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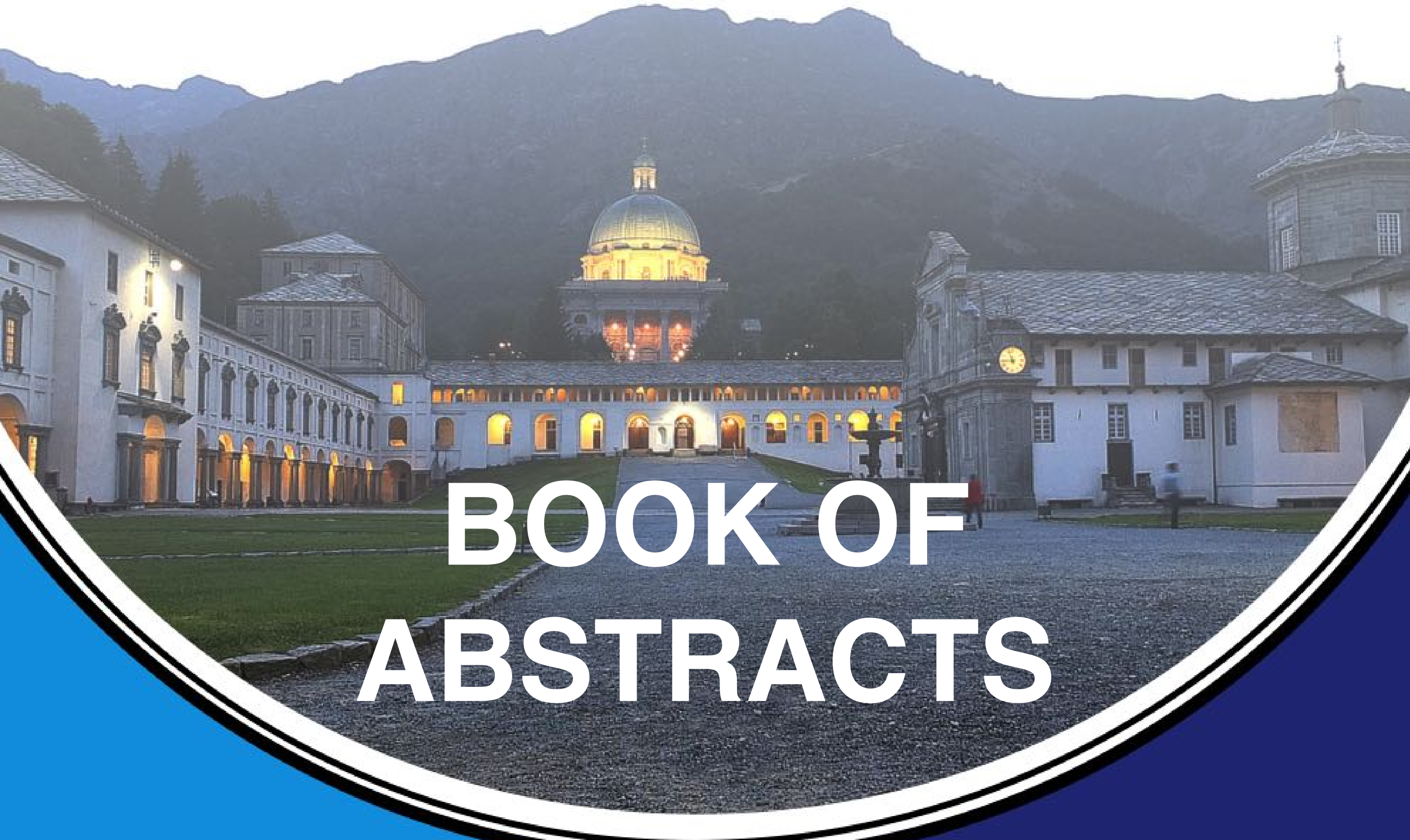

BOOK OF ABSTRACTS

May 16-18, 2018

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Book of Abstracts

CONTACT MECHANICS INTERNATIONAL SYMPOSIUM

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16-18 May 2018

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KEYNOTES

J. J. Moreau's mathematical toolkit for Nonsmooth Mechanics

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Summary: Several mathematical tools developed by Jean Jacques Moreau will be reviewed and an example of its use in nonsmooth dynamics will be provided.

On J. J. Moreau's Mathematics for Mechanics

The place of Jean Jacques Moreau as a towering figure in the fields of Mathematics and Mechanics is firmly established. The special issue of *Comptes Rendus Mécanique* is a much deserved recent celebration of his work in Mechanics. It also contains a publication list, as complete as possible ([1] p. 157-163), as well as a very nice preface by the editors, on the "legacy of a deep thinker".

The scope of this talk is more restricted: to present very briefly some of the mathematical tools which Jean Jacques Moreau created or developed in order to study the mechanical problems that were his main interest. The focus will be in a few topics that many, if not all the researchers in solid mechanics should now be acquainted with, namely

- Convex Analysis [2 a,b]
- Evolution problems, such as the sweeping process [3]
- Functions of bounded variation [4].

Ideally, these mathematical tools would be mentioned in context, that is, with some discussion of their usefulness in the re-formulation and the study of mechanical problems. In any case, for that purpose, the audience is invited to:
1) read from the source, say, from [5] [6]; or
2) to read from the special issue [1]; or simply
3) to follow this meeting, while paying attention to J. J. Moreau's pervasive influence in the fields of Mathematics and Mechanics.

A reference could also be made to

- Numerical aspects [7] [8] [9],

in which J. J. Moreau also excelled (not to mention many other works on e.g. fluid mechanics and granular materials), but these are essentially outside my reach.

An application to dynamics

An outline of a discretization method for a model problem of the dynamics of a particle or a simple system (e.g. [10]) is presented to give a specific example of the application of J. J. Moreau's ideas.

References

[1] E. Villermaux and P. Suquet (Eds.), *Comptes Rendus Mécanique*, 346:153-277, 2018.

[2a] J.J. Moreau, Fonctionnelles convexes, in: J. Leray (Ed.), *Séminaire sur les équations aux dérivées partielles n. 2*, pp. 1-108, 1966. http://www.numdam.org/article/SJL_1966-1967___2_1_0.pdf

[2b] J.J. Moreau, *Fonctionnelles convexes* (new edition of [2a]), Universitá di Roma Tor Vergata, dipartimento di ingegneria civile, Roma, 2003.

[3] J.J. Moreau, Evolution problem associated with a moving convex set in a Hilbert space, *J. Differ. Equ.* 26:347-374, 1977.

[4] J.J. Moreau, Bounded variation in time, in: J.J. Moreau, P.D. Panagiotopoulos and G. Strang (Eds.), *Topics in Nonsmooth Mechanics*, pp. 1-74, Birkhäuser, 1988.

[5] J.J. Moreau, On unilateral constraints, friction and plasticity, in: G. Capriz and G. Stampacchia (Eds.), *New Variational Techniques in Mathematical Physics*, pp. 171-322, Ed. Cremonese, 1974; and also: Springer, 2011.

[6] J.J. Moreau, Unilateral contact and dry friction in finite freedom dynamics, in: J.J. Moreau, P.D. Panagiotopoulos (Eds.), *Non-Smooth Mechanics and Applications*, pp. 1-82, Springer, 1988.

[7] M. Jean and J.J. Moreau, Dynamics in the presence of unilateral contacts and dry friction: a numerical approach, in: G.D. Piero, F. Maceri (Eds.), *Unilateral Problems in Structural Analysis 2*, pp. 151-196, Springer, 1985.

[8] J.J. Moreau, Numerical aspects of the sweeping process, *Comput. Methods Appl. Mech. Eng.* 177:329-349, 1999.

[9] J.J. Moreau, Some numerical methods in multibody dynamics: application to granular materials, *Eur. J. Mech. A, Solids* 4:93-114, 1994.

[10] M. D.P. Monteiro Marques and R. Dzonou, Dynamics of a particle with friction and delay, *Comptes Rendus Mécanique* 346:237-246, 2018.

FRICTION AND WEAR ACROSS SCALES

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Around mid-twentieth century tremendous progress was made in Tribology, the science of interacting surfaces in relative motion. Scientific advances explained the intimate relationship between surface roughness, load, and the real contact area. Due to the complexity of wear mechanisms, scientific progress has arguably slowed down ever since, although there has been a rapid increase in the number of empirical models describing friction and various forms of wear. Recently, with the advent of nanotribology, fundamental discoveries were made regarding friction mechanisms at nanoscale asperities. However, by and large, the dots remain unconnected and our macroscopic engineering-scale understanding of tribological mechanisms remains limited. We present our recent attempts at developing a fundamental, mechanistic, across scales, understanding of friction and adhesive wear.

I will begin by summarizing numerical simulation results, based on coarse-grained atomistic potentials [1,2], that capture debris formation at a contact junction. The two mechanisms at play in our simple numerical model are plastic shearing of contacting asperities, and (if enough elastic energy is available) crack propagation and coalescence leading to debris creation. This ductile to brittle transition was shown to occur at a material-dependent critical contact-junction size [2]. We also show that, in the simple situation of an isolated micro contact, the final debris size scales with the maximum junction size attained upon shear, and with the total shear-load mechanical work. This permits to draw analogies with Archard adhesive wear model [3]. I will also discuss recent results regarding the long term evolution of surface roughness, and will attempt to draw analogies with field observations of fault roughness and gouge.

References

[1] Aghababaei, R., Warner, D.H., Molinari, J.F., "On the debris-level origins of adhesive wear", PNAS, **114**(30), pp. 7935-7940 , 2017.

[2] Aghababaei, R., Warner, D.H., Molinari, J.F., "Critical length scale controls adhesive wear mechanisms", Nature Communications, 11816, 2016.

[3] Archard, J.F., "Contact and rubbing of flat surfaces", Journal of Applied Physics, **24**, 981, 1953.

On the coupling between dry friction and linear elasticity

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The steady sliding frictional contact problem is studied in the framework of linear elasticity and the Coulomb law of dry friction.

First, the model problem of the frictional equilibrium of a point particle in an quadratic elastic potential and in contact with a moving straight obstacle, as represented on figure 1, is studied. For low friction coefficient $\mathcal{F}$, it is proved that there is a unique equilibrium, and that this equilibrium is stable (at least with respect to tangential dynamics). When the friction coefficient $\mathcal{F}$ is increasing, three destabilisation scenarii come into competition: a pitchfork bifurcation, a supercritical Hopf bifurcation and a subcritical Hopf bifurcation. According to the (three-dimensional) stiffness matrix, any of them can be first one which is encountered.

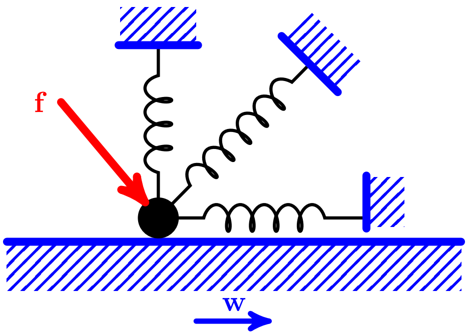

Figure 1: The steady sliding frictional contact problem for a point particle.

Then, the continuum problem is considered, as represented on figure 2. The obstacle is moving in such a way that its overall geometry remain invariant with respect to time. For

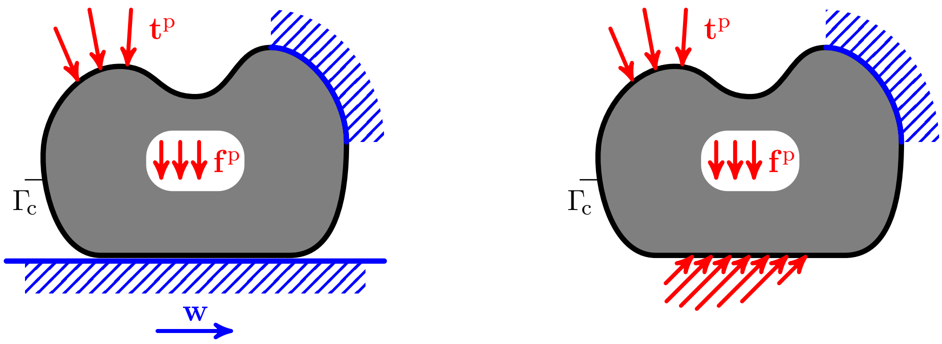

Figure 2: The steady sliding frictional contact problem for a continuum.

three-dimensional problems, some examples are that of plane translating alongside to itself, that of a surface of revolution rotating around its own axis, or that of an infinite screw rotating around its own axis. The problem is that of finding an equilibrium displacement field, that is, which remains invariant with respect to time. It is claimed that this problem is the generic abstract problem raised by any machine having moving parts in frictional contact and which is expected to run smoothly, that is, steadily. In the case of an homogeneous friction coefficient $\mathcal{F}$, the main result which is proved is that there exists a critical friction coefficient $0 < \mathcal{F}_c \leq +\infty$, such that, for all $0 \leq \mathcal{F} < \mathcal{F}_c$, the steady sliding frictional contact problems admits one and only one equilibrium solution. An example is provided of a steady sliding frictional contact problem admitting infinitely many solutions, so that $\mathcal{F}_c$ is finite. An example is also provided in which $\mathcal{F}_c$ is infinite. Hence, taking the friction coefficient $\mathcal{F}$ as a control parameter in the steady sliding frictional contact problem, it is observed that the steady sliding frictional contact problem may display a bifurcation, or not.

Finally, the case where the elastic body is a two-dimensional half-space (plane strain) is more particularly studied. It is proved that the corresponding steady sliding frictional contact problem has one and only one solution for any friction coefficient $\mathcal{F}$ ($\mathcal{F}_c = +\infty$). The solution displays a variety of universal singularities that are explicitly exhibited. One striking example is the singularity of the reaction force induced by a a jump in the friction coefficient, as represented

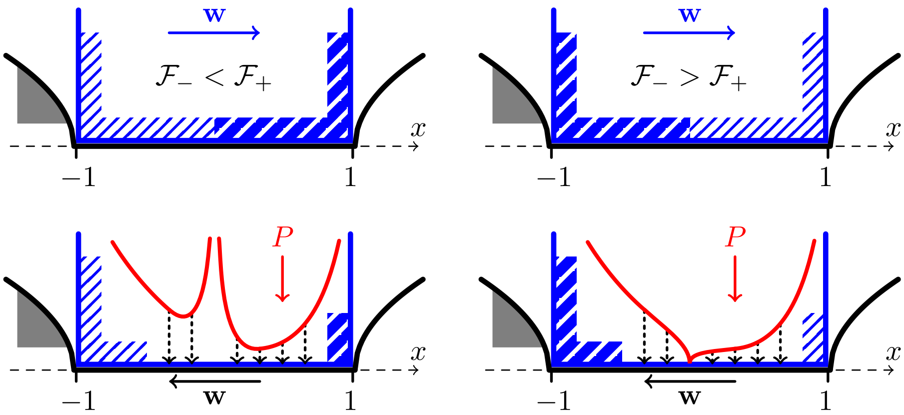

Figure 3: Normal component of the surface traction when the larger friction coefficient is front (left) or rear (right).

on figure 3. In the case where the largest friction coefficient is front, the reaction force goes to infinity at the jump (meaning that the deformable body is strongly pressed against the obstacle). On the opposite, in the case where the largest friction coefficient is rear, the reaction force goes to zero at the jump (meaning that the deformable body is locally unloaded at the jump).

References

[1] Patrick Ballard (2016), Can the 'stick-slip' phenomenon be explained by a bifurcation in the steady sliding frictional contact problem? *Discrete and Continuous Dynamical Systems Series S* **9**, pp. 363–381.

[2] Patrick Ballard (2016), Steady sliding frictional contact problem for a 2d elastic half-space with a discontinuous friction coefficient and related stress singularities. *Journal of the Mechanics and Physics of Solids* **97**, pp. 225–259.

[3] Patrick Ballard (2013) Steady sliding frictional contact problems in linear elasticity. *Journal of Elasticity* **110**, pp. 33–61.

Toward solving contact problems with trillion variables

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Summary: We shall present our results in the development of massively parallel algorithms with optimal (linear) complexity which have potential to solve extremely large contact problems. After giving an overview of the basic algorithms, theoretical results, and improvements, we shall present numerical experiments demonstrating the scope of their applicability.

We start with a short overview of scalable algorithms for the solution of contact problems and assessment of their capability to solve effectively extremely large problems. Then we shall report both our earlier and recent results. The scalability results cover 2D and 3D problems discretized by the finite element and/or boundary element [8] method, possibly with "floating" bodies, including the multibody frictionless problems, both static [5] and dynamic [2], and the problems with a given (Tresca) friction [3], including anisotropic friction. The algorithms are based the TFETI/TBETI (total finite/boundary element tearing and interconnecting) based domain decomposition methodology introduced by C. Farhat and F.-X. Roux [6] adapted to the solution of contact problems of elasticity. Recall that TFETI [1] differs from the classical FETI by imposing the prescribed displacements by means of Lagrange multipliers and treating all subdomains as "floating". A comprehensive description of all algorithms and results can be found in [4]

A special attention is paid to the implementation details, especially to those related to the massively parallel implementation. We give some hints concerning the effective parallel implementation of FETI-type algorithms for the solution of very large problems, including the implementation of the action of a generalized inverse of the stiffness matrix and projector avoiding implementation. We briefly discuss the possibility to increase the scope of parallel scalability by introducing the third level grid by a variant of HTFETI (Hybrid TFETI). The third level is introduced by the decomposition of TFETI subdomains into smaller subdomains that are partly glued in corners or by averages at the primal level (see, e.g., Klawonn and Rheinbach [7]). Let us mention that the conditioning of the dual matrix generated by small clusters glued by averages is qualitatively the same as that arising from standard TFETI [9]. We also briefly mention the packages that were used for the solution of the benchmark, namely PERMON based on PETSc (`permon.it4i.cz`) and ESPRESO based on Intel MKL and Cilk (`espreso.it4i.cz`).

We illustrate the performance of the rigid punch problem described in [4, Chap. 19]. Each subdomain was assigned to a single core. The stopping criterion was defined by the relative precision of the projected gradient and the feasibility error equal to 10^{-4} (measured in the Euclidean norm and compared with the Euclidean norm of the dual linear term. The results of computations are in Table 1.

References

[1] Z. Dostál, D. Horák and R. Kučera,Total FETI - an easier implementable variant of the FETI method for numerical solution of elliptic PDE, *Commun. Numer. Methods Eng.*, 22:1155–1162, 2006.

N_S	n [10^6]	outer iters	Hessian mult.	iter. time [sec]
64	6	8	73	258
1,728	154	8	83	311
8,000	715,0	8	117	364
21,952	1,962	8	142	418
64,000	5,712	8	149	497

Table 1: Results of the benchmark – cube contact linear elasticity problem.

[2] Dostál, Z., Kozubek, T., Brzobohatý, T., Markopoulos, A., Vlach, O., Scalable TFETI with optional preconditioning by conjugate projector for transient contact problems of elasticity, *Computer Methods in Applied Mechanics and Engineering*, 247–248: 37–50, 2012.

[3] Z. Dostál, T. Kozubek, A. Markopoulos, T. Brzobohat, V. Vondrák and P. Horyl, A theoretically supported scalable TFETI algorithm for the solution of multibody 3D contact problems with friction, *Computer methods in applied mechanics and engineering*, 205:110–120, 2012.

[4] Z. Dostál, T. Kozubek, M. Sadowská and V. Vondrák. *Scalable Algorithms for Contact Problems*, Springer, AMM 36, 20016.

[5] Dostál, Z., Kozubek, T., Vondrák, V., Brzobohatý, T., Markopoulos, A., Scalable TFETI algorithm for the solution of multibody contact problems of elasticity, *Int. J. Numer. Methods Engng.*, 82: 1384–1405, 2010.

[6] C. Farhat, F.-X. Roux, A method of Finite Element Tearing and Interconnecting and its parallel solution algorithm, *Int. J. Numer. Methods Engng.*, 32: 1205–1227, 1991.

[7] A. Klawonn, O. Rheinbach, Highly scalable parallel domain decomposition methods with an application to biomechanics, *Z. Angew. Math. Mech.* 90: 5–32, 2010.

[8] M. Sadowská, Z. Dostál, T. Kozubek, A. Markopoulos, J. Bouchala, Engineering multibody contact problems solved by scalable TBETI, in *Fast Boundary Element Methods in Engineering and Industrial Applications*, Springer, Berlin: 241–269, 2012.

[9] P. Vodstrčil, J. Bouchala, M. Jarošová and Z. Dostál, On conditioning of Schur complements of H-TFETI clusters for 2D problems governed by Laplacian, *Appl. Math.* 62:699–718, 2017.

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Multiscale frictional effects in rough soft contacts

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Summary: Non-trivial macroscopic frictional response in rough contacts can be observed even for purely elastic bodies with local Amonton-Coulomb friction law. The effect have been analyzed by the series of FEM-based numerical experiments.

Introduction

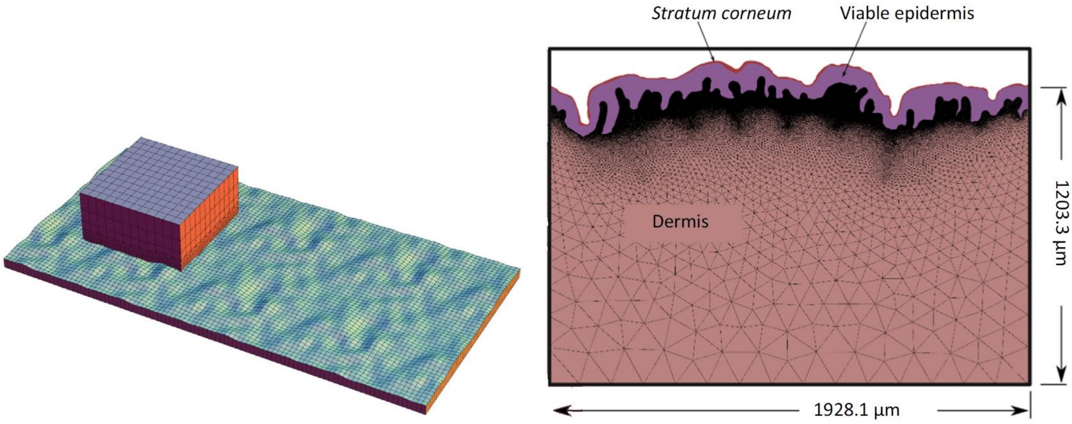
In tribology, the question of how to describe the sliding of rough contacts is still viewed as one of frontiers for modeling [1]. This is due to a number of possible mechanisms and sources of dissipation at different length- and time scales, which contribute to the observed macroscopic frictional response. Quite conveniently, despite their limitations, multiscale approaches are commonly applied to understanding and effectively modeling the friction.

Of our particular interest is the specific class of contact systems in which one or both surfaces are compliant. These can be, for instance, rubber-like materials like elastomeric seals or biological contacts like the skin. For such systems, the viscoelastic hysteresis induced by non-homogeneous contact loading due to roughness is usually viewed as a dominant effect that modifies the macroscopic frictional response versus the microscopic one [2, 3]. However, even when only considering purely elastic contacts and the simple Amonton-Coulomb friction model at the micro scale, one can observe non-trivial frictional effects at the macro scale [4–6]. The latter case is more deeply analyzed and discussed in this work.

Macroscopic friction of soft rough elastic contacts

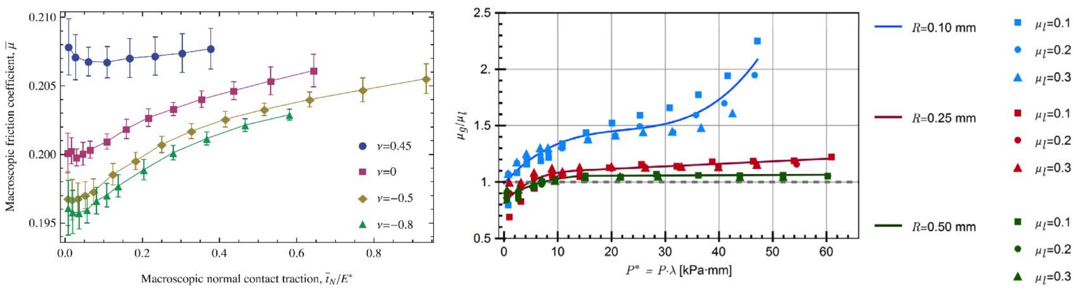
Two types of rough elastic contact systems have been considered. First systems (Case 1) are based on randomly rough periodic surfaces [4], see Figure 1a, in which the asperities' heights/slopes are relatively low. Second systems (Case 2) are based on anatomical model of the skin section, which is characterized by a complicated surface topography at the microscopic scale and additionally by a layered structure [5, 6], see Figure 1b. In the case of skin, a simplified counter-surface has been considered, represented by the isolated rigid cylinders (not shown in the Figure).

