov-Witten theory of the complex projective line, arXiv:1312.5336 [math-ph].

L.D. Faddeev works on the inverse scattering problem

L. Takhtajan
Stony Brook University, NY (US)

I will review L.D. Faddeev's contribution to the inverse scattering method with the emphasis on the Sine-Gordon equation.

Haantjes Manifolds of the classical Lagrange top

G. Tondo
Dep. of Mathematics and Earth Sciences, University of Trieste, Italy

June 17-24, 2017

It is known that the classical Lagrange top admits a multi–Hamiltonian structure that can be formulated in term of a Gelfand-Zakarevich system of co-rank two [1]. By reducing such system to the symplectic leaves of one of its three degenerate Poisson tensors, one obtains symplectic–Nijenhuis structures, generalized Lenard chains of gradients and separation variables [2].

In this talk, we will present novel geometrical structures [3, 4] for the reduced system, namely, symplectic–Haantjes manifolds and Magri-Haantjes chains [5, 6]. Such structures are reductions of (the recently introduced) Poisson-Haantjes structures, living in the same phase space of the original Gelfand-Zakarevich system.

References

[1] C. Morosi and G. Tondo, *The quasi-bi-Hamiltonian formulation of the Lagrange top*, J. Phys. A, **35** (2002),1741–1750.

[2] C. Morosi and G. Tondo, *Separation of variables in multi-Hamiltonian systems: an application to the Lagrange top*, Theor. Math. Phys., **137** (2003),1550–1560.

[3] J. Haantjes. On X_{n-1}–forming sets of eigenvectors. Indag. Mathematicae, **17** (1955), pp. 158–162.

[4] F. Magri, *Haantjes manifolds*, Journal of Physics: Conference Series, **8** (2014), paper 012028, 10 pages.

[5] P. Tempesta and G. Tondo, *Haantjes Manifolds of Classical Integrable Systems*, arXiv:1405.5118 (2016).

[6] P. Tempesta and G. Tondo, *Haantjes Structures for the Jacobi-Calogero Model and the Benenti Systems*, SIGMA, **12** (2016), paper 023, 18 pages.

Continuum limits of pluri-Lagrangian systems

Mats Vermeeren,
Technische Universität Berlin

The *Lagrangian multiform* or *pluri-Lagrangian* structure of integrable quad equations is well-studied by now, see for example [1, 2, 3, 7]. It was observed early in these studies that the lattice parameters of a discrete pluri-Lagrangian system may play the role of independent variables in a corresponding continuous pluri-Lagrangian system of non-autonomous differential equations.

This contribution presents a different connection between discrete and continuous pluri-Lagrangian systems, where the continuous variables interpolate the discrete ones. In this context, the lattice parameters describe the size and shape of the mesh on which the discrete system lives, and thus they disappear in the continuum limit. The continuous systems found this way are hierarchies of autonomous differential equations. Pluri-Lagrangian systems of this type were studied independently of the discrete case in [4].

Some continuum limits in this sense can be found in the literature, most notably in [6], where the lattice potential KdV equation is shown to produce the potential KdV hierarchy in a suitable limit. The complicated double limit procedure from that work can be presented in a simplified form using Miwa variables. In this form, the procedure is easily adapted to some other quad equations, at least on the level of the equations themselves.

On the level of the pluri-Lagrangian structure, the problem is essentially that of Lagrangian interpolation of discrete variational systems. This was studied in [5] because of its relevance in numerical analysis for backward error analysis of variational integrators. We build on the ideas from that work to construct a pluri-Lagrangian structure for several hierarchies of differential equations that appear as continuum limits of quad equations.

References

[1] A.I. Bobenko, Yu.B. Suris. On the Lagrangian structure of integrable quad-equations. Lett. Math. Phys., 92, No. 1 (2010): 17–31.

[2] R. Boll, M. Petrera, Yu.B. Suris. What is integrability of discrete variational systems? Proc. Royal Soc. A, 470, No. 2162 (2014): 20130550.

[3] S. Lobb, F. Nijhoff. Lagrangian multiforms and multidimensional consistency. Journal of Physics A: Mathematical and Theoretical 42.45 (2009): 454013.

[4] Yu.B. Suris, M. Vermeeren. On the variational structure of integrable hierarchies. In: Advances in Discrete Differential Geometry, Ed. A.I. Bobenko, Springer (2016): 347–378.

[5] M. Vermeeren. Modified equations for variational integrators. arXiv:1505.05411 (2015).

[6] G.L. Wiersma, H.W. Capel. "Lattice equations, hierarchies and Hamiltonian structures." Physica A: Statistical Mechanics and its Applications 142.1 (1987): 199–244.

[7] P. Xenitidis, F. Nijhoff, S. Lobb. On the Lagrangian formulation of multidimensionally consistent systems. Proc. R. Soc. A. Vol. 467. No. 2135 (2011): 3295–3317.

Solitons, Huygens' principle and von Karman vortex streets

A.P. Veselov
Loughborough University, UK

The soliton theory produced a tremendous wave of research, which reached almost all corners of Mathematics and Physics. I will talk about two remarkable examples of its impact: on the Hadamard theory of Huygens' principle and, more recently, on the classical theory of vortices. In both cases the explanation is based on the analytic theory of the equations in the complex domain.