Both cases have been analyzed using FEM-based contact homogenization procedure (different for each case). The main observation is that the macroscopic friction coefficient can differ from the microscopic one, and moreover can significantly depend on normal contact pressure. The further study has been performed in both cases to analyze how the friction-pressure relationship depends on various problem parameters. In the Case 1, for Poisson ratio $\nu \leq 0$, a counter-intuitive effect has been observed, in which the macroscopic friction coefficient drops below the microscopic one, see Figure 2a. In the Case 2, the global-to-local friction coefficient ratio is higher than in the Case 1, and it possibly depends on the asperity radius on the counter-surface, see Figure 2b.



(a) (b)

Figure 1: Two rough contact systems: (a) randomly rough periodic surfaces, (b) 2D anatomical model of the skin.



(a) (b)

Figure 2: Macroscopic friction-pressure relationship: (a) in Case 1, for different values of Poisson ratio ν, (b) in Case 2, for different cylinder radii R and different local friction coefficient μ_l.

References

[1] R.W. Carpick, The contact sport of rough surfaces, *Science*, 359:38-38, 2018.

[2] P. Wagner, P. Wriggers, C. Klapproth, C. Prange, B. Wies, Multiscale FEM approach for hysteresis friction of rubber on rough surfaces, *Computer Methods in Applied Mechanics and Engineering*, 296:150-168, 2015.

[3] K.I. Kilic, I. Temizer, Tuning macroscopic sliding friction at soft contact interfaces: interaction of bulk and surface heterogeneities. *Tribol. Intern.*, 104:8397, 2016.

[4] S. Stupkiewicz, M. Lewandowski and J. Lengiewicz, Micromechanical analysis of friction anisotropy in rough elastic contacts, *Int. J. Solids Struct.*, 51:3931-3943, 2014.

[5] M.F. Leyva-Mendivil, J. Lengiewicz, A. Page, N.W. Bressloff and G. Limbert, Skin microstructure is a key contributor to its friction behaviour, *Tribol. Lett.* 65, 2017.

[6] M.F. Leyva-Mendivil, J. Lengiewicz and G. Limbert, Skin friction under pressure. The role of micromechanics, *Surf. Topogr.: Metrol. Prop.* (in review).

Explicit integrators for the impact of elastic solids: a comparison of a Nitsche-based approach with existing ones

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Summary: the aim of the presentation is to revisit the analysis of explicit integration of the dynamic with impact of deformable solids by existing strategies (explicit schemes for penalized contact, Moreau's NSCD schemes, Paoli-Schatzman scheme, mass redistribution method, etc.), by comparing them to a Nitsche-based approximation of the contact condition described in [1–3] for the static case and implicit schemes. Comparisons will be provided in term of energy conservation, convergence and occurrence of spurious oscillations.

Introduction

Due to the difficulty to obtain reliable simulations, the explicit integration of the dynamic with impact of deformable solids has already been the subject of an important literature. To mention some of the main approaches, we can say that precursory and widely resumed work in this area is due to J.J. Moreau [4] in the context of the impact of rigid body systems. The schemes proposed by J.J. Moreau have been extended quite naturally to the elasticity case via finite element semi-discretization (for instance in [5]) which transforms the continuous impact problem into a discrete one very close to a rigid body system. These discrete impact problems, governed by a so-called measure differential inclusion are notoriously ill-posed and of very low regularity.

The ill-posedness can be (for the most part) fixed by the addition of an impact law with a restitution coefficient. A valuable scheme in this context is that of Paoli and Schatzman [6] who implicitly takes into account this restitution coefficient. However, the addition of a restitution coefficient is somewhat artificial in the context of deformable solids.

As it is noticed in [7], one of the important difficulties introduced by the semi-discretization by finite elements is that it transforms a well-posed continuous problem (it is well posed at least in the 1D case, see [9]) into an ill-posed measure differential inclusion having an infinite number of solutions, depending on the choice of a restitution coefficient on each node of the contact boundary. Moreover, it is not possible to decide which solution is more suitable than other. Indeed, the two most remarkable solutions are, first, the one for a unitary restitution coefficient which is energy conserving but which causes very important spurious oscillations of the contact nodes and unexploitable contact stress, and also the solution for a vanishing restitution coefficient which ensures stability and a better approximation of the contact stress but is energy dissipative, while the continuous problem is not. This resulted in [7] to propose the mass redistribution method (generalized in [8]) which allows an interesting compromise in this context, i.e. a conservation of energy and a good quality of the contact stress. However, it introduces an implicit computation on the contact nodes even when an explicit time-marching scheme is used.

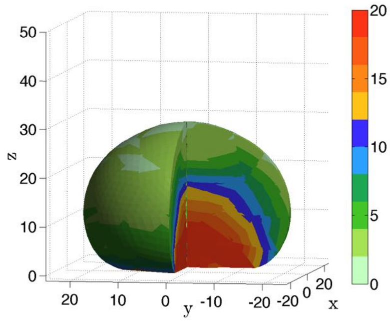
Nitsche-based method

The aim of this presentation is to compare the approximation by explicit schemes of the Nitsche method developed in [1–3] with existing methods above and with the approximation by penalized contact. The principle of the Nitsche method is the use of the following well known relation which is equivalent to the contact condition

$$\sigma_n = -(\gamma u_n - \sigma_n)_+, \quad \gamma > 0$$

by plugging it into the weak formulation, which makes the Nitsche-based method a consistent and primal one.

The comparison will be mainly performed on the one-dimensional problem introduced in [9] whose advantage is to present a known periodic solution and to make clear the occurrence of parasitic oscillations, the convergence and energy conservation properties. Comparisons in dimension 2 and 3 will also be presented (see the example on Fig. 1).



(a)

Figure 1: Example of the impact of a sphere on a rigid ground, approximated with a Nitsche-based method.

References

[1] F. Chouly, P. Hild, Y. Renard. Symmetric and non-symmetric variants of Nitsche's method for contact problems in elasticity: theory and numerical experiments. *Math. Comp.*, 84:1089–1112, 2015.

[2] F. Chouly, P. Hild, Y. Renard. A Nitsche finite element method for dynamic contact: 1. Semi-discrete problem analysis and time-marching schemes. *ESAIM Math. Model. Numer. Anal.*, 49:481–502, 2015.

[3] F. Chouly, P. Hild, Y. Renard. A Nitsche finite element method for dynamic contact: 2. Stability of the schemes and numerical experiment. *ESAIM Math. Model. Numer. Anal.*, 49:503–528, 2015.

[4] J.J. Moreau, Unilateral contact and dry friction in finite freedom dynamics, in *Nonsmooth mechanics and Applications*, Springer, pp. 1–82, 1988.

[5] D. Vola, M. Raous, J.A.C. Martins, Friction and instability of steady sliding: squeal of a rubber/glass contact, *Int. J. Numer. Meth. Engng.*, 46, 1699–1720, 1999.

[6] L. Paoli and M. Schatzman, A Numerical Scheme for Impact Problems, *SIAM J. Numer. Anal.*, 40, pp. 702–768, 2002.

[7] H. Khenous, P. Laborde, Y. Renard. Mass redistribution method for finite element contact problems in elastodynamics. *Eur. J. Mech., A/Solids*, 27(5):918–932, 2008.

[8] Y. Renard. The singular dynamic method for constrained second order hyperbolic equations. application to dynamic contact problems. *J. Comput. Appl. Math.*, 234(3):906–923, 2010.

[9] F. Dabaghi, A. Petrov, J. Pousin, Y. Renard. A robust finite element redistribution approach for elastodynamic contact problems. *Appl. Numer. Math.*, 103:48–71, 2016.

SESSION 1

Formulations and extensive comparisons of 3D frictional contact solvers based on performance profiles

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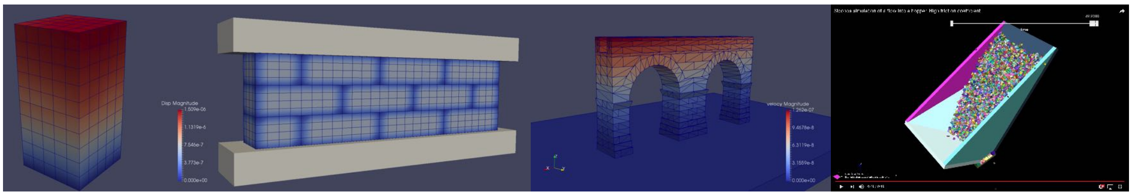
This work reviews, details and compares several numerical algorithms to solve 3D frictional contact problems. The comparisons are made on a benchmark of over 2500 instances, and performance profiles unveil the trends.



(a) Cubes_H8 (b) LowWall_FEM (c) Aqueduct_PR (d) Chute_4000

Figure 1: Illustrations of the FClib test problems

Introduction

Let $n_c \in \mathbb{N}$ be the number of contact points and $n \in \mathbb{N}$ the number of degree of freedom. Given a symmetric positive (semi-)definite matrix $M \in \mathbb{R}^{n\times n}$, a vector $f \in \mathbb{R}^{n}$, a matrix $H \in \mathbb{R}^{n\times m}$ with $m = 3n_c$, a vector $w \in \mathbb{R}^{m}$ and a vector of coefficients of friction $\mu \in \mathbb{R}^{n_c}$, the discrete frictional contact problem is to find three vectors $v \in \mathbb{R}^{n}$, $u \in \mathbb{R}^{m}$ and $r \in \mathbb{R}^{m}$ such that

$$Mv = Hr + f, \quad u = H^\top v + w, \quad \hat{u} = u + g(u), \quad K^\star \ni \hat{u} \perp r \in K, \tag{1}$$

where $g(u)$ is a nonsmooth function and $K \subset \mathbb{R}^{3n_c}$ is a Cartesian product of second order cone in $\mathbb{R}^3$. For each contact α, the unknown variables $u^\alpha \in \mathbb{R}^3$ (velocity or gap at the contact point) and $r^\alpha \in \mathbb{R}^3$ (reaction or impulse) are decomposed in a contact local frame $(O^\alpha, \mathsf{N}^\alpha, \mathsf{T}^\alpha)$ such that $u^\alpha = u^\alpha_{\mathrm{N}}\mathsf{N}^\alpha + u^\alpha_{\mathrm{T}}\mathsf{T}^\alpha, u^\alpha_{\mathrm{N}} \in \mathbb{R}, \mathrm{u}^\alpha_{\mathrm{T}} \in \mathbb{R}^2$ and $r^\alpha = r^\alpha_{\mathrm{N}}\mathsf{N}^\alpha + r^\alpha_{\mathrm{T}}\mathsf{T}^\alpha, r^\alpha_{\mathrm{N}} \in \mathbb{R}, \mathrm{r}^\alpha_{\mathrm{T}} \in \mathbb{R}^2$. The set K is the cartesian product of Coulomb's friction cone at each contact, that is

$$K = \prod_{\alpha=1\ldots n_c} K^\alpha = \prod_{\alpha=1\ldots n_c} \{r^\alpha, \|r^\alpha_{\mathrm{T}}\| \leq \mu^\alpha |r^\alpha_{\mathrm{N}}|\}. \tag{2}$$

The function g is defined as $g(u) = [[\mu^\alpha \|u^\alpha_{\mathrm{T}}\| \mathsf{N}^\alpha]^\top, \alpha = 1\ldots n_c]^\top$. For more details, we refer to [1]. In this work, we discuss and compare the numerical solution procedures for solving the discrete frictional contact problem.

Second Order Cone Complementarity Problem (SOCCP)

From the mathematical programming point of view, the problem (1) is a SOCCP. If the nonlinear part of the problem is neglected ($g(u) = 0$), the problem is an associated friction problem with dilatation, and by the way, is a gentle SOCLCP with a positive matrix $H^\top M^{-1} H$ (possibly semi–definite). When the non-associated character of the friction is taken into account through $g(u)$, the problem is non-monotone and nonsmooth, and then very hard to solve efficiently. This generic problem is at the heart of most of the simulation techniques of mechanical systems with 3D Coulomb's friction and unilateral constraints. as discussed in [2], it might be the result of

- a time–discretization scheme by event–capturing time–stepping methods or event–detecting (event–driven) techniques of dynamical systems
- a space–discretization (by FEM for instance) of the quasi-static problems of frictional contact mechanics.

Formulations based on numerical optimization

In this talk we will recall a result for the SOCCP (1) which ensures that a solution exists [3]. Then, we present several classes of algorithms that have been previously developed for solving this problem:

- Variational inequalities solvers: fixed point with projection and extragradient techniques with self-adapting step rule.
- Nonsmooth equations solvers: semi–smooth and generalized Newton methods with line-searches
- Block–splitting (Gauss-Seidel Like) and projected overrelaxation (PSOR).
- Proximal point algorithms
- Optimization based solvers: Panagatiopolous approach, Czech school approach (Tresca successive approximations) and convex SOCQP relaxation.

Extensive comparisons

The goal of this work is to compare, on a large set of problems, the methods found in the literature and to propose some new approaches. To this end, we build an open collection of discrete frictional contact problems called FCLIB[1] in order to offer a large library of problems to compare algorithms on a fair basis. In this work, this collection is solved with the software Siconos and its component Siconos/Numerics[2].

Conclusions

On one hand, we will show that algorithms based on Newton methods for nonsmooth equations solve quickly the problem when they succeed, but suffer from robustness issues mainly when the matrix H has not full rank. On the other hand, the iterative methods dedicated to solving variational inequalities are quite robust but with an extremely slow rate of convergence. To sum up, as far as we know there is no option that combines time efficiency and robustness. This presentation will be a summary of the work detailed in a recent technical report [4]

References

[1] V. Acary and B. Brogliato. *Numerical methods for nonsmooth dynamical systems. Applications in mechanics and electronics.* Lecture Notes in Applied and Computational Mechanics 35. Berlin: Springer. xxi, 525 p. , 2008.

[2] V. Acary and F. Cadoux. *Recent Advances in Contact Mechanics, Stavroulakis, Georgios E. (Ed.)*, volume 56 of *Lecture Notes in Applied and Computational Mechanics*, chapter Applications of an existence result for the Coulomb friction problem. Springer Verlag, 2013.

[3] V. Acary, F. Cadoux, C. Lemaréchal, and J. Malick. A formulation of the linear discrete coulomb friction problem via convex optimization. *ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik*, 91(2):155–175, 2011.

[4] V. Acary, M. Brémond, and O. Huber. On solving contact problems with Coulomb friction: formulations and numerical comparisons. Research Report 9118, INRIA, November 2017. https://hal.inria.fr/hal-01630836.

[1] https://frictionalcontactlibrary.github.io/index.html
[2] http://siconos.gforge.inria.fr

A local multi-physical approach to model braking materials

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Summary: Local investigations are managed to understand the multi-physical complexity of braking materials. After a determination of the Representative Elementary Volume, measures of friction, damage and temperature are related to global solicitations and the deceleration. The impact of wear on the different equilibrium is also presented.

Introduction

The relative motion of two bodies in contact is the seat of several dissipative phenomena. In particular, part of the mechanical energy necessary to rub bodies against each other is converted into thermal energy (from 80 to 95% according to the literature [1]). Under dry contact conditions, this conversion can lead to hot spot localization, thermoelastic instabilities, etc [2]. The literature proposes a large variety of analytical models trying to represent the contact complexity [3] but it appears that there are only few models accounting for the dynamic evolution of the contact, leading to the creation of an interfacial layer composed for the most part of the transformation and the degradation of debris particles issued from the bodies in contact. This layer, usually called the third body in reference of the two bodies in contact [4], is well known to its mechanical roles (velocity accommodation, load transmission,...) but less for its thermal ones.

With the development of discrete element methods (DEM) [5] and their extensions to thermo-mechanical behavior of contact interface, it is possible to analyze numerically the life of a contact. Several results have been observed [6] as, for example, the localization of the maximal temperature within the thickness of the third body as a function of its internal cohesion. Nevertheless, the approach stays at the scale of the third body, and the influence of first bodies is related to some specific thermal boundary conditions.

Based on recent works [7], a new reflection on the wear process including mechanical and thermal effects, is proposed. Using an extended discrete element approach, a discussion is proposed around the evolution of friction, temperature and wear in terms of energy balance.

Numerical Framework

The method used to simulate evolution of a discrete equivalent continuous media is the Non-Smooth Contact Dynamics (NSCD) approach, developed by Moreau and Jean [5]. Recently, the approach has been extended and used as a mesh-less approach to model equivalent continuous media under tribological solicitations where cohesive zone models (CZMs) [8] have been used to confer with the whole packing a continuous behavior (cf. Figure 1) equivalent from a mechanical and a thermal point of view [7, 9].

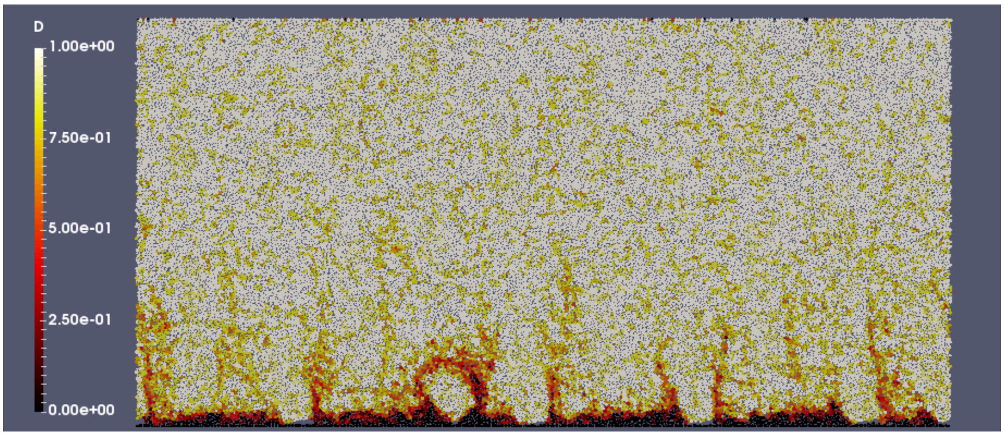

Figure 1: Evolution of the damage in a discrete structure

Results

Among the different results, impact of debris properties will be presented and how the internal cohesion of the tribological layer can be related to damage processes. Moreover, the introduction of wear flows are also discussed. They underline the fact that a controlled wear flow could be benefit for the evolution of the global damage of the REV.

References

[1] F.E. Kennedy, Single pass rub phenomena-analysis and experiment, *J. Lubr. Technol*, 104:582–588, 1982.

[2] J.R. Barber, Thermoelastic instabilities in the sliding of com- forming solids, *Proc. R. Soc. Lond. A Math*, 312:381–394, 1969.

[3] F.E. Kennedy, Surface temperature in sliding systems - a finite element analysis, , *J. Lubr. Technol.*, 103:90–96, 1981.

[4] M. Godet, The third-body approach: a mechanical view of wear, *Wear*, 100:437–452, 1984.

[5] M. Jean, The non-smooth contact dynamics method, *Comput. Methods Appl. Mech. Eng.*, 177:235–257, 1999.

[6] D. Richard, I. Iordanoff, M. Renouf, Y. Berthier, Thermal study of the dry sliding contact with third-body presence, *ASME. J. Tribol.*, 130:031404, 2008.

[7] M. Champagne, M. Renouf, Y. Berthier, Modeling wear for heterogeneous bi-phasic materials using discrete elements approach, *ASME J. Tribol.*, 136(2):021603, 2014.

[8] M. Raous, L. Cangémi, M. Cocu, A consistent model coupling adhesion, friction, and unilateral contact, *Comput. Methods Appl. Mech. Eng.*, 177:383–399,1999.

[9] J. Rivière, M. Renouf, Y. Berthier, Thermo-Mechanical Investigations of a Tribological Interface, *Tribol. Lett.* 58:48–59, 2015.

Heterogeneous Asynchronous time integrators for non-smooth dynamics

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Keywords: explicit-implicit time integrators, energy-based methods, contact and impact dynamics

Abstract

For non-smooth transient structural dynamics, the choice of the time step and the time integrator has a critical impact on the feasibility of the simulation. For instance, during an earthquake, a bridge crane, usually located overhead in buildings, may be subjected to multiple impacts between crane wheels and rail. These multiple impacts cause significant damage in the structure. Then the qualification of these structures with respect to normative seismic design requirements, which are continuously developing and becoming more and more stringent, requires strengthened simulation techniques especially to model the impact phenomenon. Furthermore, multiple time-scales coexist in a bridge crane under seismic loading. In that case, the use of multi-time scale methods is suitable. Here, we propose a new explicit-implicit heterogeneous asynchronous time integrator (HATI) for non-smooth transient dynamics with possible contacts and impacts. In a first step we introduce a Moreau-based event-capturing explicit time integrator for contact/impact problems. In a second step, a two time scales explicit-implicit HATI is developed: it consists in using an explicit time integrator with a fine time scale in the contact area, while an implicit time integrator is adopted in the other parts in order to capture the low frequency content of the solution and to optimize the CPU time. 3D Transient dynamics applications illustrate the robustness and the efficiency of the proposed approach.

References

[1] Fekak F., Brun M., Gravouil A., Depale B., a new heterogeneous asynchronous explicit-implicit time integrator for nonsmooth dynamics, *Computational Mechanics*, 50:199-225, 2017

[2] Gravouil A., Combescure A., Brun M., Heterogeneous asynchronous time integrators for computational structural dynamics. *International Journal for Numerical Methods in Engineering,* 102:202–232, 2015

[3] Acary, V., Energy conservation and dissipation properties of time-integration methods for non smooth elastodynamics with contact. ZAMM-Journal of Applied Mathematics and Mechanics/ 96, 585–603, 2016

[4] Deuflhard, P., Krause, R., Ertel, S., A contact-stabilized newmark method for dynamical contact problems. International Journal for Numerical Methods in Engineering 73(9), 1274–1290, 2008

[5] Hesch, C., Betsch, P., Transient 3d contact problems-nts method: mixed methods and conserving integration. Computational Mechanics 48(4), 437–449, 2011

[6] Lew, A., Marsden, J., Ortiz, M., West, M., Variational time integrators. International Journal for Numerical Methods in Engineering 60(1), 153–212, 2004

[7] Moreau, J.J., Numerical aspects of the sweeping process. Computer Methods in Applied Mechanics and Engineering 177(3), 329–349, 1999

[8] Jean, M., The non-smooth contact dynamics method. Computer methods in applied mechanics and engineering 177(3), 235–257, 1999

[9] Laursen, T.A., Computational Contact and Impact Mechanics: Fundamentals of Modeling Interfacial Phenomena in Nonlinear Finite Element Analysis. Springer Science & Business Media, 2002
[10] De Saxcé, G., Feng, Z.Q., The bipotential method: a constructive approach to design the complete contact law with friction and improved numerical algorithms. Mathematical and Computer Modeling 28(4), 225–245, 1998
[11] Armero, F., Petocz, E., Formulation and analysis of conserving algorithms for frictionless dynamic contact/impact problems. Computer methods in applied mechanics and engineering 158(3), 269–300, 1998
[12] Alart, P.,Curnier,A., A mixed formulation for frictional contact problems prone to Newton like solution methods. Computer Methods in Applied Mechanics and Engineering 92(3), 353–375, 1991
[13] Belytschko, T., Neal, M., Contact-impact by the pinball algorithm with penalty and Lagrangian methods. International Journal for Numerical Methods in Engineering 31, 547– 572, 1991

SESSION 2

Derivation of a model of soft imperfect interface with non local damage and unilateral contact conditions

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Summary: In this presentation, a model describing a layered structure composed by two elastic adherents and a thin adhesive subject to a degradation process and two different regimes, one in traction and one in compression, is introduced. By a matched asymptotic expansion method, a model of imperfect interface taken into account unilateral conditions and non local damage is derived.

Introduction

The problem of obtaining efficient models for imperfect interfaces is a subject of great interest in engineering [1–3]. Clearly, there are many applications of this kind of problems, in particular related to the development of composite structures. Moreover, it is known that interface zones between materials (see fig. 1) are fundamental to ensure strength and stability of structural elements.
In this presentation, the derivation of a model of imperfect interface, coupling damage and unilateral conditions, as the formal asymptotic limit [4] of a model of a composite body made by two adherents with an adhesive substance located between them, is performed.

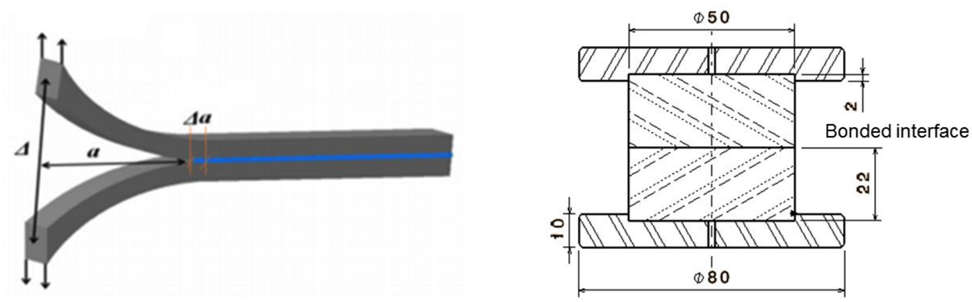

Figure 1: Examples of bonded structures.