On the invariants of integrable maps

Claude M. Viallet
CNRS & UPMC Sorbonne Universités

The most popular autonomous integrable maps have simple rational invariants, the paradigmatic examples being the so-called QRT maps, (birational maps in two dimensions, related to difference equations of order two). They define invariant pencils of elliptic curves, and are the autonomous limits of discrete Painlevé equations, a very rich domain, with links to analysis, algebraic geometry, number theory.

I will give some generalisations to higher dimension, using the algebraic entropy as a characterisation of their integrability, and show that we are naturally lead to models with a mixture of algebraic and non-algebraic invariants, opening yet more the scope of the subject.

Higher rank solutions for the periodic two dimensional Volterra system

Rhys Bury, Alexander V. Mikhailov and Jing Ping Wang
Applied Mathematics Department, University of Leeds, UK
School of Mathematics, Statistics & Actuarial Science,
University of Kent, UK

In this talk, we first discuss the dressing method for the solution of the two-dimensional periodic Volterra system with a period N. We then derive soliton solutions of arbitrary rank k and give a full classification of rank 1 solutions. We have found a new class of exact solutions corresponding to wave fronts which represent smooth interfaces between two nonlinear periodic waves or a periodic wave and a trivial (zero) solution. The wave fronts are non-stationary and they propagate with a constant average velocity. The system also has soliton solutions similar to breathers, which resembles soliton webs in the KP theory. We associate the classification of soliton solutions with the Schubert decomposition of the Grassmannians $\mathrm{Gr}_{\mathbb{R}}(k, N)$ and $\mathrm{Gr}_{\mathbb{C}}(k, N)$. These results have recently been published in [1].

References

[1] Rhys Bury and Alexander V. Mikhailov and Jing Ping Wang. Wave fronts and cascades of soliton interactions in the periodic two dimensional Volterra system. Physica D: Nonlinear Phenomena 347(2017): 21–41.

Superintegrable systems in quantum mechanics and the Painleve transcendents

Pavel Winternitz
Centre de recherches mathematiques, Universite de Montreal

Superintegrable systems in classical and quantum mechanics are finite dimensional analogs of soliton equations. For a system with N degrees of freedom they allow between N+1 and 2N-1 independent integrals of motion. The integrals generate a non-Abelian algebra, usually a polynomial one, rather than a Lie algebra. An N-dimensional Abelian subalgebra always exists, guaranteeing integrability. As in soliton theory the Painleve property plays an important role in superintegrability. It will be shown that if at least one of the integrals is a polynomial of order larger or equal to three in the momenta, superintegrable quantum potentials appear that are expressed in terms of the Painleve transcendents.

Solitons and their interactions in the PT-symmetric nonlocal nonlinear Schrödinger models

Tao Xu[1], Min Li[2], Hengji Li[1], Dexin Meng[1]

1. College of Science, China University of Petroleum, Beijing 102249, China.

2. Department of Mathematics and Physics, North China Electric Power University, Beijing 102206, China.

The continuous and discrete PT-symmetric nonlocal nonlinear Schrödinger (NNLS) models are both found to be integrable in the sense of admitting the Lax pair and an infinite number of conservation laws [1, 2]. By using the Darboux transformation and starting from a plane-wave seed, we construct a chain of nonsingular exponential and rational soliton solutions of the defocusing NNLS model [3, 4]. Via the asymptotic analysis, we show that the N-th iterated exponential solutions in general display a variety of elastic interactions among $2N$ solitons, and each interacting soliton could be either the dark or anti-dark type [3]; and that the asymptotic solitons of higher-order ($N \geq 2$) rational solutions have the non-constant propagation velocities [4]. With numerical simulations, we examine the stability of the exponential and rational soliton solutions when the initial values have a small shift from the center of the PT symmetry. Also, we find that the discrete NNLS model of the defocusing type admits the nonsingular rational soliton solutions like the continuous case [5]. In addition, we obtain the nonsingular bright- and dark-soliton solutions of the focusing NNLS model and the related parametric conditions.

References

[1] M. J. Ablowitz and Z. H. Musslimani, Phys. Rev. Lett. **110**, 064105 (2013).

[2] M. J. Ablowitz and Z. H. Musslimani, Phys. Rev. E **90**, 032912 (2014).

[3] M. Li and T. Xu, Phys. Rev. E **91**, 033202 (2015).

[4] M. Li, T. Xu and D. Meng, J. Phys. Soc. Jpn. **85**, 124001 (2016).

[5] T. Xu, H. Li, M. Li, et al, Appl. Math. Lett. **63**, 88-94 (2017).

Matsuo-Cherednik type correspondence and quantum-classical duality of integrable systems

A. Zabrodin
Higher School of Economics, Moscow, Russia

We discuss the Matsuo–Cherednik type correspondence between the Knizhnik–Zamolodchikov equations associated with $GL(N)$ and the n-particle models of the Calogero type, with n being not necessarily equal to N. The quasiclassical limit of this construction yields the quantum-classical correspondence between the quantum spin chains and the classical Calogero or Ruijsenaars models.

Unresolved problems in the theory of integrable systems

V.E. Zakharov
University of Arizona, USA; Novosibirsk University, RF

In spite of enormous success of the theory of integrable systems, at least three important problems are not resolved yet or are resolved only partly. They are the following:

1. The IST in the case of arbitrary bounded initial data.

2. The statistical description of the systems integrable by the IST. Albeit, the development of the theory of integrable turbulence.

3. Integrability of the deep water equations.

These three problems will be discussed in the talk.