The mechanical problem

A composite body made of three different materials, two materials are known as adherents and the third one as the adhesive, is considered. Adhesive whose thickness is ε is very thin as compared to the adherents. The body occupying the total bounded smooth domain is denoted by $\Omega^\varepsilon \in R^3$. We introduce an orthogonal frame of reference (O, e_1, e_2, e_3). Let (x_1, x_2, x_3) denotes the particle in three dimensional frame. The center of the inter-phase mid-plane is the origin of the frame of reference and the x_3-axis runs perpendicular to the open bounded set S where $S = \{(x_1, x_2, x_3) \in \Omega^\varepsilon \;\; x_3 = 0\}$. The adherents are supposed to be elastic. It is considered that the adhesive is a generalized Kachanov-type soft material with the elastic coefficients depends upon a damage parameter χ [5]. Moreover to avoid interpenetration between the adherents [6], and using standard notations, two regimes, one in traction, one in compression, are defined as follows

$$\begin{cases} \sigma^\varepsilon &= \varepsilon\tilde{\lambda}(\chi)(tr(e(u^\varepsilon)))I_2 + 2\varepsilon\mu(\chi)(e(u^\varepsilon)) \quad if \quad e(u^\varepsilon) \geq 0 \\ \sigma^\varepsilon &= \lambda(\chi)(tr(e(u^\varepsilon)))I_2 + 2\varepsilon\mu(\chi)(e(u^\varepsilon)) \quad if \quad e(u^\varepsilon) \leq 0 \end{cases} \tag{1}$$

where I_2 is the second order identity tensor and λ, μ the Lamé coefficients. The non local damage evolution is given by

$$\begin{cases} \eta^\varepsilon \dot{\chi} = \omega^\varepsilon + \alpha\Delta\chi - \varepsilon\lambda_{,\chi}(tr(e(u^\varepsilon)))^2 I_2 - 2\varepsilon\mu_{,\chi}(e(u^\varepsilon))^2 \quad if \;\; e(u^\varepsilon) \geq 0 \\ \eta^\varepsilon \dot{\chi} = \omega^\varepsilon + \alpha\Delta\chi - \lambda_{,\chi}(tr(e(u^\varepsilon)))^2 I_2 - 2\varepsilon\mu_{,\chi}(e(u^\varepsilon))^2 \quad if \;\; e(u^\varepsilon) \leq 0 \end{cases} \tag{2}$$

In the lecture, it will be shown that using an asymptotic theory [7, 8], it is possible to obtain the following limit problem on S

$$\begin{cases} [\sigma_{i3}] = 0 \\ \sigma_{\alpha 3} = \mu(\chi)\,[u_\alpha]\,, \;\alpha = 1, 2 \\ \sigma_{33} = (\tilde{\lambda}(\chi) + 2\mu(\chi))[u_3]_+ + \tau \\ [u_3] \geq 0, \;\tau \leq 0\, [u_3]\,\tau = 0 \\ \chi \in [0, 1] \\ \bar{\eta}\dot{\chi} = \Big(\bar{\omega} + \bar{\alpha}\Delta_2\chi - \Big(\mu(\chi)\,[u_1]^2 + \mu(\chi)\,[u_2]^2 + (\tilde{\lambda}(\chi) + 2\mu(\chi))\,[u_3]_+^2\Big)\Big)_- \end{cases} \tag{3}$$

where $()_+$ (resp. $()_-$) denotes the positive (resp. negative) part of a function and $\bar{}$ denotes the term of order -1 in the expansions of the considered parameters. Δ_2 is the laplacian restricted to the plane. This model of imperfect interface takes into account damage evolution and unilateral conditions.

References

[1] Y. Benveniste and T. Miloh, Imperfect soft and stiff interfaces in two-dimensional elasticity, *Mechanics of Materials*, 33:309-323, 2001.

[2] N. Challamel and U.A. Girhammar, Boundary-layer effect in composite beams with interlayer slip, *Journal of Aerospace Engineering*, 24:199-209, 2011.

[3] Fouchal F., Lebon F., Raffa M.L., Vairo G., *An Interface Model Including Cracks and Roughness Applied to Masonry*, The Open Civ. Eng. J., 8, 263-271 (2014).

[4] P.G. Ciarlet, *Mathematical Elasticity. Volume I: Three-Dimensional Elasticity*, , North-Holland, Amsterdam, 1988.

[5] Bonetti E., Bonfanti G., Lebon F., Rizzoni R., *A model of imperfect interface with damage*, Meccanica, 52 (8), 1911-1922, (2017).

[6] Mielke A., Roubiceck T., Thomas M., *From damage to delamination in nonlinear elastic materials at small strains*, J. Elast., 109 (2), 235-273, (2012).

[7] Klarbring A., *Derivation of the adhesively bonded joints by the asymptotic expansion method*, International Journal of Engineering Science, 29, 493-512 (1991).

[8] Dumont, S.,Lebon, F., Rizzoni, R., *An asymptotic approach to the adhesion of thin stiff films*, Mech. Res. Com., 58, 24-35 (2014).

Numerical study of the mixed-mode delamination of composite interfaces

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In this work we propose a new numerical formulation to compute the delamination onset and propagation along weak interfaces under mixed-mode conditions. The interfacial problem is addressed through a cohesive crack model, concentrating all the non-linearities at the interface. The accuracy of the proposed formulation is verified against predictions of a combined contact-delamination algorithm.

Introduction

Nowadays, many engineering components are made of high performance laminated composites and adhesively bonded interfaces. One of the most important damage modes of laminated structures is related to the non-linear and irreversible delamination process, including the formation and propagation of inter-laminar cracks, up to the complete detachment of the adherends. The delamination process falls within a fracture mechanics framework, for which several test configurations under transverse forces have been developed and standardized to measure the delamination strength and/or toughness. Among such test we would like to cite the symmetric and asymmetric Double Cantilever Beam (DCB), the End-Notched Flexure (ENF), the Crack Lap Shear (CLS), the Mixed-Mode Bending (MMB).

In this context, we address the interfacial delamination problem through an innovative cohesive formulation, named as Enhanced Beam Theory (EBT). Here the specimen is considered as an assemblage of two sublaminates, partly bonded together by an elastic interface. Such interface is represented by a continuous distribution of elastic springs acting along the normal and/or tangential direction, depending on the interfacial mixed-mode condition. This generalizes the idea suggested recently in [1] for a single mode-I delamination, and extended in [2] to include mixed loading, geometrical and mechanical conditions.

Numerical solutions

A parametric analysis of the problem is perfomed, both locally and globally, in terms of interfacial stresses, internal forces and displacements, as well as in terms of compliance, energy release rate, mode mixity angle, and global load-displacement response. The proposed EBT is compared to some mechanical models available in the literature based on a Simple Beam Theory (SBT), or a Local Method (LM). A further validation of the proposed approach is based on a comparative assessment of our results with respect to predictions based on a frictional contact formulation. The contact algorithm is generalized to handle cohesive forces along the normal and tangential directions, as employed in [3], (see the global response of a Moment Loaded DCB in mode-I loading condition in Figure 1).

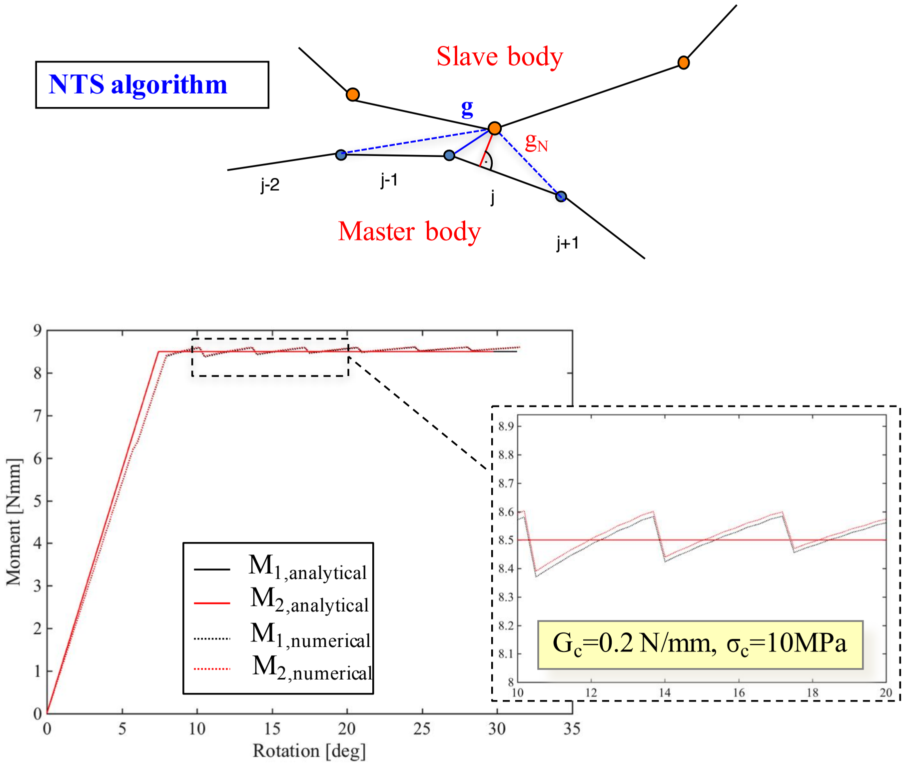

Figure 1: Global response of a Moment Loaded DCB in mode-I loading condition.

References

[1] R. Dimitri, P. Cornetti, V. Mantič, M. Trullo, L. De Lorenzis, Mode-I debonding of a double cantilever beam: a compariosn between cohesive crack modeling and Finite Fracture Mechanics, *Int. J. Solids Struct.*, 124:57-72, 2017.

[2] R. Dimitri, F. Tornabene and G. Zavarise, Analytical and numerical modeling of the mixed-mode delamination process for composite moment-loaded double cantilever beams, *Comp. Struct.*, in press, DOI: 10.1016/j.compstruct.2017.11.039.

[3] R. Dimitri, M. Trullo, L. De Lorenzis and G. Zavarise, Coupled cohesive zone models for mixed-mode fracture: A comparative study, *Eng. Fract. Mech.*, 148:145-179, 2015.

Frictional Receding Contact Problems With Separation of Interface At Remote Points

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Summary: Frictional receding contact problems involving a semi-infinite linear elastic layer in contact with a half-plane of the same material are solved by the insertion of distributions of dislocations (or eigenstrains). A new formulation is presented which takes into account the expected displacements at remote points.

Introduction

Receding contacts are an important class of contacts found in many components, including bolted joints, for which the contact area reduces with the application of any finite normal load. The reduction in many cases is non-continuous, — with the application of any finite normal load, however small, the contact surfaces snap into a new configuration with a reduced contact area. This was shown to be the case for frictionless elastic receding contacts by Dundurs and Strippes [1] and the result is valid for frictional elastic interfaces when the loading is monotonic [2].

Despite receding contacts being present in many mechanical components, such as bolted joints, their properties are largely unknown and are difficult to model using finite elements. Modelling receding contacts with distributions of dislocations is a promising analytical method [3]. However current methods require the inserted distributed dislocations to be bounded-bounded. This is not appropriate for many receding contact situations where the contact surfaces separate even at infinite distances away from the region of the application of the load, as in the problem studied by Chaise et al. [4], which is revisited here. The problem geometry studied is given in Figure 1.

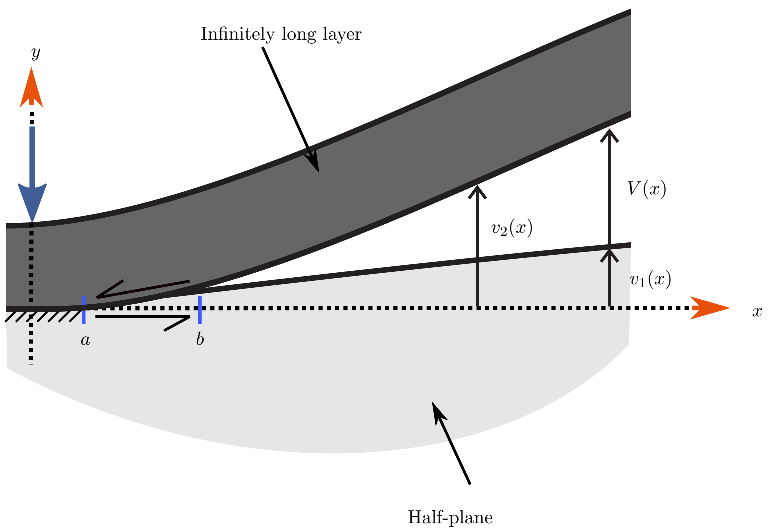

Figure 1: The problem geometry.

Overview of the method

Preformed dislocation distributions (or eigenstrains) are inserted along the location of the contact interface which take into account the expected displacements of the contact interfaces at remote points, and then corrective bounded-bounded distributions of dislocations are superimposed.

Results

The displacements of the layer and the half-plane interfaces as well as the contact tractions are found for different coefficients of friction and for different loading regimes. Figure 2 shows the relative displacement along the contact interface found from this model compared with the Chaise model. Results of other problems with different loading have been obtained as well.

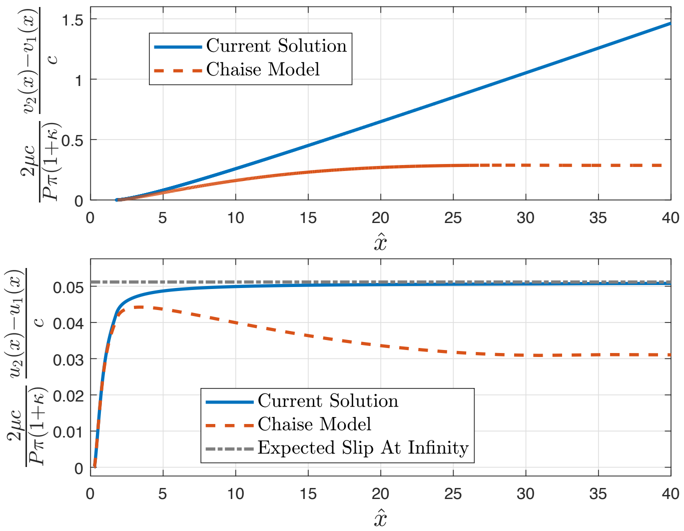

Figure 2: For $f = 0.5$, **Top**: Separation along the contact interface. **Bottom**: Relative slip along the interface. Results from the current model are compared with the results from the Chaise model.

References

[1] J Dundurs and M Stippes. Role of elastic constants in certain contact problems. *Journal of Applied Mechanics*, 37(4):965–970, 1970.

[2] Young Ju Ahn and JR Barber. Response of frictional receding contact problems to cyclic loading. *International Journal of Mechanical Sciences*, 50(10):1519–1525, 2008.

[3] KS Parel and DA Hills. Frictional receding contact analysis of a layer on a half-plane subjected to semi-infinite surface pressure. *International Journal of Mechanical Sciences*, 2016.

[4] T Chaise, RJH Paynter, and DA Hills. Contact analysis of a semi-infinite strip pressed onto a half plane by a line force. *International Journal of Mechanical Sciences*, 81:60–64, 2014.

Are mode II and mode III fracture energies real material properties? A response based on a 3D DCB FEA with frictional multiplane CZMs

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Summary: The meaningfulness of mode II and mode III fracture energies as objective test setup-independent material properties of structural interfaces in quasi-brittle materials, like Fiber-Reinforce Polymers (FRPs) and concrete, has long been debated, in consideration of the spurious effects that friction may play over the total amount of fracture toughness in mixed-mode decohesion processes [1,2]. Meso-scale irregularities, characteristic of fracture surfaces in concrete (and also observable in FRP fracture process zones), represent a further complexity multi-scale factor which hinders the design of experimental standards and makes the mechanical analysis theoretically and computationally challenging.

In this contribution the results are presented of a recent campaign of numerical-experimental Finite Element (FE) multi-scale analyses of mixed-mode FRP delamination, tested by a Double Cantilever Beam setup with Uneven Bending Moments (DCB-UBM) [3,4]. The FE model employs Multi-plane Cohesive-Zone Models (M-CZMs) to describe decohesion in the process zone. M-CZMs, based on the concept of Representative Multiplane Element (RME) [3], are found to be a computationally convenient option, which is intermediate between phenomenological macroscale CZMs and full multiscale models, for analysing mixed-mode fracture over micro-structured interfaces. By employing a small-scale elementary cohesive-frictional response formulated within the mechanics of generalized continua, M-CZMs formulations capture the increase in measured fracture energy, under increasing mode II and mode III components, as a natural effect of multiscale coupling between cohesion, friction and interlocking.

Numerical results bring compelling evidence, relevant to debonding and delamination processes in which friction and mesoscale irregularities are significant in the process zone, for giving a possible answer to the opening question on whether mode II and mode III fracture energies can be treated as real material properties or not.

References

[1] Carpinteri, A. et al. "Is mode II fracture energy a real material property?" *Computers & Structures*, 48, 397-413 (1993).

[2] Brunner, A. et al. "A status report on delamination resistance testing of polymer–matrix composites" *Engineering Fracture Mechanics* 75, 2779-2794 (2008).

[3] Sørensen, B.F., Jacobsen, T.K. "Characterizing delamination of fibre composites by mixed mode cohesive laws", *Compos. Sci. Technol.* 69, 445–456 (2009).

[4] Serpieri, R., Albarella, M., Sacco, E. "A 3D two-scale multiplane cohesive-zone model for mixed-mode fracture with finite dilation", *CMAME*, 313, 857-888 (2017).

Non-local interface model based on fractional operators

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Keywords: fractional long-range interaction, cohesive zone model, mode-I interface debonding.

Cohesive zone models (CZMs) have been recently used to describe the failure processes and the debonding phenomena within materials and interfaces [1,2]. In this work we apply an enhanced CZM, where a displacement-based non-local elastic interaction is introduced in the interfacial constitutive law. In this way, the debonding process is described taking into account the non-linear elastic local forces and the long-range interactions depending on the relative displacements. In order to model the non-local effects, we introduce in the constitutive equation of the interface, the elastic interaction forces depending on the relative displacements and distance-decaying functions ruling the amount of the interface interactions. In several works from the literature a power-law is adopted as distance-decaying function [3-5]. Such choice leads to fractional order operators in the non-local constitutive law.
In the present study, a structural element composed by two beams partly bonded together by a non-local elastic interface is considered. For this case, a numerical solution is achieved by a finite element analysis introducing the proposed non-local constitutive law of the interface element.

References

[1] Dimitri R., Cornetti P., Mantič, Trullo M., De Lorenzis L., "Mode-I debonding of a double cantilever beam: A comparison between cohesive crack modeling and Finite Fracture Mechanics" in International Journal of Solids and Structures, 124:57-72 (2017).
[2] Dimitri R., Trullo M., G. Zavarise G., De Lorenzis L., "A consistency assessment of coupled cohesive zone models for mixed-mode debonding problems" in Frattura ed Integrità Strutturale, 8(29):266-283 (2014).
[3] Di Paola M., Zingales M., "Long-range cohesive interactions of non-local continuum faced by fractional calculus" in International Journal of Solids and Structures, 45:5642-5659 (2008).
[4] Di Paola M., Failla G., Zingales M., "Physically-Based Approach to the Mechanics of Strong Non-Local Linear Elasticity Theory" in Journal of Elasticity, 97:103-130 (2008).
[5] Failla G., Santini A., Zingales M., "A non-local two-dimensional foundation model" in Archive of Applied Mechanics, 83:253-272 (2013).

SESSION 3

An Interplay of Nayak and Hurst Parameters in Mechanical Contact of Rough Surfaces

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Surface roughness is responsible for numerous interfacial contact properties. The roughness can be characterized through different parameters: for example, the Hurst exponent, root mean squared gradient, and rarely used Nayak parameter. In this contribution we discuss the interplay between these characteristics and propose a simple phenomenological model of rough contact.

Introduction

Contact and friction interactions play an essential role in many quotidian contexts, including those related to industry (e.g., tire-road and wheel-rail contacts, electric switches, bearings, and brake systems), everyday human activity (e.g., walking, handling, touching, and sitting) and natural phenomena (e.g., earthquakes, landslides, and glacier motion). Regardless of such prevalence, contact-related mechanisms (friction, adhesion, and wear) are still not fully understood and thus are among the most cutting edge research topics in the mechanical community.

Numerous models of contact-related mechanisms exist at structural scale. They serve to model interfacial normal and tangential stiffness, frictional resistance, material removal on rubbing surfaces (wear), heat transfer between contacting solids, contact electric resistance, adhesion, interfacial fluid flow, microstructural changes in near-contact material layers, fretting life-cycle, debris generation, lubrication, especially in mixed regime, and other mechanisms. Admittedly, all the aforementioned phenomena are strongly related to the surface roughness. The associated models can be incorporated in a macroscopic/structural model via constitutive interfacial equations. These equations can be based either on experimental data, and thus remain purely phenomenological, or can take the microscopic roughness as the starting point. The latter class of models shall have a greater predictive power, and potentially can be used for a large spectrum of applications. However, because of the strong non-linearity of the contact/friction mechanisms and extreme complexity of surface roughness, construction of a reliable analytical micro-mechanical model presents a serious challenge. Nevertheless, reliable phenomenological models can be constructed on the data obtained with microscopic-scale numerical simulations as it is done in this contribution.

Methods

We used a spectral-based boundary element method [1] to solve friction- and adhesionless normal contact between two elastic half-spaces with rough surfaces in the framework of infinitesimal deformations. Numerical simulations of contact were carried out on synthetic rough [2] surfaces with controlled spectra. The study is focused on the analysis of the evolution of the true contact area under increasing squeezing pressure since the true contact area is one of the most important characteristics of the "rough contact". To improve the accuracy of the contact area simulations, we used a simple technique which is based on estimation and correction of the discretization-error; and includes the evaluation not only of the contact area but also of the contact perimeter [3]. This technique enables us to study rough surfaces within unprecedently *wide* range of parameters without loss of accuracy. Therefore, this analysis could be done using moderate computational grids of only 2048×2048 points on surface.

Results

Apart from the classical scaling of the squeezing (nominal) pressure by the root mean squared roughness gradient (square root of the doubled second spectral moment), we identified new trends. First, we demonstrated a weak but persistent dependence of the contact area on Nayak parameter. This parameter, rarely used in characterization of surface roughness, is central in Nayak's random process model of rough surface [4]. Apart from the zero-th and second moment, this dimensionless parameter includes the forth moment and determines the breadth of the roughness spectrum. Second, in the literature on the topic, the role of the Hurst exponent (related to the fractal dimension) was studied intensively, it characterizes the decay of the spectrum with the increasing wavenumber, and was believed to determine the contact area. We showed that for the same Nayak parameter but different Hurst exponents, the contact area is the same, whereas for the same Hurst exponent and different Nayak parameters, the results are different. Thus, we demonstrated that the believed dependence on the Hurst exponent is explained by the underlying dependence of the latter on Nayak parameter. Third, we deduced a phenomenological, weak logarithmic dependence of the contact area on the Nayak parameter and formulated a pressure-dependent friction law with physically meaningful parameters [5].

Discussion

The presented study deals with artificially synthesized "rough" surfaces, which should be obligatory smooth at small scale in order to permit accurate resolution of continuum mechanics equations. The necessity to introduce a high-frequency cut-off in the spectral representation, or equivalently keep the surface smoothness is physically questionable and calls for a critical discussion. We plan to address these questions and suggest possible research directions.

References

[1] H. M. Stanley and T. Kato. An FFT-based method for rough surface contact. *J Tribol-T ASME*, 119:481–485, 1997.

[2] Y. Z. Hu and K. Tonder. Simulation of 3-D random rough surface by 2-D digital filter and fourier analysis. *Int J Mach Tool Manu*, 32:83–90, 1992.

[3] V.A. Yastrebov, G. Anciaux, and J.F. Molinari. On the accurate computation of the true contact-area in mechanical contact of random rough surfaces. *Tribol Int*, 114:161–171, 2017.

[4] P. R. Nayak. Random process model of rough surfaces. *J Lubr Technol (ASME)*, 93:398–407, 1971.

[5] V.A. Yastrebov, G. Anciaux, and J.F. Molinari. The role of the roughness spectral breadth in elastic contact of rough surfaces. *J Mech Phys Solids*, 107:469 – 493, 2017.

A 2-D model for friction of complex anisotropic surfaces

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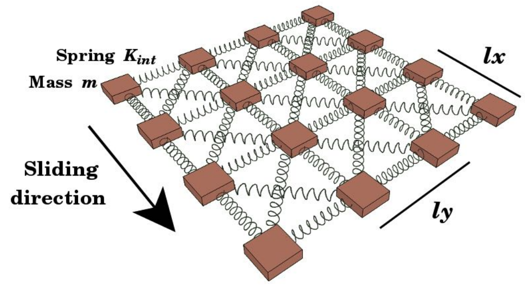
Summary: Microscopic surface structures can modify the emergent friction properties at macroscale, especially in the case of anisotropic designs. Here, we present a two-dimensional version of the spring-block model to investigate the effects of patterns such as grooves, cavities, pillars and complex structures on the surface frictional properties.

Abstract

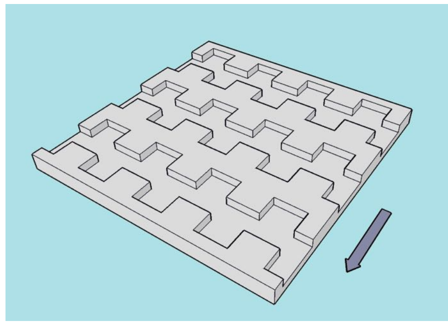
The frictional behavior of macroscopic bodies arises from various types of interactions occurring at different length scales between contact surfaces in relative motion. While it is clear that their ultimate origin lies in inter atomic forces, it is difficult to scale these up to the macroscopic level, including other typical phenomena such as surface roughness, elasticity or plasticity, wear and specific surface structures. For this reasons, simplified models have been developed to address specific friction problems [1]. In particular, one of the most used models is the spring-block model, which has been adopted to investigate many aspects of dry friction of elastic materials [2][3].

In this presentation, we illustrate a two-dimensional version of the spring-block model (figure 1a) to describe the frictional behavior of an elastic patterned surfaces sliding on a rigid substrate [4]. Our principal aim is to compare the results with those obtained in the one-dimensional model [5][6] and to extend our study to more complex patterns, e.g. arrangements of cavities or anisotropic structures like those found in biological materials, and to two-dimensional surfaces with functionally graded material properties. This formulation of the spring-block model allows to consider a more realistic situation and captures a variety of behaviors that can be interesting for practical applications.

We show how static friction can be effectively tuned by appropriate design of surface features and we identify some mechanisms that modify the global behavior during the transition from static to dynamic friction. Some of these effects appear to be universal, in the sense that they take place regardless of the specific configuration of the surface. We investigate the role of the geometry of the structures, showing that friction can be considerably reduced by increasing their perimeter (figure 1b). Finally, we illustrate how the friction coefficients of anisotropic surface structures depends non-trivially on the sliding direction. Overall, we find that the two-dimensional spring-block model is able to capture these effects due to complex structures, similar to those commonly observed in nature or employed in technological fields.



(a)



(b)

Figure 1: (a) Discretization of a square surface into a 2-D spring-bock model, showing the mesh of the internal springs. (b) Example of winding tread patterns analyzed with the spring-block model allowing to reduce the static friction coefficient.

References

[1] A. Vanossi, N. Manini, M. Urbakh, S. Zapperi, E. Tosatti, *Colloquium: Modeling friction: From nanoscale to mesoscale*, Rev. Mod. Phys. 85 (2013) 529

[2] O.M. Braun, I. Barel, M. Urbakh, *Dynamics of Transition from Static to Kinetic Friction*, Phys. Rev. Lett. 103 (2009) 194301

[3] J. Trømborg, J. Scheibert, D.S. Amundsen, K. Thøgersen, A.Malthe-Sørenssen, *Transition from Static to Kinetic Friction: Insights from a 2D Model*, Phys. Rev. Lett. 107 (2011) 074301

[4] G. Costagliola, F. Bosia, N.M. Pugno,*A 2D model for friction of complex anisotropic surfaces*, Jour. Mech. Phys. Solids 112 (2018) 50

[5] G. Costagliola, F. Bosia, N.M. Pugno, *Static and dynamic friction of hierarchical surfaces*, Phys. Rev. E 94 (2016) 063003

[6] G. Costagliola, F. Bosia, N.M. Pugno, *Tuning friction with composite hierarchical surfaces*, Tribol. Int. 115 (2017) 261

Strong adhesive performance of indenters on thin layers

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In this work we study the adhesive behaviour of probes, which indent a thin layer of micro/nanometric thickness coated on a rigid foundation. It is shown that ultrastrong adherence can be obtained. The study is particularly suited for polymeric coatings over metals.

Introduction

Adhesion is a very debated topic in contact mechanics, which spans different fields of application, from adhesion of rough surfaces to bioinspired adhesive mechanisms. Different researchers have tried to imitate the design strategies adopted by insects (e.g. geckos, ants) and develop "optimal" surface profile to enhance adhesion. Nanopatterned surfaces, with repeating pillars or dimples are nowadays commonly adopted by many researchers who aim to develop pressure sensitive adhesives. In the present work we study the adhesive behaviour of rigid probes with power law profile that indent thin elastic layers of micro/nanometric thickness coated on a rigid foundation. The adhesive solution is obtained generalizing the adhesiveless solution obtained by Johnson-Jaffar-Barber [1-3], to the case of short-range adhesion (JKR type) retaining only the original "thin layer" approximation proposed by Johnson [3], who assumed that the layer thickness b is much smaller than the radius of contact a, i.e. $b<<a$, so that plane sections remain plane upon deformation. Plane and axisymmetric problems are handled for both cases of frictionless unbounded and bounded compressible layer. We show that very strong adherence (up to the theoretical strength) can be reached both in line contact and in axisymmetric contact for thin layers, typically of nanoscale size. We give analytical predictions of the loading curves and provide indentation, load and contact radius at the pull-off. In line contact adhesion enhancement occurs as an increase of the actual pull-off force, while for the axisymmetric case, we show that the adhesive behaviour is strongly affected by the indenter shape. Nevertheless below a critical thickness of the layer (typically below 1 μm) the theoretical strength of the material is reached. This in contrast with the axisymmetric Hertzian case, which has been shown to be insensitive to the layer thickness. This suggests a new possible strategy for "optimal adhesion". It is shown that due to Poisson effects the case of compressible confined layer is more efficient that the case of frictionless layer as the same detachment force is reached with smaller contact area. High sensitive micro-/nanoindentation tests may be performed using probes with different power law profiles for characterization of adhesive and elastic properties of micro-/nanolayers.

References

[1] Jaffar, M. J. (1989) Asymptotic behaviour of thin elastic layers bonded and unbonded to a rigid foundation, *Int. J.Mech. Sci.*, 31, 229.

[2] Barber, J. R. (1990). Contact problems for the thin elastic layer, *International journal of mechanical sciences*, 32(2), 129-132.

[3] Johnson, K. L., *Contact Mechanics*, Cambridge University Press, Cambridge (1985).

Usage of an asperity-free formulation in asperity models

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Summary: We start with a simple observation on the nature of nominally flat, fractal surfaces and give an argument in favor of asperity models, and suggest a semi-analytical approach. We then apply a relatively new idea of an asperity-free formulation of junction sizes in debris formulation to the model asperity.

Abstract

Rough surface topography is often described using the fractal model, wherein the power spectral density follows the law $C_{2D} \propto |q|^{-2(H+1)}$, with H being the Hurst exponent, q is the wave vector. This also assures the property of self-affinity. Below a certain wavevector $q_{\min}$, the power spectrum is assumed to be either constant (in which case this number is called 'roll-off') or zero ('cut-off'). This minimum wave-vector is necessary to make the surface appear nominally flat. Its corresponding wavelength λ is a characteristic length scale of the surface, at the macroscopic end of the scale. In the current talk, we focus on these large-scale structures. It is easy to see that a nominally flat surface of the type in question with spatial extent L should exhibit approximately $N = (L/\lambda)^2$ fractal asperity peaks. The nature of those $q_{\min}$-free asperities has been studied in detail by the author in [1]. In [2], it was shown that for a number of macroscopic quantities, the fractal asperity can replaced with a single, regular indenter of radially symmetric shape, described by the shape function $z(r) = c \cdot r^H$.

This description does however not truly eliminate the fractal paradox, which now appears in form of a stress singulary at $r = 0$.

In the talk, we will further analyze indenters of this kind of indenter, in single contact and in larger numbers with a given height distribution.

Special focus is put on the question of the feasibility of wear particle formation. Following the works of Rabinovicz [3], the recent works from Molinaris group [4] and the very fresh idea of an asperity-free size criterion, based on energy-density [5], we investigate under which conditions such indenters could indeed form a particle. To assess the applicability to rough surfaces, some results of numerical BEM-simulations are given for comparison.

References

[1] R. Pohrt, Normal Stiffness of Multiscale Rough Surfaces in Elastic Contact, PhD thesis, TU Berlin, 2013.

[2] V. Popov, M. Heß, *Methode der Dimensionsreduktion in Kontaktmechanik und Reibung. Eine Berechnungsmethode im Mikro- und Makrobereich*, chapter 10, Springer, 2013.

[3] E. Rabinowicz, The effect of size on the looseness of wear fragments, *Wear*, 2:4–8, 1958.

[4] R. Aghababaei, D. H. Warner, J.-F. Molinari, Critical length scale controls adhesive wear mechanisms, *Nature Comm*. 7, 11816, 2016.

[5] V. Popov, Adhesive wear: generalized Rabinowicz criteria, *Facta Universitatis: Mech. Eng. Vol. 16*, in press (2018).

SESSION 4

Optimal control approach to simulating wear under cyclic loading

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A control optimal approach is developed to estimate the asymptotic state reached by a solid continuum subjected to wear contact submitted to a cyclic loading. The stabilized state is the solution of a minimization problem in order to define the stabilized geometry for wear problem of a half-plane under contact with a cyclically moving indenter.

Introduction

We consider a cylindrical indenter with radius R in contact on an half-plane. The indenter has a cyclic horizontal motion with amplitude U and an imposed vertical displacement δ. As the displacement is imposed, the loss of matter is finite and the thickness of the loss volume is less than δ, it tends to a stabilized value. The presentation proposes an estimation of the loss of matter using two main ideas. The simulation of the contact using the Galin's equation, and their variations with respect to the geometry of the contact with the indenter, and the determination of the stabilized state with respect to a wear-contact criterion written in terms of energy.

Variations of the shape, and state of the system

The surface of the half-plane evolves and its shape is defined by the position $x, \theta(x,t)$ of the surface point. Because of the loss of matter the vertical position θ is a function of time. For an half plane in linear elasticity, the displacement along the boundary is determined by the distribution of the traction imposed on the surface by the relation of Galin's [3]:

$$c_1 u_{,x}(x) = c_2 \sigma^{yy}(x) + \frac{1}{\pi} Fp \int_{-a}^{+a} \frac{\sigma^{xy}(s)}{x-s}\, \mathrm{d}s, \tag{1}$$

$$c_1 v_{,x}(x) = -c_2 \sigma^{xy}(x) + \frac{1}{\pi} Fp \int_{-a}^{+a} \frac{\sigma^{yy}(s)}{x-s}\, \mathrm{d}s, \tag{2}$$

where $c_1 = \dfrac{E}{2(1-\nu^2)}, c_2 = \dfrac{1-2\nu}{2(1-\nu)}$, E is the Young's modulus, ν is the Poisson's ratio, Fpf is the principal value of f in the sense of Cauchy. The traction on the surface is given in terms of the Cauchy stress tensor : $\mathbf{T}^o = \boldsymbol{\sigma}^o.\, \boldsymbol{e}_y = \sigma^{yy}_o\, \boldsymbol{e}_y + \sigma^{xy}_o\, \boldsymbol{e}_x$. If during the change of geometry, the traction $\mathbf{T}$ is conserved, the variation of the displacement on the upper surface is:

$$c_1 u^1_{,x}(x) = -c_2 \theta T^o_x - \frac{1}{\pi} Fp \int_{-a}^{+a} \frac{\theta T^o_y}{x-s}\, \mathrm{d}s - \frac{2}{\pi^2} \int_{-a}^{+a} \frac{\theta}{x-s} \int_{-a}^{+a} \frac{T^o_x(t)}{s-t} dt\, \mathrm{d}s$$

$$c_1 v^1_{,x}(x) = c_2 \Big(\theta T^o_y + \frac{2\,\theta}{\pi} Fp \int_{-a}^{+a} \frac{T^o_x}{x-s}\, \mathrm{d}s\Big) - \frac{1}{\pi} Fp \int_{-a}^{+a} \frac{\theta T^o_x}{x-s}\, \mathrm{d}s$$

The conservation of f is given by $D_\theta f = \lim_{\epsilon\to 0}(f(x,\epsilon\theta,\epsilon) - f(x,0,\epsilon))/\epsilon) = 0$. As $D_\theta \boldsymbol{n} = \theta_{,x}\ \boldsymbol{e}_x$, $\dot{T}_x = (\theta\sigma_o^{xx})_{,x}$ and $\dot{T}_y = (\theta\sigma_o^{xy})_{,x}$. These variations of traction are applied at point $(x, y = 0)$ to conserve the value T^o of traction on the surface at point $(x, y = \theta(x))$. These equations are different from those proposed in [4]. They can be obtained directly applying classical integral equations.

The contact condition and the wear process

Denoting the gap to contact by $w = \delta - v - \theta(x) + g(x,t)$. The contact condition is defined by

$$T_y \geq 0, w \leq 0, \quad T_y w = 0. \tag{3}$$

δ is the vertical position of the cylindrical indenter of radius R, g describes the shape of the indenter center at point $(X(t) = A\sin\pi\dfrac{t}{T})$, $g(x,t) = \dfrac{(x - X(t))^2}{2R}$. This condition can be replaced by a regularized form : $T_y = \gamma(w)$, where γ is a convex and increasing function.

The optimal condition

The loss of matter is governed by a wear criterion expressed in terms of local energy release rate as proposed in [1][5]

$$G = W(\boldsymbol{\varepsilon}(u)) - \boldsymbol{n}.\boldsymbol{\sigma}.\nabla\boldsymbol{u}.\boldsymbol{n} \leq G_c; \quad W = \frac{1}{2}\boldsymbol{\varepsilon} : \mathbb{C} : \boldsymbol{\varepsilon}. \tag{4}$$

The optimal shape $(x, \theta_o(x))$ is given by the optimal condition

$$\min_{\theta,B} J; \quad J(\theta, B) = G_c \int_B^B \theta \;\mathrm{d}x + \alpha \int_T \int_B^B < G(x,t) - G_c >_+ \;\mathrm{d}x\mathrm{dt}. \tag{5}$$

where $< f >_+ = (f + |f|)/2$. The first term corresponds to the dissipation of the system. The last one implies that the criterion is obtained at a minimum set of point during the peroid T of the loading. Due to interaction with the geometry change, G is an implicit fonction of θ.

Results and conclusion

The properties of the problem of optimisation is discussed and an adapted algorithm of simulation is proposed. In particular the condition of contact is regulatized as proposed in [4] The influence of the shape variations on the distribution of pressure is analysed.

References

[1] C. Stolz, Thermodynamical description of running discontinuities: application to friction and wear, *Entropy*, 12:1418-1439, 2010.

[2] M. Dragon-Louiset, On a predictive macroscopic contact sliding wear model based on micromechanical considerations., *Int. J. solids Struct.* 38(9):1625-1639, 2001

[3] L.A. Galin, Contact problems of the theory of elasticity in the presence of wear, *J. Appl. Math. Mech.*, 40:981-986

[4] M. Peigney, Simulating wear under cyclic loading by a minimization approach, *Int. J. Solids Struct.*, 41:6783-6799, 2004

[5] M. Dragon-Louiset, C. Stolz, A thermodynamical approach to contact wear, *C.R. Acad. Sci. Paris* 327:1275-1280, 1999

Frictional contact and wear along virtual interfaces

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Summary: In this work, features of surface-to-surface contact discretization are combined with the extended finite element method to handle contact problems along virtual surfaces in 2D. We focus on the incorporating of geometrical changes, which result from wear process and are taken into account by mobile virtual interfaces.

Interactions between solids involving contact, friction, adhesion and wear are complex both with regard to their mathematical description and numerical treatment. The interfacial nature of these phenomena lays a strong emphasis on the interface discretization schemes. Stability and appropriate patch-test performance of these schemes are necessary ingredients to ensure the overall accuracy and robustness of the associated numerical treatment.

A relative motion between contacting bodies can lead to material removal (wear) on rubbing surfaces. The resulting geometrical changes along the interface affect the distribution of contact pressures, leading to a modification of the wear evolution and thus determines the lifespan of the system. Standard numerical treatment of wear uses the assumption of a slow surface change and thus relies on the notion of equivalent wear cycle. It also involves: (i) constitutive local wear laws determining the wear depth evolution at every effective cycle, (ii) remeshing procedures to capture the geometrical changes at the interface, and (iii) field remapping of history variables, if their storage is required in used nonlinear material models. A different approach to this problem is elaborated in this work.

Methodology

The surface-to-surface discretization scheme combined with penalty/Lagrange based resolution techniques for the frictional contact constraints, treat accurately the contact problems along non-conformal interfaces [1]. The extended finite element method (X-FEM) enables to handle intra-mesh discontinuities: voids, cracks, material interfaces [2]. In this work we extend the idea of embedded interfaces for tying problems [3]: Combining the embedded interfaces within the X-FEM framework with the surface-to-surface contact formulations, results in a simplified numerical framework to treat contact problems along virtual interfaces. These virtual interfaces passing through the mesh volume (not necessarily through interfaces delimited by element boundaries) can incorporate enriched geometrical features, such as surface evolution due to wear. In addition to handling surface evolution, this computational scheme ensures accurate representation of surface tractions, which is essential for wear simulation.

As illustrated in Figure. 1, we propose to use the embedded/virtual interface Γ_v, to describe the geometrical changes resulting from wear. Surface Γ_v coincides with the unworn surface $\Gamma_{c2} \subset \partial\Omega_2$ at the beginning of the simulation. Upon loading, the material of Ω_2 in contact with Ω_1 experiences wear and the surface Γ_v propagates into the bulk to capture the surface evolution. The approach includes: (i) mortar method to treat frictional contact between the real interface pair ($\Gamma_{c1} - \Gamma_{c2}$) initially and between virtual-real interface pair ($\Gamma_v - \Gamma_{c1}$) subsequently (ii) an energy based wear law to determine the wear depth (wear depth is proportional to the dissipated energy per equivalent wear cycle) (iii) virtual interface Γ_v to locate the evolving worn-surface and (iv) selective integration within blending elements intersected by the worn-surface.

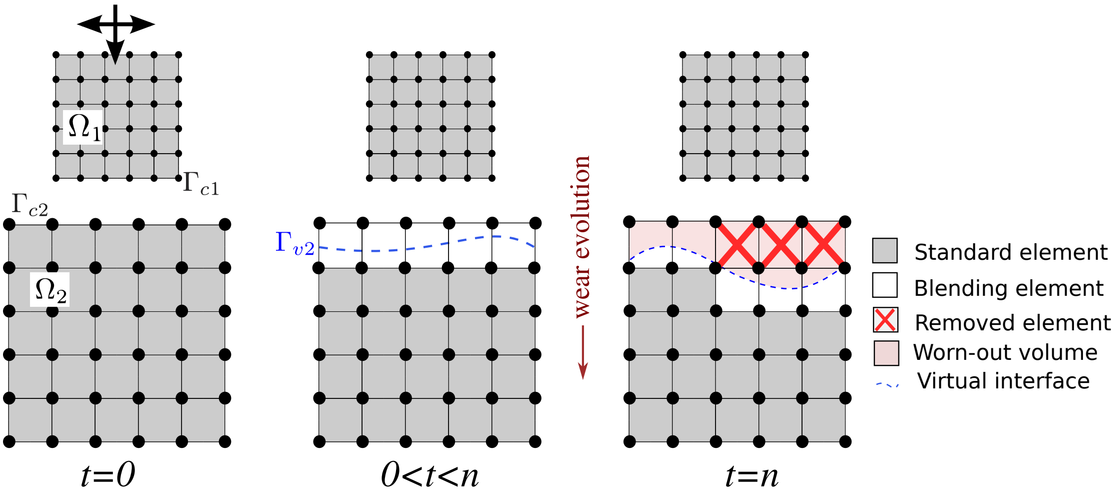

Figure 1: Wear at the interface, modeled using virtual interface Γ_v (total simulation time $t = n$).

Conclusion

The proposed method provides a simplified solution to the numerical treatment of wear compared to conventional ones. It can be applied typically for fretting of disk-blade assemblies in aircraft engines and power generating turbines, wear of the cylinder-liner system in automotive applications and other areas in which wear plays a significant role. In perspective, this method will be tested on various configurations to quantify its performance in regard to numerical aspects such as convergence.

References

[1] M. Glitterle, A. Popp, M.W. Gee, W.A. Wall. Finite deformation frictional mortar contact using a semi-smooth Newton method with consistent linearization, *Int. J. Num. Meth. Engng.*, 84 (5), 543-571 (2010).

[2] N. Sukumar, D.L. Chopp, N. Moës, T. Belytschko. Modeling holes and inclusions by level sets in the extended finite-element method, *Comput. Methods in Appl. Mech. Eng.*, 46, 6183–6200 (2001).

[3] J.D. Sanders, T.A. Laursen, M.A. Puso. A Nitsche embedded mesh method, *Comput. Mech.*, 49 (2), 243–257, (2012).

Modeling of wear considering heterogeneous friction material

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Summary: This paper focuses on the methodology leading to simulate wear generation in a heterogeneous material submitted to contact sliding. The strategy consists to enrich the contact stiffness in terms of gap and behavior. The non-linear behavior of the contact stiffness is obtained using a homogenization method near contact interface (FE2).

Introduction

Nowadays, many industrial structures are more and more made of highly heterogeneous materials due to their good mechanical behavior. For example, in the transportation domain, frictional materials are of a big interest. A major problem in transportation, in particular for braking applications, is the durability of components subjected to rubbing contact, mainly wear and its consequences in terms of generation and particles emission. The latter one represents a major ecological stake. The materials of friction used are heterogeneous and the solicitations applied lead to the formation of interface layer, called third body [1], from the wear particles. The latter one plays an important role in the phenomenon of friction influencing the system performances (energetics, noise pollution, rate of wear). The current modeling of the rubbing systems considers generally materials in the contact as homogeneous and interface as continuous, what is insufficient to understand wear phenomenon. The major difficulty is to consider at the same time the contact and the system scale. Considering the problem of contact simulation, Finite element method (FEM) is almost used to model such structures. Usually, the whole structure and all heterogeneities are meshed explicitly. Doing so is very high demanding numerically.Some authors use analytical methods, see the work of Leroux and al[2] and among all, because of their fast computation time but they are very restrictive. Because of high computation time induced by contact problem and material heterogeneity, taking the latter one into account in a global structure requires multiscales methods. In [3], Temizer and al., improve a multiscale finite element method to integrate material heterogeneity and material damage in a contact problem. Many authors have shown that surface instabilities have an impact on the contact pressure repartition, see the work of Dufrénoy and al.[4]. These above developments assume the material in contact as homogeneous and most of them don't take wear mechanisms into account.

Strategy

In this paper we propose using a multiscale strategy(FE2) to simulate wear generation in a heterogeneous material context submitted to contact sliding. Thus, the strategy incorporates two main steps. The first one is to discretize the whole heterogeneous structure into a set of patches, calculate the contact stiffness of each patch by the way of a homogenization method and to embed it into a large scale numerical model. The second step aims to compute the wear lost using an Archard wear law [5], to integrate it in the homogenization strategy and to embed it into the large scale numerical model in terms of gap. The gap is updated in function of the lost wear. In this paper the main wear mechanism considered is the flow debit in the concept of tribological circuit [1]. The whole strategy is summarized in the figure1 below. One of the key contribution of the strategy proposed in this work is the reducing of computation time compared to the traditional FEM method. Also, not only the flow debit in the concept of tribological circuit can be considered but also other mechanisms can be introduced such as crack and particle decohesion at the microscale.

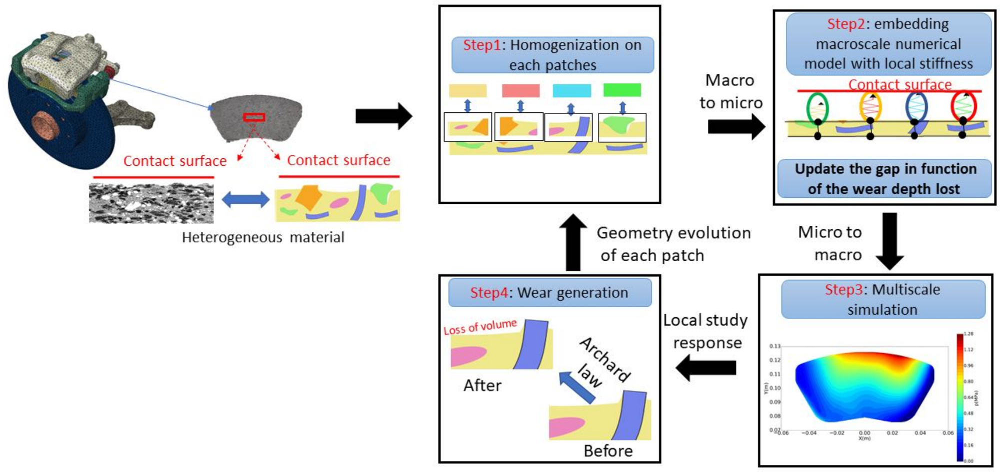

Figure 1: The global strategy showing the wear modeling considering heterogeneous friction material

First results

A comparison between an explicit and a homogenized problem considering a heterogeneous friction material rubbing on a disk is performed. Our first results obtained via the multiscale strategy explained above are in a good agreement with the traditional FEM method. Advanced results will be presented at the conference.

References

[1] Y. Berthier. Maurice Godet's Third Body *Tribology Series*, vol. 31, p. 21-30, janv. 1996

[2] J. Leroux. Contact analysis in presence of spherical inhomogeneities within a half-space *International Journal of Solids and Structures*, 2010, 47, 30343049.

[3] I. Temizer and P. Wriggers. On the computation of the macroscopic tangent for multiscale volumetric homogenization problems *Comput. Methods Appl. Mech. Engrg.*, 198 (2008) 495-510.

[4] P. Dufrénoy and al, A multiscale model of a disc brake including material and surface heterogeneities, *2016, SAE Int. J. of Passenger Cars-Mechanical Systems*, 9(2016-01-1911), 1136-1143.

[5] Archard and al, Contact and Rubbing of Flat Surface, *J.Appl. Phis.*, 24 (8): 981-988.

Wear Analysis of a Multiphasic Heterogeneous Cylinder

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The wear of a cylindrical punch composed of multiphasic materials is studied under the assumption of Archard's law of wear. In the case of annular cylinder with rings of alternating material, the changes in surface topography and pressure distribution during the wear process is obtained and validated by the Boundary Element Method (BEM). For the cylinder with randomly distributed multiphases, the limiting profile in steady wear state as well as the root mean square (RMS) of surface gradient is numerically calculated with the BEM based on a theoretical analysis.

Introduction

The friction between a vehicle's tire and the road is an everyday contact problem. The pavement surface topography and mixture of aggregates and asphalt binder play a significant role in the skid resistance as well as tribological behavior of rubber sliding contact. It is also known that the friction coefficient in the contact between an elastomer and a rigid rough surface is roughly in the order of the mean slope of the surface [1]. In this talk we consider wear of a multiphasic composite in sliding contact with an elastic half space under a constant normal load. Two cases are investigated to study the effects of material composition under the assumption of Archard's law of wear. One is an ideal heterogeneous annular cylinder with rings of alternating material (e.g. representing aggregate and binder of the asphalt respectively). The Method of Dimensionality Reduction (MDR) is used to numerically calculate the wear of the surface and the pressure distribution in each time iteration. The other one is a cylinder with randomly distributed multiphase materials having different wear coefficients. We proposed a simple theoretical solution for the limiting profile at the stationary state. Based on that, the final surface topography with any combination of multiphases can be calculated with the BEM. The dependence of surface gradient on the ratio of wear coefficients of biphasic materials for both cases is discussed.

Case of annular cylinder

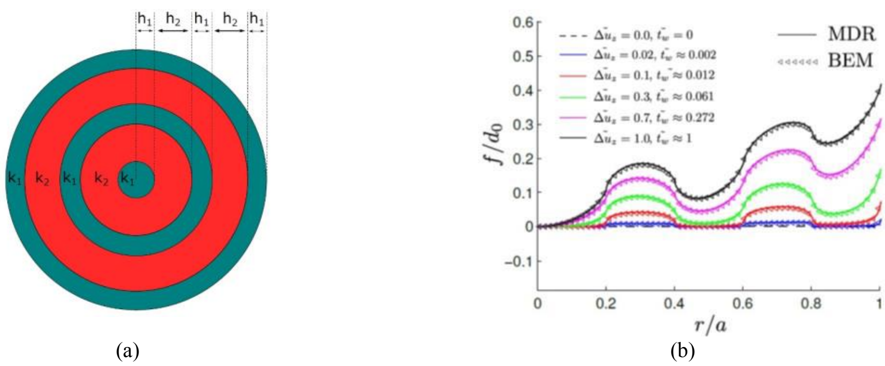


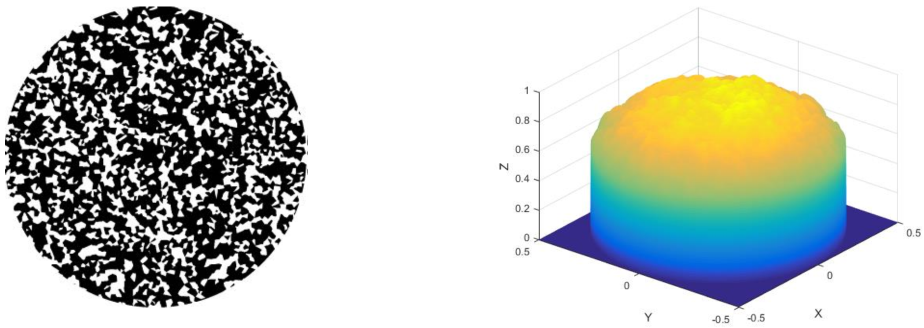
Figure 1: Wear of an annular cylinder (a) with 5 rings having different wear coefficient k_2/k_1=10, (b) worn profiles during the wear process (ratio of ring width h_2/h_1=1).

The algorithm employed to solve the wear problem with the MDR can be found in [2], which gives exact solution of axially symmetric contact problems [3], including the case of this annular cylinder with rings. The Figure 1 shows an example of the time-dependence of wear of the indented cylinder with five rings: 3D profiles $f(r)$ at distinct time steps. The numerical results from the simulation with the BEM are added in this figure for comparison. However the numerical calculation with the MDR is much faster than the BEM.

For the piecewise constant distribution of wear coefficients, the resulting stress distribution of the limiting profile will be piecewise constant. It is found that the constant stress level of rings with the higher wear coefficient is reached significantly faster. Therefore, the time needed to reach the limiting profile is mostly dependent on the lowest wear coefficient.

Case of randomly distributed multiphase materials

Theoretical analysis shows that the stress in the areas with the same phase (wear coefficient) will be constant and same at the final stationary state. Therefore, the final deformation as well as the surface topography of a multiphasic composite can be calculated using the theory of contact mechanics, or numerically using the BEM. Figure 2 is an example of biphasic composite with wear coefficient k_2/k_1=10.



(a) (b)

Figure 2: An example of biphasic composite: (a) distribution of two phases with wear coefficient k_2/k_1=10 (white and black); (b) the worn surface topography at the stationary state.

In this talk the theoretical solution for limiting profile of homogeneous material will be provided, then the numerical results for biphasic composite using the BEM will be presented. The dependence of the RMS of surface gradient on wear coefficient k_2/k_1 and area ratio of two phases will be discussed. The experimental investigation for wear of the steel-brass composite are carried out and will be compared with the numerical results.

References

[1] V.L. Popov. *Contact Mechanics and Friction*, 3nd ed., Springer, 2017.

[2] A.V. Dimaki, A.I. Dmitriev, N. Menga, A. Papangelo, M. Ciavarella, V.L. Popov, Fast High-Resolution Simulation of the Gross Slip Wear of Axially Symmetric Contacts, *Tribol. Trans.*, 2016, doi:10.1080/10402004.2015.1065529.

[3] A.V. Dimaki, A.I. Dmitriev, Y.S. Chai, V.L. Popov, Rapid Simulation Procedure for Fretting Wear on the basis of the method of dimensionality reduction, *Int. J. Solids Struct*., 2014.

Error estimation and computational simulation of contact problem with wear

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Summary: We present a quasistatic problem of an elastic body in frictional contact with a moving foundation. The main aim of this talk is to introduce a fully discrete scheme for numerical approximation and an error estimation of a solution to this problem. Finally, computational simulations are performed to illustrate these results.

Introduction

For any given mechanical contact problem from mathematical point of view we are interested in obtaining existence and uniqueness of the solution to the model. Theory of variational or hemivariational inequalities is useful in the approach to obtain these results. In many cases the proof of existence and uniqueness of the solution is not constructive. The next step in dealing with these cases is usually presenting the discrete numerical scheme and estimation of finite element method error. In this talk we focus on mechanical contact problem with wear modeled by Archard's law. The friction between body and the foundation can cause the foundation to wear over time.

Mechanical contact problem

An elastic body occupies a domain $\Omega \subset \mathbb{R}^d$ with the boundary divided into three disjoint measurable parts Γ_D, Γ_C, Γ_N, where the part Γ_D has a positive measure. The body is clamped on Γ_D with a displacement $\boldsymbol{u}$ equal to 0. The forces $\boldsymbol{f}_N$ and $\boldsymbol{f}_0$ act on Γ_N and in Ω, respectively. We assume that the acceleration of the body is almost zero, so our problem is quasistatic. In our model framework of the small strain theory is employed. We are interested in the body displacement $\boldsymbol{u}$ and foundation wear w in the time interval $[0, T]$, with $T > 0$.

From the viewpoint of Contact Mechanics, the intrinsic conditions describe the relations between normal and tangential components of displacement $\boldsymbol{u}$ and stress $\boldsymbol{\sigma}$, and take into consideration the wear effect w caused by the moving foundation with a given velocity $\boldsymbol{v}^*$. We consider the following conditions on Γ_C:

- a normal compliance condition with unilateral constant

$$\left.\begin{aligned} u_\nu(t) \le g,\ \sigma_\nu(t) + p(u_\nu(t) - w(t)) \le 0, \\ (u_\nu(t) - g)\Big(\sigma_\nu(t) + p(u_\nu(t) - w(t))\Big) = 0 \end{aligned}\right\}$$

- a type of Coulomb's law of dry friction

$$\boldsymbol{\sigma}_{\boldsymbol{\tau}}(t) = \mu\, p(u_\nu(t) - w(t))\boldsymbol{v}^*(t)\|\boldsymbol{v}^*(t)\|^{-1}$$

- the evolution of the wear function

$$w'(t) = \kappa\|\boldsymbol{v}^*(t)\|\, p(u_\nu(t) - w(t)), \quad w(0) = 0.$$

We assume that the foundation is made of a hard, perfectly rigid material covered by a soft, wearable layer of thickness $g > 0$. Detailed formulation and derivation of these conditions is presented in [2].

Variational formulation

Using the standard procedure and Green's formula we obtain the variational formulation of considered problem in the following form.

Find $\boldsymbol{u}\colon [0,T] \to U$ and $w\colon [0,T] \to L^2(\Gamma_C)$ such that for all $t \in [0,T]$

$$\begin{aligned}
&\langle F\boldsymbol{u}(t), \boldsymbol{v} - \boldsymbol{u}(t)\rangle_{V^*\times V} + \varphi(t, w(t), \boldsymbol{u}(t), \boldsymbol{v}) - \varphi(t, w(t), \boldsymbol{u}(t), \boldsymbol{u}(t)) \\
&\qquad \geq \langle \boldsymbol{f}(t), \boldsymbol{v} - \boldsymbol{u}(t)\rangle_{V^*\times V} \qquad \text{for all } \boldsymbol{v} \in U, \\
&w(t) = \int_0^t \kappa \|\boldsymbol{v}^*(s)\|\, p(u_\nu(s) - w(s))\, ds.
\end{aligned}$$

Here $V = \{\boldsymbol{v} \in H^1(\Omega)^d \mid \boldsymbol{v} = 0 \text{ on } \Gamma_D\}$, $U = \{\boldsymbol{v} \in V \mid v_\nu \leq g \text{ on } \Gamma_C\}$, the operator $F\colon V \to V^*$ is related to a certain elasticity operator and functions $\varphi(t, w, \boldsymbol{u}, \boldsymbol{v})$, $\boldsymbol{f}(t)$ are connected with the other data of the contact problem.

Let us note that variational formulation consists of a variational inequality and an integral equation that are coupled and cannot be considered separately. The existence and uniqueness of solution to this problem was also presented in [2].

Error estimation and simulations

We introduce a fully discrete scheme (both spatial variables and time are discretized) for the variational formulation. This discretization is studied in order to employ finite element method framework and estimate solutions of considered problem. In the talk we introduce this discretization as well as a numerical error estimation which guarantees a convergence of estimated solutions to original one. Finally, we present results of computational simulations showing the evolution of displacement of the body and wear of the foundation for a set of sample data. The details are contained in [1].

References

[1] M. Jureczka and A. Ochal, Numerical analysis and simulations of contact problem with wear, submitted to *Computers and Mathematics with Applications*, 2017.

[2] M. Sofonea, F. Pătrulescu and Y. Souleiman, Analysis of a contact problem with wear and unilateral constraint, *Applicable Analysis*, 95(11):2590-2607, 2017.

SESSION 5

Non-classical contact conditions and size effects in Cosserat solids

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Summary: A generalized contact model is developed for the Cosserat continuum. Based on micromechanical considerations, the model provides a link between the classical contact variables, i.e., normal gap and contact traction, and additional contact variables characteristic for the Cosserat continuum, i.e., micro-rotations and micro-moments.

Microstructured materials may exhibit size effects when the characteristic dimension of the body is sufficiently small compared to the characteristic size of the microstructure. The classical continuum theory is not capable of predicting the related size effects as it possesses no intrinsic length. One of the feasible approaches to the modelling of the size effects is to resort to generalized continuum theories that include gradient-type terms and the related intrinsic length-scale parameters. In this work, we focus on the Cosserat continuum because in this model the additional degrees of freedom have a clear physical interpretation and can be directly related to the rotations of the microstructural elements [1].

Micro-/nano-indentation is a typical example of a contact problem in which size effects can be observed experimentally. Formulation of the related boundary value problem requires that adequate boundary conditions are specified on the contact surface. The main issue here is that application of a generalized continuum model, e.g., the Cosserat model, implies that boundary conditions involve not only the displacements and surface tractions, as in the classical continuum, but also additional boundary conditions that are related to the additional unknowns or gradient terms. The simplest choice is to assume that the related generalized tractions are equal to zero, e.g., [2], but this choice is not necessarily justified from the physical point of view.

In this work, we develop a generalized contact model for the Cosserat continuum. The model is based on simple micromechanical considerations inspired by masonry-like structures. The essential feature of the model is that contact tractions and micro-moments are linked in a consistent manner. In particular, for an elastic microstructured solid, the potential structure of the problem is preserved. The model has been implemented in the finite-element method, and illustrative examples are provided.

References

[1] S. Casolo, Macroscopic modelling of structured materials: Relationship between orthotropic Cosserat continuum and rigid elements, *Int. J. Solids Struct.*, 43:475–496, 2006.

[2] Th. Zisis, P.A. Gourgiotis, K.P. Baxevanakis and H.G. Georgiadis, Some basic contact problems in couple stress elasticity, *Int. J. Solids Struct.*, 51:2084–2095, 2014.

Equilibrium solutions in structural mechanics with a small Lipchitz continuous or with a monotone non linearity

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We address the equilibrium of structures which involve unilateral contacts which may be of two types. A first case involves unilateral springs, often simplified models of bumpers. The second case involves rigid contacts and their approximation with hardening barrier springs. The Newton-Raphson algorithm may be accomodated to these cases (using respectively Lipschitz continuity or monotony). Asymptotic expansions as well as numerical algorithms are provided.

Introduction

We study equilibrium solutions of some non linear systems of structural mechanics: we address the case of a soft contact such as a weak unilateral spring but also the case of rigid contacts (for example involved in backlash); the structure may involve rigid movements;

Unilateral springs are often simplified models of bumpers usually made of viscoelastic materials; the model has been addressed in the dynamic case in [1] and [2]: asymptotic expansion of periodic solutions have been obtained. This can be considered as an extension of the use of normal modes of a linear free system in order to study the dynamics of the forced associated linear system. In [1], the Lindstedt-Poincaré method was used in order to derive approximate non linear normal modes (a periodic solution close to a linear normal mode) for small non linearity.

In the smooth case the well known Newton-Raphson algorithm provides a sequence to approximate the solution of non linear equations or implicitely defined functions $p \mapsto u$ solution of $F(u, p) = 0$.

$$\frac{\partial F}{\partial u}|_{|(u_k,p)}(u_{k+1} - u_k) = -F(u_k, p)$$

Usually, we start from a reference solution u_0 of $F(u_0, p_0)$; and the algorithm converges for p close to p_0. A key hypothesis is that the norm of the inverse of the linear map $v \mapsto \frac{\partial F(u,p)}{\partial u} v$ should be not too big. We use some notions presented in Dontchev-Rockafellar [3]; more precisely we use ideas going back to Hildebrand-Graves, 1927.

We no longer assume that the function F used in the equilibrium equation $F(u, p) = 0$ is differentiable but that there exists a **strict estimator** H of F for u close to a reference solution u_0: $F(u_0, p_0)$; the function H is replacing the use of $\frac{\partial F}{\partial u}$. Strict estimator means that $E = F - H$ is Lipschitz continuous function with lipshitz constant μ; moreover H should be invertible (analogous to the invertibility of $\frac{\partial F}{\partial u}$); the inverse H^{-1} should be Lipschitz continuous with Lipshitz coefficient λ. Finally the following inequality should hold

$$\lambda\mu < 1$$

This assumption is analogous to the assumption on the norm of the inverse of the linear map $v \mapsto \frac{\partial F(u,p)}{\partial u} v$.

An example of a system of this case in in fig. 1 ; an example of the next case is in fig. 2.

We study the approximation of the second case involving rigid contacts with a structure which involves nonlinear springs which harden in compression; I call these springs, hardening barrier springs; the stress-strain law involves a function like $\frac{x}{1-x}$; this function is used in optimization and is an example of a barrier penalty function; in structural mechanics, it replaces a rigid contact with a hardening spring. Here, the study is restricted to the static case; the dynamic situation will be addrressed in a forthcoming paper. The idea is not of solving the regularised problem but to use them to obtain a solution of the original problem Here we intend to use ideas from non smooth optimization to prove existence of solutions and to derive a numerical algorithm to find them.

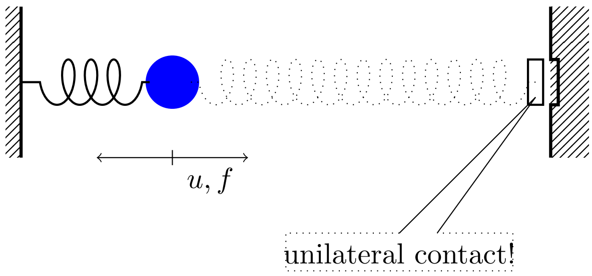

Figure 1: One mass with 2 springs with a weak unilateral one, load f

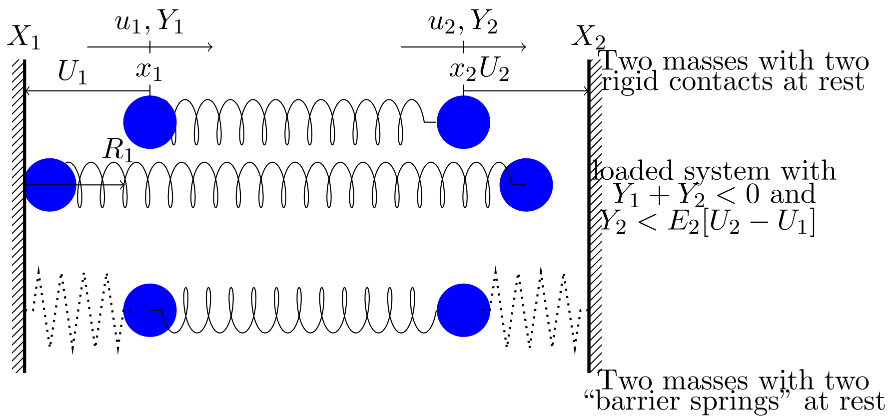

Figure 2: Two masses with 2 rigid contacts, loads Y_1, Y_2

[1] S. Junca and B. Rousselet. The method of strained coordinates for vibrations with weak unilateral springs. *The IMA Journal of Applied Mathematics*, 2010. http://hal-unice.archives-ouvertes.fr/hal-00395351/fr/.

[2] H. Hazim, N. Fergusson, and B. Rousselet. Numerical and experimental study for a beam system with local unilateral contact modeling satellite solar arrays. In *Proceedings of the 11th european spacecraft structures, materials and mechanical testing conference (ECSSMMT 11)*, 2009. http://hal-unice.archives-ouvertes.fr/hal-00418509/fr/.

[3] Asen L. Dontchev and R. Tyrrell Rockafellar. *Implicit functions and solution mappings: : a view from variational analysis.* Springer, 2009.

A class of contact problems with rate-dependent surface constraints

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Summary: The aim of this work is to study the variational formulations in nonlinear small strains (visco)elasticity of some rate-dependent interactions between the boundaries of two bodies. The corresponding evolution boundary value problems constitute a unified approach to study various interface models and their analysis is based on approximation results for evolution variational inequalities. Several examples are presented to illustrate the capability of the proposed model.

Introduction

This work is concerned with the study of some nonsmooth evolution contact problems which describe various complex interactions between the boundaries of two nonlinear (visco)elastic bodies, including relaxed unilateral contact conditions, adhesion and (non)local friction laws, in quasistatic or dynamic processes.

Quasistatic elastic problems with unilateral contact conditions and Coulomb friction law have been analyzed in [1, 2] and with an adhesion law in [3, 4], where an evolution of the intensity of adhesion was also considered. The normal compliance model has been investigated by several authors, see e.g. [5, 6] and references therein.

Dynamic viscoelastic problems with nonlocal friction laws, obtained by suitable regularizations of the normal component of the stress vector appearing in the Coulomb friction law, were analyzed in [7] and dynamic problems coupling unilateral contact, recoverable adhesion and nonlocal friction were studied in [8].

A quasistatic contact problem

In this section, the results described in [9], where a static contact problem with relaxed unilateral conditions and Coulomb friction was studied, are extended to an evolution variational inequality involving a differentiable functional. First, an implicit variational inequality is analyzed by an incremental method. Second, applications to two-field formulations of some nonlinear elastic quasistatic contact problems with friction are presented. These results have been partially published in [10].

A dynamic contact problem

In this section is presented the extension of the contact conditions considered in [11] to the case of a contact condition that contains not only the gap function but also the velocity. The corresponding variational problem has a three-field formulation and the applications include nonlinear constitutive laws of Kelvin-Voigt type.

These formulations enable a direct and simpler approach to study some complex surface interactions, including normal compliance, limited interpenetration, unilateral contact, adhesion, local and nonlocal friction laws.

References

[1] L.E. Andersson, Existence results for quasistatic contact problems with Coulomb friction, *Appl. Math. Optim.*, 42:169–202, 2000.

[2] R. Rocca and M. Cocou, Existence and approximation of a solution to quasistatic Signorini problem with local friction, *Int. J. Engrg. Sci.*, 39:1233–1255, 2001.

[3] M. Raous, L. Cangémi and M. Cocou, A consistent model coupling adhesion, friction, and unilateral contact, *Comput. Meth. Appl. Mech. Engrg.*, 177:383–399, 1999.

[4] M. Cocou and R. Rocca, Existence results for unilateral quasistatic contact problems with friction and adhesion, *Math. Modelling and Num. Analysis*, 34:981–1001, 2000.

[5] N. Kikuchi and J.T. Oden. *Contact Problems in Elasticity : A Study of Variational Inequalities and Finite Element Methods. SIAM Studies in Applied Mathematics*, SIAM, 1988.

[6] M. Shillor, M. Sofonea and J.J. Telega. *Models and Analysis of Quasistatic Contact, Lect. Notes Phys. 655*, Springer, 2004.

[7] M. Cocou and G. Scarella, Analysis of a dynamic unilateral contact problem for a cracked viscoelastic body, *Z. Angew. Math. Phys.*, 57:523–546, 2006.

[8] M. Cocou, M. Schryve and M. Raous, A dynamic unilateral contact problem with adhesion and friction in viscoelasticity, *Z. Angew. Math. Phys.*, 61:721–743, 2010.

[9] P.J. Rabier and O.V. Savin, Fixed points of multi-valued maps and static Coulomb friction problems, *J. Elasticity*, 58:155–176, 2000.

[10] M. Cocou, A variational inequality and applications to quasistatic problems with Coulomb friction, *Applicable Analysis*, published online, 2017.

[11] M. Cocou, A class of dynamic contact problems with Coulomb friction in viscoelasticity, *Nonlinear Analysis: RWA*, 22:508–519, 2015.

The discrete quasi-static incremental frictional contact problem: an alternative formulation

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Summary: A coefficient of friction free determinant consistent Jacobian matrix is used in the Newton-Raphson method for the numerical resolution of the discrete quasi-static incremental frictional contact problem. A mixed static-kinematic formulation smoothed by the use of a smoothing function is adopted which guarantees a local quadratic convergence rate.

Given a sequence of external applied forces or prescribed displacements, the quasi-static incremental problem (QI) consists of solving the system of equilibrium equations together with the unilateral contact conditions and the Coulomb friction law in which the tangent velocity is approximated by the backward-Euler method. The discrete quasi-static incremental frictional contact problem is formulated in a mixed manner; the advantage of dealing with such an enlarged system of equations is the coefficient of friction free Jacobian matrix determinant. In this static-kinematic mixed formulation there are 5 unknowns per node in plane problems or 7 unknowns per node in spatial problems: the increments of the relative tangent displacements, the normal gap, the increment of a friction multiplier, and the local Cartesian components of the contact force [1].

Both the unilateral contact and the 2D or 3D Coulomb friction (non-smooth) laws may be expressed in terms of the non-smooth projection function onto the positive part of the real line (the "plus function"), $x_+ = \frac{x+|x|}{2} = \mathrm{proj}_{\mathbb{R}_+}(x)$, which has a "kink" at the origin. This function may be approximated by the smooth function

$$S(x, \texttt{Error}) = \begin{cases} x + \frac{1}{\alpha(\texttt{Error})} \ln(1 + \mathrm{e}^{-\alpha(\texttt{Error})x}), & \text{if } x \geq 0, \\ x + \frac{1}{\alpha(\texttt{Error})} \left[-x\alpha(\texttt{Error}) + \ln(1 + \mathrm{e}^{\alpha(\texttt{Error})x}) \right], & \text{if } x \leq 0, \end{cases} \tag{1}$$

where $\alpha(\texttt{Error}) = \frac{\ln 2}{\texttt{Error}}$, and the sufficiently small quantity $\texttt{Error} = \max_{x \in \mathbb{R}} \{S(x,\alpha) - x_+\} = S(0,\alpha)$ controls the degree of approximation [2]. The non-smooth system of equations to be solved in the end of each load increment of problem QI is smoothed by the use of the family of smoothing functions (1) which guarantees a local quadratic convergence rate to the Newton-Raphson method. The resulting system of (non-symmetric) algebraic linear equations defining the iterates is well conditioned because, for not too small values of $\texttt{Error}$, function $S(x, \alpha(\texttt{Error}))$ is a good approximation to the plus function [2]. The algorithm is able to deal with large coefficients of friction; for cases in which problem QI exhibits multiple solutions the algorithm resolves that ambiguity by choosing the sticking solution.

Solutions to the QI problem for several finite element models of solids are presented. As an example, Figure 1(a) shows the finite element discretization of a $80 \times 40 \times 9.6$ mm^3 elastic block with a modulus of elasticity $E = 5$ MPa, a Poisson's ratio $\nu = 0.48$ and a mass per unit volume $\varrho = 1.2 \times 10^{-3}$ g/mm^2. The lower surface of the block is pressed against an horizontal rigid obstacle. The upper surface of this model is submitted to a uniform downward displacement until the total reaction from the obstacle reaches 55 N; then it is submitted to an horizontal motion towards the right until a steady sliding state is reached. The coefficient of friction is $\mu = 1.1$. Figure 1(b) illustrates the Cauchy stresses during the steady sliding phase (x and y are the horizontal and vertical coordinate axes, respectively). A three dimensional example will be addressed as well.

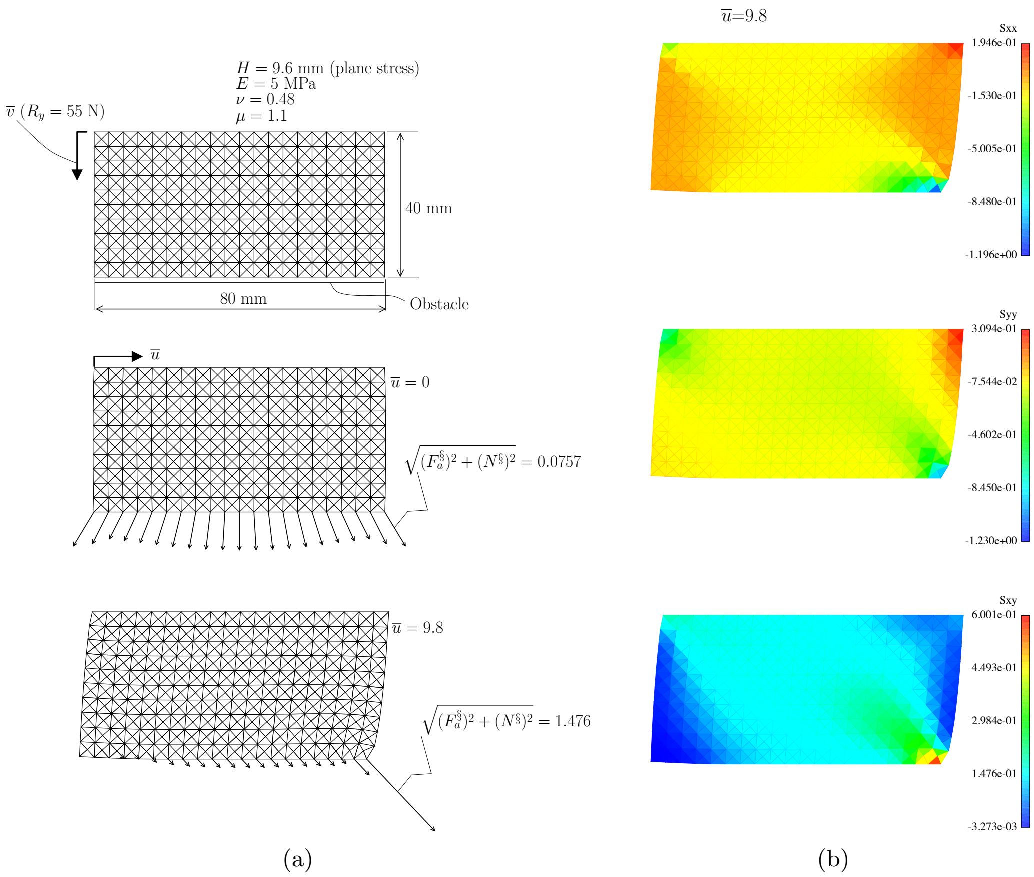

Figure 1: A tangentially driven elastic discretized block: (a) Finite element mesh, type of loading and actions on the obstacle, (b) σ_{xx}, σ_{yy} and σ_{xy} (MPa) components of the Cauchy stress tensor during the steady sliding regime.

References

[1] P. Areias, A. Pinto da Costa, T. Rabczuk, F.J.M. Queirós de Melo, D. Dias-da-Costa and Mourad Bezzeghoud. An alternative formulation for quasi-static and cohesive contact problems, *Computational Mechanics*, 53:807–824, 2014.

[2] C. Chen and O.L. Mangasarian. Smoothing methods for convex inequalities and linear complementarity problems, *Mathematical Programming*, 71(1):51–69, 1995.

[3] S. Barbarin. *Instabilité et frottement en élasticité: application à un problème d'ondes de contrainte*, PhD thesis, Laboratoire de Mécanique et d'Acoustique, Marseille, France, 1997.

A new lengthscale in wear problems with adhesion

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In this work we propose a new lengthscale for wear problems with adhesion. Under the assumption of short range adhesion, minimum size of adhesive contacts is derived. The effect of adhesion is that of increasing the contact area, so the wear coefficient, with respect to the non-adhesive case.

Introduction

Recently Carpick [1] commented in Science the big effort of the contact mechanics community to predict the contact behaviour of rough surfaces even in presence of adhesion. Despite the great effort the “contact challenge” showed that this area of tribology is still “challenging” (!) and many researchers are actively working on it. Nevertheless the present level of understanding does not allow predicting very important quantities in engineering design such us friction coefficient and wear. Those two phenomena, essential in the most of engineering applications, have not experienced any significative improvement in the last decades. Leonardo Da Vinci already 500 years ago estimated the friction coefficient to be roughly 0.25, quite a good approximation! Similarly, the “academic sport” of studying rough surfaces, did not get us any closer to being able to predict the coefficient of proportionality between wear loss and friction dissipation which was observed already by Reye in 1860 [2]. Later Archard [3] proposed the wear loss to be inverse proportional to the hardness of the material, but recent studies seem to suggest Reye’s hypothesis to be more general. In a recent paper Aghababaei, Warner and Molinari [4] have confirmed a criterion for formation of debris of a single particle, proposed in 1958 by Rabinowicz [5], which is based on consideration of competition of adhesion and plasticity. In the paper so far, and in the review of previous models, an important ingredient for "adhesive wear" is surprisingly missing. They do not consider that at the contact interface the average size of the micro contacts will strongly depend on the strength of adhesion, which is a contradiction to the idea that plasticity junction should be formed in the first place, that is before possibly breaking in a wear particle. We have introduced effectively another critical length scale in the problem, which is the minimum size of adhesive contacts based on JKR theory. The effect of adhesion is, as intuitively expected, that of increasing the contact areas, and hence the wear coefficient with respect to the non-adhesive case. We derive an upper bound to the radius of asperities, above which very high wear is predicted. As we estimate this upper bound to be quite high (hundreds of microns) but not incompatible with worn particle size measurements, we find reason for possible further discussion.

References

[1] Carpick, R. W., 2018, The contact sport of rough surfaces, Science, 359(6371), 38-38.

[2] Reye, Th., 1860, Zur Theorie der Zapfenreibung, J. der Civilingenieur. 4, pp. 235-255.

[3] Archard, J. F., 1953, Contact and Rubbing of Flat Surfaces, Journal of Applied Physics, 24, pp. 981-988.

[4] Aghababaei, R., Warner, D.H., Molinari, J.-F., 2016, Critical length scale controls adhesive wear mechanisms, Nature Communications, 7, 11816.

[5] Rabinowicz, E., 1958, The effect of size on the looseness of wear fragments, Wear, 2, pp. 4–8.

SESSION 6

Beam-to-solid contact interaction in stent graft modeling for endovascular repair

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In recent years, contact interactions of solid-to-solid as well as beam-to-beam type have been an active field of research. In this talk, a connection between the two in the form of a new beam-to-solid contact framework will be presented. Moreover, its practical application for stent graft placement during vascular surgery will be illustrated.

Arterial stent placement has become a very successful intervention in vascular surgery. One of the most common scenarios includes self-expandable stent grafts composed of a fabric graft and a metal stent mesh. Such stent grafts are used in endovascular aortic repair (EVAR) to support aneurysms, e.g. abdominal aortic aneurysms (AAA), that are at risk of rupture. Over the last years, a thrust of research in vascular mechanics and AAA-related topics has taken place. While significant progress has been made, the computational analysis of stent placement procedures using finite element methods (FEM) is still not predictive enough to give specific advice to vascular surgeons on how to optimally place the device during EVAR. Possible risks, which are still far from being fully understood, include a movement of stents away from the desired location (migration), leaking of blood around stent grafts (endoleakage) and damage of the arterial wall.

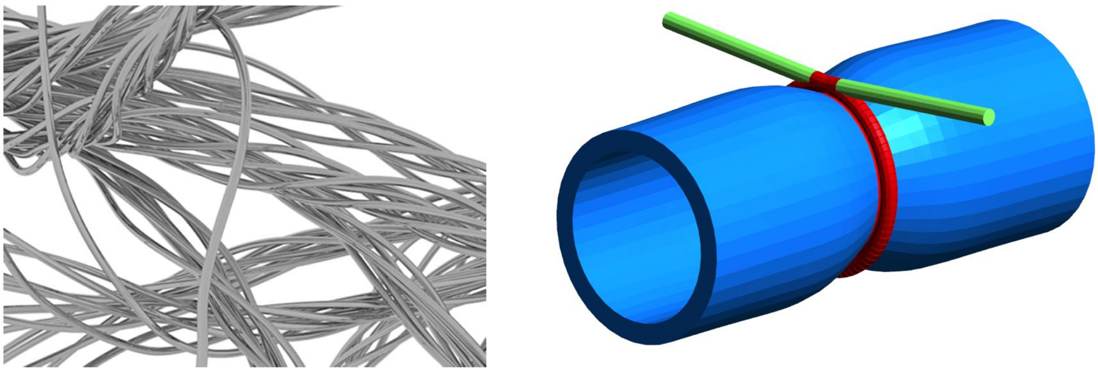

Figure 1: Beam-to-beam contact (left) and beam-to-solid contact (right).

From a mechanical point of view, the metal stent can conveniently be represented by 1D beam elements based on the geometrically exact Kirchoff theory of thin rods [1], thus significantly increasing the computational efficiency as compared with 3D solid-based approaches. The beam formulation is based on a C^1-continuous centerline interpolation using third-order Hermite polynomials, therefore allowing for coarse discretizations. Contact interaction between the beam elements is modeled using the so-called all-angle beam contact (ABC) formulation [2], which incorporates a variationally consistent model transition approach to connect formulations for point and line contact between the individual beams and, consequently, to allow for arbitrary contact angles. Going beyond purely beam-based discretizations, many engineering applications also require accurate models for the fully nonlinear contact interaction between beams and solid bodies. To this end, two new formulations for beam-to-solid mesh tying (i.e. tied contact) and beam-to-solid contact (i.e. unilateral contact) will be presented. Both formulations make extensive use of cutting-edge mortar finite element methods that have become the state-of-the-art in finite deformation contact mechanics of deformable solids [3]. Among the new topics addressed here are biorthogonal Lagrange multiplier bases for third-order Hermite polynomials as used for the beam centerline interpolation, efficient numerical integration strategies and the treatment of strong discontinuities when beams reach over sharp edges of the solid bodies. In Fig. 1 two typical beam-to-beam and beam-to-solid contact scenarios are shown.

The modeling of beam-to-solid mesh tying as well as beam-to-solid contact is also ubiquitous in the case of stent graft placement during EVAR. Beam-to-solid mesh tying is applied to couple the metal stent wire with the synthetic graft material in typical devices for AAA, see Fig. 2 (left). This is necessary because the employed Hermite polynomial interpolation for the beam centerline is not compatible with standard FEM shape functions, and a matching mesh discretization is therefore impossible to achieve. Stent grafts are usually implanted by crimping them into a very thin catheter. Next the catheter is used for a minimally invasive stent placement at the desired position in the AAA before the stent graft is eventually released. A mechanical model of the releasing process is shown in Fig. 2 (right) and it is clear that this process involves some serious challenges for the unilateral beam-to-solid contact formulation.

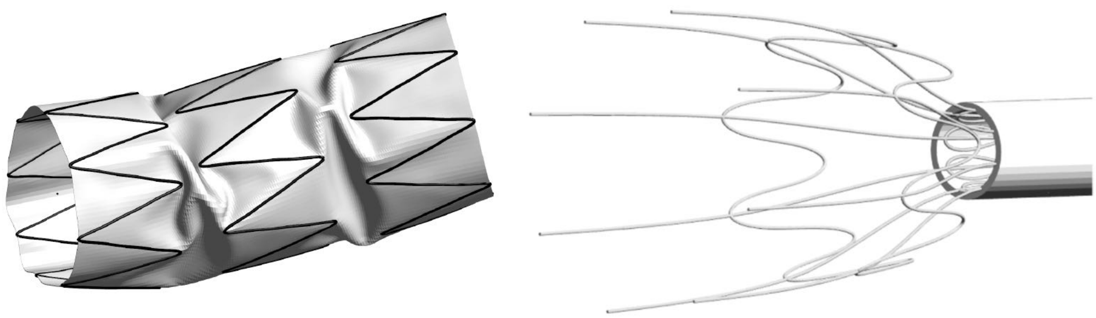

Figure 2: Beam-to-solid interaction: stent graft as an example for mesh tying (left) and bare-metal stent in catheter as an example for contact (right).

References

[1] C. Meier, A. Popp, W.A. Wall. *An objective 3D large deformation finite element formulation for geometrically exact curved Kirchhoff rods*, Comput. Method. Appl. M., 278, 445-478 (2014).

[2] C. Meier, W.A. Wall., A. Popp *A unified approach for beam-to-beam contact*, Comput. Method. Appl. M., 315:972-1010, (2017).

[3] A. Popp, A. Seitz, M.W Gee and W.A. Wall, *Improved robustness and consistency of 3D contact algorithms based on a dual mortar approach*, Comput. Method. Appl. M., 264, 67-80 (2013).

A new analysis for parallel beams contact

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Summary: The full existence and uniqueness criterion of the closest point procedure is analyzed in detail for the curve-to-curve situation in the case of the multiple solutions. Point-to-curve and curve-to-surface (as curve to solid beam) contact algorithms are compared. The reduced and ad-hoc criteria used in earlier publications are revised and discussed in numerical examples.

Introduction

The existence and uniqueness criterion of the Closest Point Procedure (CPP) is the first and the most important step in order to formulate the corresponding contact algorithm for any geometrical contact pair such as surface-to-surface (STS), curve-to-curve (CTS), curve-to-surface (CTS) etc. A beam-to-beam contact algorithm is normally arising from the curve-to-curve closest point procedure, and it is well known that the solution is multiple for the parallel straight beams. Here we are analyzing in detail the full criterion for curve-to-curve CPP and studying especially the case of the multiple solutions. This case geometrically leads to the parallel (offset) curves or mechanically to the parallel beams contact. This includes a wide range of situations: wire ropes, knots etc. The careful analysis recovers that the contact algorithm which is capable to solve this problem includes the point-to-curve, surface-to-surface and the curve-to-surface algorithms.

Closest point projection procedure for curve-to-curve contact

In order to analyze the full criteria for the existence and uniqueness, the CPP procedure

$$\mathbf{F}(s_1, s_2) = \frac{1}{2}\|\boldsymbol{\rho}_1(s_1) - \boldsymbol{\rho}_2(s_2)\|^2 \equiv \frac{1}{2}\|\boldsymbol{\rho}_1(\xi^1) - \boldsymbol{\rho}_2(\xi^2)\|^2 \longrightarrow \min . \qquad (1)$$

is formulated dually in the coordinate system attached to both curves, see Fig. 1:

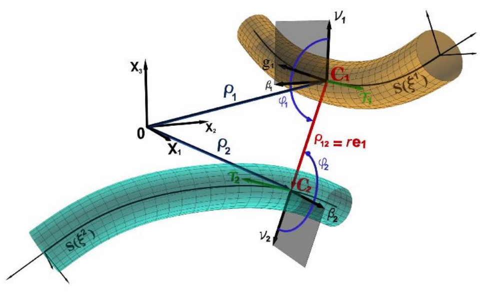

Figure 1: Definition of local coordinate systems $(\boldsymbol{\tau}_i, \boldsymbol{\nu}_i, \boldsymbol{\beta}_i)$ and $(\boldsymbol{\tau}_i, \mathbf{e}_i, \mathbf{g}_i)$

Here $\boldsymbol{\tau}_i$, $\boldsymbol{\nu}_i$, $\boldsymbol{\beta}_i$ are Frenet vectors, $\mathbf{e}_i$ is a unit vector of the closest distance vector, $\mathbf{g}_i = \boldsymbol{\tau}_i \times \mathbf{e}_i$. If the second derivative for $\mathbf{F}$ in eqn. (1) is positively determined, then solution exists and is unique. A Sylvester criterion showing the positivity of the matrix $\mathbf{F}^{''}$ recovers that the determinant should be positive

$$\det \mathbf{F}^{''} = (1 - k_1 r \cos\varphi_1)(1 - k_2 r \cos\varphi_2) - \cos^2\psi > 0 \qquad (2)$$

with $\cos\psi = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$. From the common view, it seams that if the angle ψ between curves is far away from zero (crossing tangents) then the closest projection procedure should be unique, which is the case for the straight beams. A significant example is a parallel (or offset) curve generated as

$$\boldsymbol{\rho}_2(s,r,\varphi_1) = \boldsymbol{\rho}_1(s) + R\mathbf{e}_1(\varphi_1) = \boldsymbol{\rho}_1(s) + R(\boldsymbol{\nu}_1 \cos\varphi_1 + \boldsymbol{\beta}_1 \sin\varphi_1) \tag{3}$$

The tangent vector of the offset curve is derived by using the Serret-Frenet formulas

$$\boldsymbol{\tau}_2 = \frac{\partial\boldsymbol{\rho}_2(s,r,\varphi_1)}{\partial s} = \frac{\partial\boldsymbol{\rho}_1(s)}{\partial s} + R\frac{\mathbf{e}_1(\varphi_1)}{\partial s} = \boldsymbol{\tau}_1(1 - k_1 R\cos\varphi_1) + R\kappa\mathbf{g}_1. \tag{4}$$

One can see, that the distance function $\mathbf{F}$ in eqn. (1) is constant for each point s along the curves and the matrix $\mathbf{F}''$ is singular, because the determinant in eqn. (2) is zero $\det\mathbf{F}'' = 0$. However, an offset curve can be generated in a wide range of angles ψ between the tangent lines. For example an offset by normal $\boldsymbol{\nu}_1$ ($\varphi_1 = 0$) recovers $\cos\psi = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 = 1 - k_1 R$ and an offset by bi-normal $\boldsymbol{\beta}_1$ ($\varphi_1 = \frac{\pi}{2}$) recovers for angle between tangent vectors $\cos\psi = 1$. Therefore, by setting an angle φ_1 the full range of angles between the tangents $-\pi \leq \psi \leq \pi$ can be generated. A set of offset curves satisfying eqn. (3) gives a tube surface $\boldsymbol{\rho}_2(s,\varphi_1)$ with a radius R for a generatrix curve $\boldsymbol{\rho}_1$, similar to the tube shown in Fig. 1. Any curve drawn on the tube surface will have a constant distance R to the generatrix curve $\boldsymbol{\rho}_1$.

Contact algorithm for parallel beams – preferences

Several algorithms, in general, are applicable for beams: these are Curve-To-Curve (CTC), Point-To-Curve (PTC) and Surface-To-Surface (STS) algorithms. However, the fastest CTC algorithm is not capable to work in the case of parallel curves. A PTC algorithm allows to avoid this problem, however, is not giving the answer about the length of contact zone. Thus, the question about the "switch criteria" between CTC and PTC algorithms is still remaining. In order to resolve this situation the Curve-To-Solid Beam (CTSB) contact algorithm together with a solid beam finite element can be proposed. The CTSB algorithm is geometrically developed as a sub-set of the Surface-To-Surface (STS) algorithm in which the slave surface is taken as a cylinder with a mid-line (non-deformable cross-section) while keeping the master surface fully inherited from the solid beam finite element. This allows to employ all necessary weak forms and their linearization from the already developed STS algorithm. The CTSB contact requires a special algorithm to compute an initial point for the CPP procedure in order to keep the uniqueness of the solution for the cyclic variable. The final computation of the weak forms as well as the tangent matrices is given via higher order integration with sub-domains. The last allows to analyze carefully the length of a contact zone for the "problem of parallel tangents". This problem in the case of contacting cylinders can be validated from the classical Hertz contact theory, namely: the contact length l is inversely proportional to the sine of the angle between the mid-lines $l \sim \frac{1}{|\sin\psi|}$. This approach has a big potential for the mechanical cases including wire ropes, coils, knots etc. as a combination of a finite element allowing an elliptical deformation of the cross section from the continuum mechanics point of view and possessing a-priori the existence and uniqueness of the solution even for the generally parallel curves case from the contact kinematics point of view.

References

[1] A. Konyukhov, O. Mrenes and K. Schweizerhof, Consistent development of a beam-to-beam contact algorithm via the curve-to-solid beam contact, *Int. J. Num. Meth. Engng.*, 113:108–1144, 2018.

Toward a two-scale modeling of spiral strand wire rope for floating offshore wind turbine to predict fatigue damages: possibilities and difficulties in using a legacy code

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Summary: Mooring systems of offshore wind turbines have to be designed to resist cyclic tension and bending. In order to take into account bending for the computation of the fatigue damage, local phenomena of fretting have to be properly predicted by means of a new numerical model.

Introduction

In the wind energy context, a recent development area aims to extend the wind site possibilities by means of floating offshore wind turbines. These structures are basically constituted by a wind turbine installed on a floater, which is linked to the sea bed by means of mooring lines. The mooring system ensures the structure reliability by limiting the floater movement.

We consider a case study of a semi-submersible floater equipped with a redundant mooring system of six catenary mooring lines. These are constituted by spiral strands (Figure 1), whose design must consider extreme and fatigue loading, leading to combined tension and bending stresses induced by the floater movements for various wind and waves conditions.

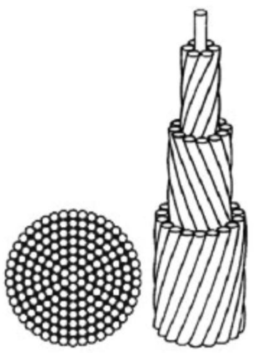

Figure 1: Spiral strand wire rope.

Wire rope behavior

During bending, relative movements among the wires take place, with consequent friction due to tension. Results from free-bending fatigue tests show the importance of these effects, since the first rupture is localized in correspondence with the bending neutral axis, where relative movements are larger [1].

Proposed approach

A global analysis alone is not sufficient to characterize and quantify the local fretting phenomenon. The proposed approach involves two scales: a macroscopic scale representing the global wind turbine system under simplified environmental loads and a local scale which models the response of the cables at the wires level.

From the global model, implemented in an industrial software, Deeplines^{TM}, tension and curvature distributions along the mooring lines are obtained.

The local model of the wire rope has been implemented in ABAQUS®v6.14. It models single wires with beam elements, in order to reduce the computational cost. Since a wire rope is made by a great number of wires (121 in our case), it seems not affordable to model all of them in a single analysis. Alternatively, we propose to decompose the local model into subparts, modeling just some wire layers at each time.

Contact modeling

Edge-to-edge, node-to-surface and surface-to-surface contact algorithms have been tested and the results have been compared with the analytical predictions proposed in [2] and [3]. In order to make use of the latter two, the idea has been to track the actual external surfaces of wires by linking shell elements to the beam nodes thanks to the use of Abaqus Multi-Point Constraints in order to follow the beam deformation. Moreover comparisons between small displacements and large sliding hypothesis have also been performed.

Local model results

The aim of the local model is to link the global quantities, namely tension and rope curvature, to local quantities that govern the fatigue damage at the wire level. In particular, we are interested in computing accurately the axial strain in the wires, the normal and tangential contact stresses and the relative movements among the wires. The performances of each local model is shown through parametric studies which enlighten the choice of the input parameters linked to contact modeling and mesh discretization which allows to obtain reliable results.

Since the problem is highly nonlinear due to the presence of friction and possibly large slidings between wires, loading/unloading behavior is studied.

The choice of the boundary conditions to prescribe on the local model is also carefully investigated in order to avoid any edge effect when coupling the two scale models.

The final objective of this work, once the fatigue law will be chosen, is to compute the cable fatigue damage for each environmental state. By considering the probability of occurence of each of these states, the fatigue damage would then have to be estimated during the structural lifespan.

References

[1] M. Raoof. *Free bending tests on large spiral strands*, Proceedings of the Institution of Civil Engineers, 87(4):605-626, 1989.

[2] K.O. Papailiou, Bending of helically twisted cables under variable bending stiffness due to internal friction, tensile force and cable curvature, *Doctor of Technical Sciences thesis, ETH, Zurich, Swiss*, 1995.

[3] K.J. Hong, A. Der Kiureghian and J.L. Sackman, Bending behavior of helically wrapped cables, *Journal of engineering mechanics*, 131(5):500-511, 2005.

Stress gradient effect in estimating fretting fatigue life

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Summary: Fretting fatigue life of a structural component under cylindrical contacts is here evaluated. The critical plane-based multiaxial fatigue criterion by Carpinteri et al. is employed, and a novel procedure is proposed to determine the point where to perform the fatigue assessment.

Abstract

In the present paper, the fretting fatigue life assessment for a metallic structural component under cylindrical contact is performed by employing the critical plane-based multiaxial fatigue criterion proposed by Carpinteri and co-workers [1,2]. Firstly, the stress state related to such a fretting fatigue configuration is analytically evaluated in a closed form [3]. Then, the critical point, i.e. is the point where to perform the above assessment, is determined by alternatively moving along three different paths with the same length *L/2* (which is a function of the material properties): a normal path, an inclined path, and a pseudo-isostatic path. The structural components examined are 2024-T351 aluminium alloy dog bone specimens under cylindrical contact [4]. Since it is observed that by using the above path length *L/2* the fretting fatigue life tends to be overestimated, a novel path length is herein proposed as a function of both the material properties and the stress gradient in the contact zone. The results obtained are quite satisfactory, and the new path length seems to be promising not only for fretting fatigue but also for other fatigue configurations with high stress gradients.

References

[1] A. Carpinteri, A. Spagnoli, S. Vantadori. Multiaxial fatigue assessment using a simplified critical plane-based criterion. *Int J Fatigue*, 33:969-976, 2011.

[2] C. Ronchei, A. Carpinteri, G. Fortese, D. Scorza, S. Vantadori. Fretting high-cycle fatigue assessment through a multiaxial critical plane-based criterion in conjunction with the Taylor's point method. *Solid State Phenom*, 258:217-220, 2016.

[3] K.L. Johnson. *Contact mechanics*. Cambridge: Cambridge University Press, 1985.

[4] M.P. Szolwinski, T.N. Farris. Observation, analysis and prediction of fretting fatigue in 2024-T351 aluminum alloy. *Wear*, 221:24-36, 1998.

Dynamical interaction of an elastic rod and rigid foundation through a viscous-elastic-plastic layer

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Summary: The problem of propagation of longitudinal shock wave in an elastic rod interacting with the rigid medium is investigated using the model of viscous-elastic-plastic resistance on the lateral surface. An exact solution of the initial-boundary problem is obtained using the Laplace transform. The asymptotic for small departure from the dry friction is given.

Introduction

Analysis of energy dissipation due to the frictional interaction of deformable contacting bodies is of great importance in the research of applied problems of the structural dynamics. In this proceeding the nonstationary dynamics of an elastic rod with viscous-elastic-plastic external resistance has been studied. This problem can be used for modelling the dynamics of drilling equipment to eliminate the sticking of the tool, the process of the pulling of rods in building or the dynamical damage of fibers in the composite with inelastic matrix. The obtained results generalize the cases of purely dry friction [1, 2], purely viscous friction [3], contact through a viscous-plastic Bingham layer [4–6] and through a rigid-plastic layer with linear hardening (softening) [7, 8].

Formulation of problem

We consider the propagation of longitudinal shock wave in a semi-infinite elastic rod with constant cross section induced by sudden loading of the end. The classical theory of rods dynamics has been used. The friction forces on the lateral surface of the contact with rigid foundation are modeled by Voigt parallel connection of Saint-Venant's, Newton's and Hook's rheological elements. We study the process of attenuation of shock waves due to action of this viscous-elastic-plastic resistance.

The initial-boundary-value problem is the following:

$$\frac{u''}{L}+\frac{\tau_x}{E}=\frac{\ddot{u}}{L}, \qquad x>0, \quad t>0; \tag{1}$$

$$\tau_x=-(\tau_c \operatorname{sgn}\dot{u}+b\dot{u}+ku), \quad \dot{u}\neq 0 \quad \text{or} \quad \dot{u}=0, \quad |\tau_x+ku|<\tau_c; \tag{2}$$

$$u(x,0)=\dot{u}(x,0)=0, \quad x>0; \tag{3}$$

$$\frac{u'(0,t)}{L}=-\frac{\sigma_0}{E}H(t), \quad u(\infty,t)=0, \quad t>0. \tag{4}$$

Here u is the axial displacement, τ_x is the shear stress, $x=X/L$, $t=cT/L$ are dimensionless coordinate and time, $L=F/\Pi$ is the characteristic size, F is the area and Π is the perimeter of cross-section, $c=\sqrt{E/\rho}$ is the wave velocity, E is Young's modulus and ρ is density of the rod material, $b=\beta_l c/(h_l L)$, $k=E_l/h_l$ are moduli of viscosity and hardening of the bed, τ_c, β_l, E_l – are the threshold friction, dynamical viscosity and secant module of material of layer accordingly, h_l is its thickness, σ_0 is the stress at the rod end and $H(t)$ is a Heaviside function. The primes and the dots denote the partial derivate with respect to the dimensionless coordinate and to the dimensionless time respectively.

Results

Divining the sign of the velocity, we linearize the nonlinear relations (2) and represent them in motion domain in the form

$$\tau_x = -\left(\tau_c H(t-x) + b\dot{u} + ku\right) . \tag{5}$$

The equation (1) taking into account the expression (5) converts to telegraph equation with inhomogeneous right side:

$$\frac{u''}{L} - \frac{\ddot{u}}{L} - B\frac{\dot{u}}{L} - K\frac{u}{L} = \frac{\tau_c}{E} H(t-x), \quad x > 0,\ t > 0 , \tag{6}$$

where $B = bL/E = \beta_l/(h_l\sqrt{E\rho})$, $K = kL/E = E_l L/(Eh_l)$ are the dimensionless parameters of viscosity and hardening.

The analytical solution of problem (6), (3) and (4) is constructed using the Laplace transform over the time coordinate and represend by Bessel's functions. A wave pattern of perturbation including the prefront zone of rest, the area of motion and the domain of stationary residual stresses is built.

The asymptotic expansion of results for small departure from the dry friction is obtained as a first approximation:

$$\frac{u(x,t)}{L} = \frac{\sigma_0}{E}\left\{ t - x - \frac{\tau_c}{\sigma_0}\frac{t^2-x^2}{4} - \right.$$

$$\left. -\left[B\frac{t^2-x^2}{4}\left(1-\frac{\tau_c}{\sigma_0}\frac{t}{4}\right) + K\frac{(t-x)^2}{4}\left(\frac{t+2x}{3} - \frac{\tau_c}{\sigma_0}\frac{(t+x)^2}{16}\right)\right] + O\left(\max\{B^2, BK, K^2\}\right)\right\} H(t-x) .$$

This expression is correct in domain of motion which bounded by characteristic and stop line:

$$x \le t \le \frac{2\sigma_0}{\tau_c}\left[1 - \left(B\frac{L_*^2 + x^2}{8L_*} + K\frac{L_*^2 - x^2}{8}\right) + O\left(\max\{B^2, BK, K^2\}\right)\right],$$

where $L_* = 2\sigma_0/\tau_c$.

References

[1] L. V. Nikitin. *Statics and dynamics of solids with external dry friction*, Mosk. Litsei, 1998 (in Russian).

[2] N. I. Aleksandrova. Numerical-analytical investigation into impact pipe driving in soil with dry friction. Part 1: Nondeformable external medium, *J. Mining Sci*., 48(5): 104–119, 2012.

[3] N. A. Veklich and B. M. Malyshev. Longitudinal waves propagation in an elastic rod placed in viscous liquid, *Mech. of Solids*., 5:174–178, 1967 (in Russian).

[4] I. P. Shatskii and V. V. Perepichka. Shock wave propagation in an elastic rod with a viscoplastic external resistance, *J. Appl. Mech. and Techn. Phys*., 54(6):1016–1020, 2013.

[5] I. Shatskyi and V. Perepichka. Dynamic problem for an elastic rod with decreasing function of viscoplastic friction. *Proc. 22nd Int. Conf. "MECHANIKA 2017" (19 May 2017, Kaunas, Lithuania),* 348–350.

[6] I. Shatskyi and V. Perepichka. Shock wave in a pile immersed into viscoplastic medium, *Trans. V?B – TU Ostrava, Civ. Eng. Ser*., 17(2):115–118, 2017.

[7] V. V. Perepichka and I. P. Shatskyi. Nonstationary interaction of an elastic rod with nondeformable foundation through a rigid-plastic layer with hardening, *Bull. of T. Shevchenko Nat. Univ. of Kyiv, Ser.: Phys. & Math*., 3:117–180, 2017 (in Ukrainian).

[8] I. Shatskyi and V. Perepichka. Dynamic problem for an elastic rod with decreasing function of elastoplastic external resistance, *14th Conf. "Dynamical Systems – Theory and Applications" (Dec. 11–14, 2017. Lodz, Poland)*, 68.

SESSION 7

Sliding contact using adaptive node insertion with the Virtual Element Method

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A node based contact procedure with adaptive inserted contact nodes is developed by making use of the Virtual Element Method and its advantages. Large deformations and sliding are treated by a node adjustment algorithm.

Introduction

The Virtual Element Method (VEM) is a generalization of the FEM for arbitrary shaped Elements (see [1]). By choosing only one approximation function per degree of freedom on each element the integration is carried out on the boundary and independent of the number of nodes. To correct the approximation error made, a stabilizing part is added. By using a special energy based stabilization [2] the Virtual Elements can be applied to large deformational problems.

The method then can be used for arbitrary elements as in the direct use of Voronoi meshes for computation. Additional it is also possible to change the element topology during a computation by adding or removing nodes. This makes it possible to adjust a contact surface discretization during the contact detection.

Contact Algorithm

The contact procedure using the VEM [3] offers a flexible node-to-node formulation. It is based on projecting and freely adding contact nodes to the original mesh to construct matching contact surfaces with nodal contact pairs. This way differences in mesh size of contacting surfaces are overcome (see Figure 1).

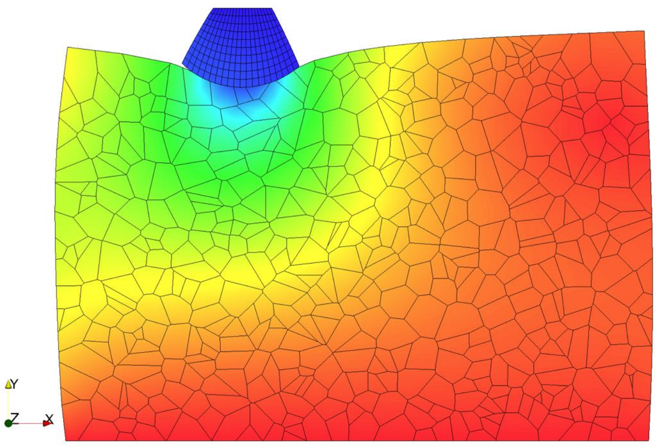

Figure 1: Ironing problem using an arbitrary mesh with complex element shapes.

Sliding Contact

In most classical contact computations nodes are projected on parametrized surfaces where then contact constraints are enforced in tangential and normal direction to differentiate between stick and slip state. As an alternative, contact can be computed without respect to the contact normal by simple coupling of the nodes. The sliding case is then considered afterwards by letting the projected node follow a friction cone defined by normal and tangential tractions. This was applied to node-to-segment contact discretizations in [4]. However these interpolations are dependent on the mesh size and relation.

In combination with the moving cone description, the VEM contact now offers a simple formulation for surfaces in sliding contact. Contrary to classical node-to-node contact, sliding movement is possible by adjusting the position of the contact nodes in the mesh according to the friction state. Using the node-to-node contact a stable method was formulated and the correct transmittance of contact forces is fulfilled.

References

[1] L. Beirão da Veiga, F. Brezzi, A. Cangiani, G. Manzini, L. D. Marini, and A. Russo. Basic principles of virtual element methods. *Mathematical Models and Methods in Applied Sciences*, 23(01):199214, 2013.

[2] P. Wriggers, B.D. Reddy ,W.T. Rust, B. Hudobivnik. Efficient virtual element formulations for compressible and incompressible finite deformations. *Computational Mechanics*, 60:253-268, 2017.

[3] P. Wriggers, W.T. Rust, and B.D. Reddy. A virtual element method for contact. *Computational Mechanics*, 58(6):10391050, 2016

[4] P. Wriggers and A. Haraldsson. A simple formulation for two-dimensional contact problems using a moving friction cone. *Communications in Numerical Methods in Engineering*, 19(04):285-295, 2003.

Computational modelling of ship collision

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Summary: In this contribution, numerical simulation of ship collision will be presented, where elasto-plastic behaviour of material along with mortar based contact formulation will be considered. Furthermore, a gradient-enhanced damage model for degradation of material will be considered.

With growth in traffic of ships, the risk of collision is increasing. Efforts are being made to improve the crashworthiness of ships and one promising design approach is to use granular materials in the cavity of double hull ships [1], where granulates are filled between the hull of a ship. This strategy provides a medium between the hull which can absorb impact energy and transfer the load to the inner hull. Consequently, impact energy is shared between two hulls, in contrast to localized impact on outer hull only.

Numerical modelling of such a problem can be very challenging as it requires implementation of a robust contact model for the interaction of the ship with the colliding body and a finite strain based elasto-plastic material model for describing the large deformation of the structure. Furthermore, a particle-based method is also required to model granular materials. In this study, a mortar based method, as implemented in [2], is used for the contact formulation which involves a segment-to-segment strategy with weak enforcement of contact constraints. Such a weak coupling leads to a rather robust formulation in case of large deformation and sliding. Regarding the material behaviour, a finite strain based elasto-plastic model is considered, where a multiplicative split of the deformation gradient is applied. Furthermore, material degradation of ship structure during collision process cannot be neglected. In order to account for this phenomenon, a gradient-enhanced damage model will be presented, by which pathological dependence of finite element mesh during degradation will be avoided.

For numerical modelling of granular materials, the discrete element method (DEM) is used where the analysis of particles is carried out at the micro-mechanical level. In this study, expanded glass granular materials are considered, where the bulk and particle level properties of such materials were studied in detail in [3, 4]. The DEM-FEM coupling, as discussed in [5], is employed to study the load transfer between granular particles and the confining structure. Finally, a homogenization technique for granular materials will be presented, where macroscopic quantities are derived from volume averaged properties of particles. Such a technique is helpful in the continuum based description of mechanical properties of granular materials.

References

[1] M. Schöttelndreyer, I. Tautz, W. Fricke, E. Lehmann. Side Structure filled with multicellular glass hollow spheres in a quasi-static collision test. Collision and Grounding of Ships and Offshore Structures (Ed: J. Amdahl, S. Ehlers, B. Leira), 101-108. Taylor & Francis Group, London, 2013.

[2] C. Weissenfels. Contact methods integrating plasticity models with application to soil mechanics. Ph.D. Thesis, Leibniz University Hannover, Hannover,Germany (2013)

[3] C. Woitzik, A. Düster. Modelling the material parameter distribution of expanded granules. J. of Gran. Matter, 2017. doi: 10.1007/s40571-017-0169-0

[4] M. A. Chaudry, et al., Experimental and numerical characterization of expanded glass granules. J. of Comp. Part. Mech, 2017. doi: 10.1007/s40571-017-0169-0.

[5] C. Wellmann, P. Wriggers. A 3D Coupled Finite-Discrete Element Method for Modeling the Interaction of Granular Materials and Solid Structures. *Particle-Based Methods, Fundamentals and Applications* pp. 72-75, 2009.

On the simulation of treaded tires in rolling contact: A coupled Lagrangian - ALE approach

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Arbitrary Lagrangian Eulerian (ALE) framework for relative kinematic description of rolling bodies is well established because of its numerical efficiency. However, this framework does not enable for circumferential discontinuities, such as real tread pattern of tires. This research is aimed on the development of a coupled Lagrangian-ALE solution strategy, which enables the solution of the tread pattern contact with the road in a transient dynamic manner, whereas the underlying tire structure remains in stationary ALE-description. The suitability of this approach will be demonstrated on academic examples first.

Introduction

The Arbitrary Lagrangian Eulerian (ALE) description has been proven as a efficient relative kinematics description for the simulation of rolling bodies [1]. For detailed simulation of tires this approach has been extended for the efficient treatment of frictional contact and inelastic material behavior [2]. Later on it has been extended for thermo-mechanical simulations [3]. Nonetheless, because material particles are moving on their trajectories in the spatially fixed mesh, this approach is limited to axi-symmetric models. In order to model the contact of tires in detail, the description of the tread pattern geometry and its transient dynamic contact with the road is needed. To avoid a computational costly full transient dynamics simulation, a coupling strategy is suggested. The tire body still remains in the efficient stationary rolling ALE-formulation, which the tread pattern is described in transient dynamics Lagrangian formulation. This presentation describes a coupling strategy which will be demonstrated on first 3D academic examples.

Theoretical Developments

Both, transient dynamics contact and stationary modeling of rolling bodies are highly developed, this presentation is devoted on the coupling strategy. The idea behind is to place a coupling interface immediate between the tread strip and the carcass structure. As the carcass is assumed to move stationary and the contact of the tread strip is transient, this approach is based on the assumption that high frequent vibration induced from the transient impact of tread block is rapidly damped out by the viscous rubber material. This assumption is underlined by simulations with visco-elastic tread blocks impacting the road. The general idea is sketched in figure 1 left. A second issue is on the coupling interface between the ALE-structure and the Lagrangian parts. Here the idea introduced first in [2] is picked up, where for the ALE-structure the movement of the material particles is described by additional degrees of freedom in the coupling interface, which enables for a robust, reliable and efficient coupling strategy. A third topic is on related temporal synchronized schemes for the coupling strategy. For that a first suggestion is made based on Courant-criterion for advective motion, but further investigations are needed to figure out the optimal parameter settings with regard to accuracy and numerical efficiency.

Results

The coupling scheme has been implemented into a matlab based finite element code. An early result is depicted in figure 1 (right), where the displacement field under rolling conditions is shown. The inner (ALE) core is discretized with quadratic shape functions. The (Lagrangian) treadblocks are discretized with linear shape functions. A smooth transition of the displacement field between the coupled parts could be observed.

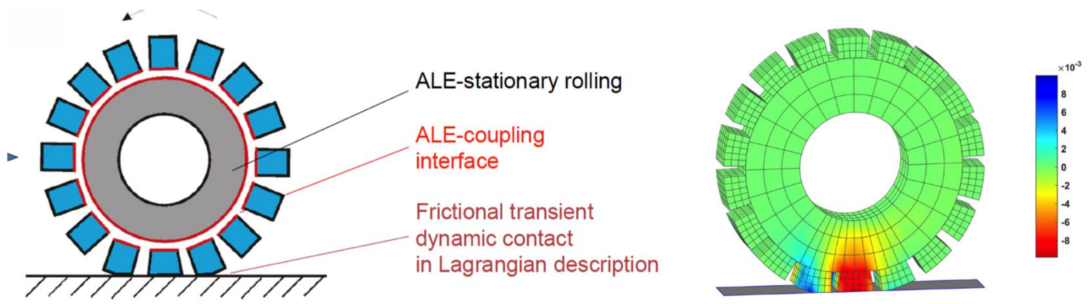

Figure 1: left) principle sketch of the Lagrangian - ALE coupling scheme for rolling contact of treaded tires; right) on a simplified test structure computed displacement field.

References

[1] U. Nackenhorst. The ALE-formulation of rolling contact, *Comp. Meth. Appl. Math. Eng.*, 193:4299–4322 2004.

[2] M. Ziefle and U. Nackenhorst, Numerical Techniques for Rolling Rubber Wheels - Treatment of Inelastic Material Properties and Frictional Contact, *Comp. Mech.*, 42:337–356, 2008.

[3] A. Suwannachit and U. Nackenhorst, A Novel Approach for Thermomechanical Analysis of Stationary Rolling Tires within an ALE-Kinematic Framework, *Tire Science and Technology*, 41:174–195, 2013.

Solving Lennard-Jones-based adhesive problems with a contact model-adaptivity methodology

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Summary: The Lennard-Jones interaction model is used for Adhesive Contact Problems (ACP). A sequence of partitions of contact models is adaptively constructed to both extend and approximate the LJ-problem. Weak formulations of associated ACP are developed within the Arlequin framework and solved numerically, in a macro-prediction and a micro-correction steps, to track critical localized adhesive forces.

Introduction

Classical macroscopic contact problems neglect adhesive phenomena. However, in applications as the ones involving thermal and electrical resistance that are present in many industrial applications like those involving design of MEMS and CNTs nanodevices, consideration of micro-attractive contact forces is essential. Analytical models like the JKR, DMT and MD models, but also semi-numerical approaches like those in the works by Muller and Greenwood, have been developed to take into account adhesive forces. Though currently used, these models lack generality. They are basically limited to particular geometries. Fully-numerical continuum-based approach for the solution of adhesive contact problems have been more recently developed [1]. For more references on this topic, we refer to a recent review [2]. Our research work is in this veine.

The goal of this work is to provide an extended and model-adaptive computational multiscale method which contributes to the enhancement of the reliability and effectiveness of the solution of adhesive contact problems, based on the surface Lennard-Jones model, used in the continuum mechanics framework. Incidentally, by some of its aspects, our approach is also appropriate for an efficient solution of classical macroscopic contact problems based on stiff normal contact relations of the form $R_n = g(d_n)$, where R_n and d_n refer to the contact force density and to the signed distance, respectively. This law could have been identified experimentally. One could mention compliance laws, barrier laws and so on.

Basic ideas

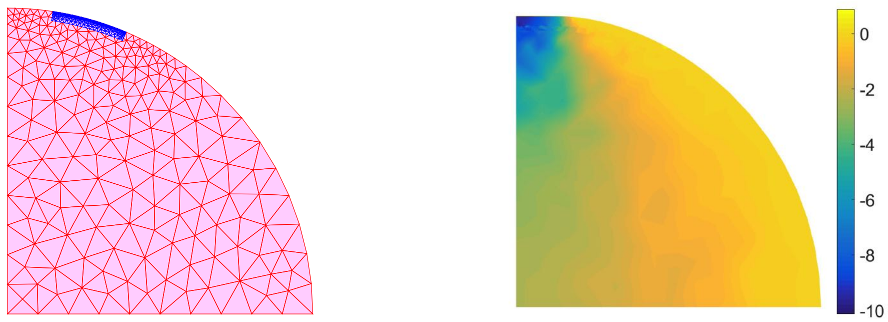
The basic ideas on which relies our methodology are the following:

- To address numerical shortcomings of a contact problem based on the surface Lennard-Jones adhesive contact potential and by taking cue from previous ideas, a sequence, checked to be convergent, of partitions of contact models constructed to both extend and approximate the LJ model. This sequence is formed by a combination of the LJ model with a sequence of shifted-Signorini (or, alternatively, -Linearized-LJ) models, indexed by a shift parameter field (and not a scalar parameter). Our idea takes cue from previous works [3, 4] for the creation of a partition of models using Shifted-Signorini contact models and [1, 5] for the Shifted Linearized-LJ model.

- For each adhesive contact model of this sequence, a weak formulation of the associated local Adhesive Contact Problem (ACP) is developed. To track critical localized adhesive areas, a two-step strategy is developed: firstly, a macroscopic contact problem (with no adhesive forces) is solved once to detect contact separation zones. Secondly, at each shift-adaptive iteration, a micro-macro ACP is re-formulated and solved within the multiscale Arlequin framework, with significant reduction of computational costs

Some numerical results

The contact between a sphere and two parallel infinite rigid surfaces is studied here. The problem is assumed to be under axisymmetric conditions. Moreover, due to symmetries, only a quarter of the a median section is considered. One of our numerical results obtained for this classical test is shown in Figure 1, corresponding to prescribed indentation by the two planes. Other results will be shown during the conference. These results are in a good agreement with other theoretical and numerical available results.



(a) (b)

Figure 1: Spherical indentation: (a) finite element global and local meshes in the undeformed configuration, (b) σ_{yy} component of the Cauchy stress tensor in the deformed configuration.

References

[1] R. Sauer and L. Shaofan, An atomic interaction-based continuum model for adhesive contact mechanics , *Finite Elements in Analysis and Design*, 43:384-396, 2007.

[2] R. Sauer, A survey of computational models for adhesion, *The Journal of Adhesion*, 92:81-120, 2016.

[3] H. Ben Dhia, Aspects géométriques numériques micro-macro des interfaces , *Workshop Proceedings, LMA-Marseilles*, 2003.

[4] H. Ben Dhia et al., Modele d'interface multi-niveau pour les problemes de contact, *proceedings of* 7^{th} *Col. Nat. Calcul Structures, France*, 2005.

[5] H. Ben Dhia, Plaques en grandes transformations élastiques sous contact hyperélastique , *proceedings of* 3^{rd} *Col. Nat. Calcul Structures, France*, 1997.

SESSION 8

Frictional energy dissipation in an incomplete contact for a tilted flat and rounded punch with varying normal load

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Summary: Frictional contacts in mechanical assemblies are often subject to cyclically varying loads, usually caused by vibration. In a gas turbine, dovetail joints will accumulate large numbers of vibration cycles during operation. Results of this study will help determining damping properties and assessing fretting damage for unsymmetrical contact interfaces of practical relevance.

Introduction

The idealised frictional contact of a tilted flat and rounded punch subject to a varying normal load is considered in this study, including effects of a shearing force insufficient to cause sliding. These give rise to slip and stick regions within the contact. The evolution of the stick-slip pattern, together with frictional energy dissipation, is studied for a wide range of loading scenarios.

The solutions are obtained assuming that the contacting bodies exhibit isotropic elastic behaviour and have similar mechanical properties and that both bodies can be modelled as half-planes.

Extending methods to unsymmetrical problems is necessary in order to broaden the understanding of the behaviour of dovetail roots of gas turbine fan blades. The flat and rounded punch, i.e. symmetrical case, has often been used to represent the dovetail flank contact. However, in the unsymmetrical case, contact pressure may be considerably higher at one of the contact edges compared to the respective edge in the symmetrical case. The severe stress concentration at the edge of the contact is extremely localised and a practical design consideration will therefore be whether these cycles can cause fretting fatigue cracks to initiate and propagate.

In previous publications it has been demonstrated that the case of a general symmetric incomplete contact subject to complex loading cycles can be solved in closed form [1, 2]. Furthermore, frictional energy dissipation was the subject of investigation for symmetrical contacts [3]. Here, parts of this approach are applied to the unsymmetrical case with the intention to provide further understanding of realistic contacts as found in gas turbine applications.

References

[1] J.R. Barber, M. Davies, D.A. Hills, Frictional elastic contact with periodic loading, *Journal of Solids and Structures,* 48:2041 – 2047, 2011.

[2] R. Ramesh, J.R. Barber, D.A. Hills, Plane incomplete contact problems subject to bulk stress with varying normal load, *Int. Journal of Mechanical Sciences*, 122:228–234, 2017.

[3] M. Davies, J.R. Barber, D.A. Hills, Energy dissipation in a frictional incomplete contact with varying normal load, *Journal of Mechanical Sciences*, 55:13–21, 2012.

Shear deformable beams in contact with an elastic half-plane

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Summary: The present work deals with the contact problem of a Timoshenko beam bonded to an elastic semi-infinite substrate under different loading conditions. The analysis allows to investigate the effects induced by shear compliance of the beam on the stress intensity factors at the beam edges as well as the singular nature of the interfacial stresses.

Formulation of the Problem

Let to consider a shear deformable beam of length $2a$ with a cross section area $A = 1{\cdot}h$, in contact with a homogeneous elastic half-space. The cover beam element is subjected to static axial (N_1, N_2) and vertical (T_1, T_2) concentrated forces and couples (M_1, M_2) acting at the beam edges, as reported in Figure 1 for the case of symmetric external concentrated loads.

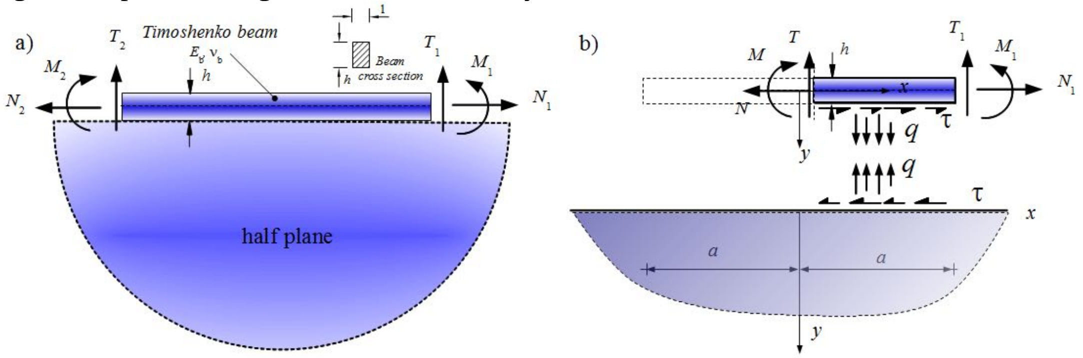

Figure 1: a) Timoshenko beam bonded to an elastic half-plane subjected to external loads; b) shear and peel stresses acting within the contact region.

Then, the strains at the lower side of the beam, for $|x| < a$, read to,

$$u'_b(x) = \frac{N_1}{E_b A} + \frac{1}{E_b A}\int_x^a \tau(s)\,ds + \frac{h}{2E_b I}[M_1 + T_1(a-x)] + \frac{h}{4E_b I}\int_x^a h\tau(s) + 2q(s)(x-s)\,ds\,,$$

$$v'_b(x) = -\frac{M_1 x}{E_b I} - \frac{T_1}{2E_b I}(2ax - x^2) - \frac{h}{2E_b I}x\int_x^a \tau(s)\,ds - \frac{h}{2E_b I}\int_0^x s\,\tau(s)\,ds + \frac{1}{2E_b I}\int_0^a s^2 q(s)\,ds \ldots$$

$$-\frac{1}{2E_b I}\int_x^a (s-x)^2 q(s)\,ds - \phi_p - \frac{\chi T_1}{G_b A} + \frac{\chi T_1}{G_b A}\int_x^a q(s)\,ds\,, \qquad (1)$$

where $E_b = E_0$ or $E_0 / (1-v_b^2)$ denotes the Young modulus of the beam in plane stress or plain strain conditions, respectively, v_b is the Poisson ratio of the beam, I represents the moment of inertia of the beam cross section, G_b is the shear modulus of the beam, whereas χ denotes the dimensionless shear factor. In eq(1), $v_b(x)$ represents the transverse deflection of the beam along the y axis, $u_b(x,y)$ is the axial displacement of the beam cross section at the interface, i.e. $u_b(x,y)|_{y=0}$, and φ_p denotes a constant of integration (i.e. the rotation of the beam cross section at x=0, positive if counterclockwise), to be determined.
The half-plane strains at the contact domain are known in closed form [2] as,

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$$u_s' = -\frac{2}{E_s\pi}\int_{-a}^{a}\frac{\tau(\xi)}{\xi-x}d\xi+\left(\frac{2}{E_s}-\frac{1}{2G_s}\right)q(x),$$
$$v_s' = -\frac{2}{E_s\pi}\int_{-a}^{a}\frac{q(\xi)}{\xi-x}d\xi-\left(\frac{2}{E_s}-\frac{1}{2G_s}\right)\tau(x), \qquad \text{for } |x|<a. \qquad (2)$$

being E_s and ν_s the Young modulus and the Poisson ratio of the half-plane, respectively, and $G_s = E_s/2(1+\nu_s)$ its shear modulus.

The strain compatibility conditions between the beam and the half-plane require:

$$u_s' = u_b' \qquad v_s' = v_b', \qquad \text{for } |x|<a. \qquad (3)$$

System (3) cannot be solved in closed form. However, an approximate solution can be straightforwardly found by expanding the unknown shear and peeling stresses in series of orthogonal polynomials, namely [3]

$$\tau(x)=(a+x)^s(a-x)^s\sum_n C_n P_n(x/a), \qquad q(x)=(a+x)^s(a-x)^s\sum_n D_n P_n(x/a), \qquad (4)$$

being $P_n(x)$ the Jacobi polynomial of order n and the index s of the polynomials denotes the singular strength of the interfacial stresses at the end of the contact region, i.e. at $x = \pm a$. The solution of system (3) is imposed at selected collocation points, thus founding the coefficients C_n and D_n.

Results

As an example, the shear and peel stresses of a Timoshenko beam under axial loads are reported in Figure 2 for some values of the parameter $\gamma=E_s\chi/G_b$. The case of an Euler-Bernoulli beam can be recovered as the limiting case of a Timoshenko beam having a vanishing value of γ.

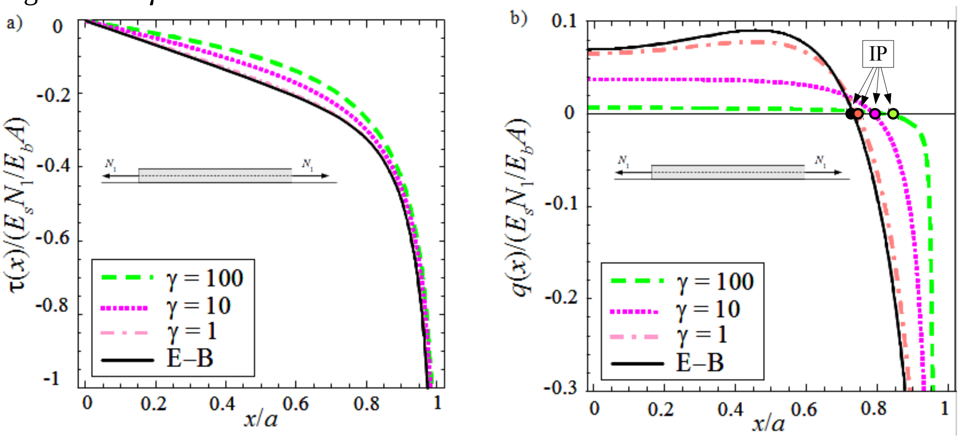

Figure 2. a) Dimensionless interfacial a) shear and b) peel stress of a Timoshenko beam subjected to two axial forces acting at the beam ends varying the parameter g.

Conclusion

The analysis of a Timoshenko beam in contact with an elastic half-plane under static loads has been performed in the present work. The investigation allows to evaluate the effects induced by the shear compliance of the beam on the behaviour of the beam-substrate system. The special case of a membrane bonded to a half plane or a beam in frictionless contact with the underlying support can be retrieved by imposing only the condition $(3)_1$ or $(3)_2$, respectively. The inversion point (IP) position, i.e. the point where the peeling component inverts its sign, moves toward the beam middle as the shear compliance increases and as the beam slenderness decreases.

References

[1] G.R. Cowper, The shear coefficients in Timoshenko's beam theory. *ASME J. Appl. Mech* 13(3):335-340, 1966.
[2] K.L. Johnson, *Contact Mechanics*, Cambridge University Press, 1985.
[3] L. Lanzoni and E. Radi, A loaded Timoshenko beam bonded to an elastic half plane. *Int. J. Solids and Structures*, 92:76–90, 2016

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Surface stretch produced by a hemispherically-ended indenter

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The tangential displacement field induced at the surface of a transversely isotropic elastic half-space by a spherical indenter is evaluated.

Introduction

It was shown by Brown et al. [1], in their *in vitro* study on articular cartilage explants, that the surface stretch of tissue due to axial loading is sensitive to early degenerative changes in the tissue. The tangential displacement (stretch) at the tissue surface, both under and surrounding a transparent (glass) indenter, was captured optically at the grid points. The effective surface stretch was introduced by normalizing the tangential displacement against the indenter characteristic size (diameter). It has been also shown [2] that the tangential displacements may be important in solving an inverse problem of determining biomechanical parameters from indentation experiments. In this study, we derive explicit formula for the surface stretch when the tested sample is represented by an elastic half-space.

Elastic constant governing the surface stretch

It is known [3], that the radial tangential displacement at the surface of a transversely isotropic elastic half-space under the action of a normal point force, applied at the origion of coordinates, can be represented in the form

$$u_r(r) = -\frac{\alpha F}{\pi M_3 r}. \tag{1}$$

Making use of the known solutions [4, 5], for a transversely isotropic material, the indentation modulus, M_3, and the dimensionless elastic constant α can be represented by the formulas

$$M_3 = 2\alpha(\sqrt{A_{11}A_{33}} + A_{13}), \quad \alpha = \frac{\sqrt{A_{44}}\sqrt{(A_{11}A_{33})^{1/2} - A_{13}}}{\sqrt{A_{11}}\sqrt{A_{13} + 2A_{44} + (A_{11}A_{33})^{1/2}}}. \tag{2}$$

Here, A_{11}, A_{13}, A_{33}, and A_{44} are the material stiffnesses of transversely isotropic material. In the case of an isotropic material, we have

$$M_3 = \frac{E}{1-\nu^2}, \quad \alpha = \frac{1-2\nu}{2(1-\nu)}, \tag{3}$$

where E and ν are Young's elastic modulus and Poisson's ratio, respectively.

Surface stretch under a spherical indenter

Making use of the known solution [6] of the contact problem in the case of the indenter shape function $\Phi(r) = R - \sqrt{R^2 - r^2}$, the following formula has been established:

$$u_r(r) = -\frac{\alpha}{\pi r}\left\{\frac{(R^2+a^2)}{2}\ln\left(\frac{R+a}{R-a}\right) - Ra + 2R\sqrt{a^2-r^2}\right.$$
$$\left. - R\sqrt{R^2-r^2}\ln\frac{\sqrt{R^2-r^2}+\sqrt{a^2-r^2}}{\sqrt{R^2-r^2}-\sqrt{a^2-r^2}} + \int_r^a \sqrt{\rho^2-r^2}\ln\left(\frac{R-\rho}{R+\rho}\right)\mathrm{d}\rho\right\}.$$

For the same ratio a/R, the surface stretch under a spherical indenter is somewhat lower than that predicted in the framework of Hertz's theory.

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References

[1] C.P. Brown, A. Oloyede, H.R. Moody and R.W. Crawford, A novel approach to the development of benchmarking parameters for characterizing cartilage health, *Connect. Tissue Res.*, 48:52–61, 2007.

[2] I.I. Argatov and G. Mishuris, Axisymmetric contact problem for a biphasic cartilage layer with allowance for tangential displacements on the contact surface, *Eur. J. Mech. A/Solids*, 29:1051–1064, 2010.

[3] V.I. Fabrikant, Elastic field around a circular punch, *J. Appl. Mech.*, 55:604–610, 1988.

[4] M.T. Hanson and I.W. Puja, Love's circular patch problem revisited closed form solutions for transverse isotropy and shear loading, *Q. Appl. Math.*, 54:359–384, 1996.

[5] A. Delafargue and F.-J. Ulm, Explicit approximations of the indentation modulus of elastically orthotropic solids for conical indenters, *Int. J. Solids Struct.*, 41:7351–7360, 2004.

[6] C.M. Segedin, The relation between load and penetration for a spherical punch, *Mathematika*, 4:156–161, 1957.

A model of friction of rough bodies taking into account adhesion and hysteresis losses at microlevel

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A contact problem is considered for an indenter sliding on a viscoelastic base with the use of a phenomenological friction law obtained taking into account the conditions of interaction at microscale level. Contributions of both adhesion forces and hysteretic losses are taken into account.

Introduction

Adhesion forces associated with molecular attraction can have substantial influence on the characteristics of contact and friction between surfaces; their action manifests itself at quite fine, microscopic levels of interaction. At these scale levels, adhesion forces can lead to a significant increase in hysteretic losses in the surface layers of a viscoelastic material subjected to friction. The combined action of the imperfect elasticity of a material and the adhesion forces acting in the direction normal to the surface leads to the friction force acting in the tangential direction [1, 2].

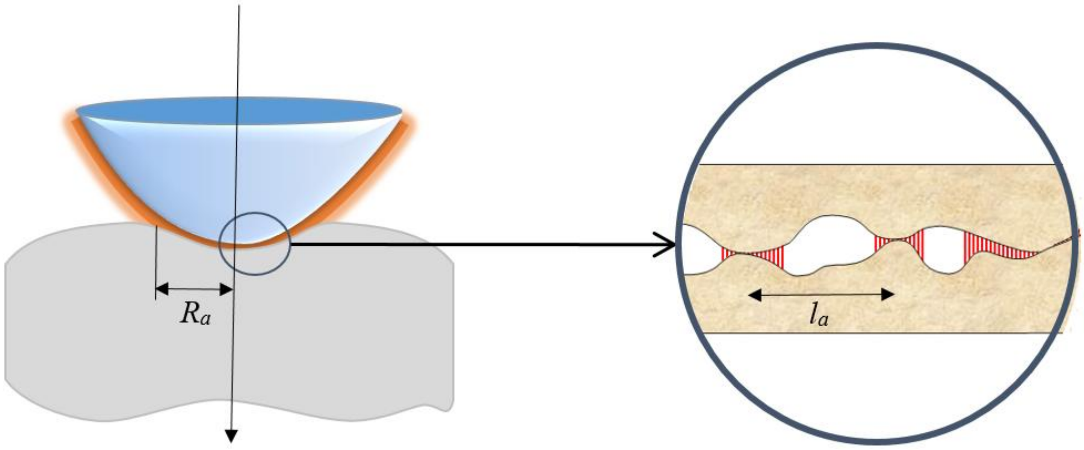

Figure 1: Scheme of contact at two scale levels.

In this study, sliding contact is considered at two scale levels characterized by two scales of length. The first scale is associated with a characteristic size of the nominal contact area R_a and the second one with characteristic distance between asperities l_a (Fig. 1). The shear stress in the contact zone is specified as a function of the pressure, velocity, and parameters of surface microgeometry (phenomenological law of friction) The phenomenological friction law thus constructed can be used not only for the formulation and solution of contact problems for viscoelastic materials, but also for the description of experimental results and comparison to results obtained with other available models.

Contact problem at macrolevel

The contact problem is considered for an indenter sliding on a viscoelastic base in the presence of shear stresses acting in the contact zone and adhesion forces acting outside the contact zone. Unlike the classical Amonton-Coulomb friction law, this relationship will contain, as parameters, characteristics of micro-roughness of the indenter surface and adhesion properties of the interacting bodies. As a result of the contact problem solution, a model of friction is constructed, which takes into account the contribution of hysteretic losses into the overall friction force at both macro- and micro-levels. The model allows one to explain how the adhesion forces acting in the direction normal to the surface contribute into the macroscopic friction force acing in the direction tangential to the surface. The contact characteristics at macrolevel – nominal pressure distribution, size and position of the nominal contact area – are calculated and analyzed depending on the parameters of micro-roughness and adhesion properties of surfaces. The overall value of the friction force is also studied as a function of input parameters of the problem, including geometric characteristics of the indenter at micro- and macro-scales, load, and sliding velocity.

Friction law at microlevel

The friction law at microlevel is constructed by following steps:

- an average shear stress at microlevel is calculated on the basis of the contact problem solution for a rough surface sliding on a viscoelastic base in the presence of adhesion forces acting outside real contact areas in the direction normal to the surface. In formulation of the contact problem, only normal forces of attraction and repulsion between the surfaces are taken into account; as a result the tangential force (friction force) is calculated, which arises in contact due to hysteretic losses occurring in cycling deformation of the viscoelastic material by sliding asperities of the rough counterbody.
- the tangential force is analyzed depending on the roughness parameters (size and shape of asperities, their density), surface energy of the viscoelastic material, its mechanical properties, sliding velocity, and external normal load.
- a friction law is constructed relating normal stresses acting in the contact zone to tangential ones.

Acknowledgements

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References

[1] I.G. Goryacheva, *Contact Mechanics in Tribology*. Dordercht: Kluwer Academic Publ, 1998.

[2] I. Goryacheva, Y. Makhovskaya, Adhesion effect in sliding of a periodic surface and an individual indenter upon a viscoelastic base. *J Strain Anal Eng Des*, 51(4): 286-293, 2016.

Partial elastic contact of nonsinusoidal wavy surfaces

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Summary: The normal contact problem for a rigid nonsinusoidal wavy surface, indenting into elastic half-plane is studied analytically. The close-form solution for a wavy profile, represented as cosine series is obtained. The approach is implemented for the case of a two-scale wavy profile. The results show oscillating character of contact characteristics.

Introduction

Regular (periodic) surface relief is often observed on some machined (e.g. on turned or milled) and natural surfaces. Also it is created on a surface for achievement of specified physical characteristics. As a first approximation the relief can be modeled as a cosine profile. Contact problems for a cosine wavy surface are intensively studied [1-3]. As contact characteristics are much sensitive to surface geometry, the contact problems for nonsinusoidal wavy surfaces, including multiscale profiles (e.g. Weierstrass profile) are of interest. For plane contact problems the analytical approaches can be applied with appropriate assumptions. The close-form solution of plane contact problem was found in [4] for parametric wavy profile, different from the sinusoidal one. Here the more general approach is considered, assuming wavy profile as a cosine trigonometric series.

Assumptions and general equations

The wavy profile is a part of a rigid body and it has an amplitude much smaller than period. The half-plane is a linear elastic isotropic material with two elastic constants - Young's modulus E, and Poisson's ratio μ (plain strain state). Only symmetrical wavy profiles, described by twice differentiable functions are considered. Friction is negligible. The main integral equation of periodic contact problem is [1-3]

$$\frac{E}{2(1-\nu^2)} h_x(x) = \frac{1}{2\pi} \int_{-a}^{a} p(\xi) \cot \frac{x-\xi}{2} d\xi, \tag{1}$$

where x is a linear coordinate; $h_x(x)$ is a derivative of a gap function; a is a contact half-length; $p(x)$ is a contact pressure. With use of variable transform [4] equation (1) is transformed to Cauchy integral equation, which solution is known [1, 2]:

$$\frac{E}{1-\nu^2} h_x(\upsilon) = \frac{2}{\pi} \int_{-\alpha}^{\alpha} \frac{p(u)}{\upsilon - u} du, \tag{2}$$

where u = tan ξ/2; υ = tan ξ/2 ; α = tan a/2. For unknown contact length 2a the equlibrium equation is used to complete the system [3].

Problem statement and method of solution

Assume that wavy profile is represented by cosine series:

$$f(x) = \Delta_1 \cos \frac{2\pi n_1 x}{\lambda_1} + \Delta_i \cos \frac{2\pi n_i x}{\lambda_1} + ... + \Delta_N \cos \frac{2\pi n_N x}{\lambda_1}, \tag{3}$$

where N is a number of wavelengths; Δ_N is an aplitude of N'th harmonic ($\Delta_i < \Delta_{i-1}$, $\Delta_i \in$ **Rj**); λ_1 is a largest wavelength of a profile; $n_i = \lambda_1 / \lambda_i$ ($n_i > n_{i-1}$; $n_i \in$ **N**).

Then derivative of a gap function for a i'th harmonic can be represented after change of variables in terms of Chebyshev polynomials of the first kind (period λ = 2π was chosen for simplicity):

$$h_x(\upsilon)_i = -2\Delta_i n_i \frac{\upsilon}{1+\upsilon^2} U_{n_i-1}\left(\frac{1-\upsilon^2}{1+\upsilon^2}\right), \qquad (4)$$

where U_n – is a Chebyshev polynomial of a second kind with degree n_i.

The solution for the contact pressure for i'th harmonic can be obtained using expansion of equation (4) into Chebyshev polynomials of the first kind and its integral relations with Cauchy singular integral.

$$p_i(x) = -\frac{\pi E \Delta_i n_i}{\left(1-\nu^2\right)\lambda_1}\sqrt{1-\left(\frac{\tan(\pi x/\lambda_1)}{\tan(\pi a/\lambda_1)}\right)^2}\sum_{j=1}^{\infty} A_j U_{j-1}\left(\frac{\tan(\pi x/\lambda_1)}{\tan(\pi a/\lambda_1)}\right), \qquad (5)$$

where

$$A_{ij} = \int_{-1}^{1}\frac{\varphi_i(s)T_j(s)}{\sqrt{1-s^2}}ds,\ j = 1, 2, \ldots; \qquad (6)$$

$$\varphi_i(s) = \frac{\tan(\pi a/\lambda_1)s}{1+\left(\tan(\pi a/\lambda_1)s\right)^2} U_{n_i-1}\left(\frac{1-\left(\tan(\pi a/\lambda_1)s\right)^2}{1+\left(\tan(\pi a/\lambda_1)s\right)^2}\right). \qquad (7)$$

The total pressure distribution $p(x)$ is given by the sum of distributions for separate harmonics $p_i(x)$. The dependences for displacements, average pressure and internal stresses can be obtained using relations for plane contact problems [1, 2].

Contact characteristics analysis

The graphs of dimensionless nominal (average) pressure $\tilde{p}_\infty = P/(\lambda_1 p^*)$ and maximum pressure $\tilde{p}_{max} = p(x=0)/p^*$; $p^* = -\pi\Delta_1 E/(\lambda_1(1-\nu^2))$ versus contact length $2a/\lambda_1$ for wavy surface with two wavelengths, having Δ_1 = 1 mm; Δ_2 = 0.025 mm; λ_1 = 10 mm; $n = \lambda_1/\lambda_2$ are presented on a figure 1.

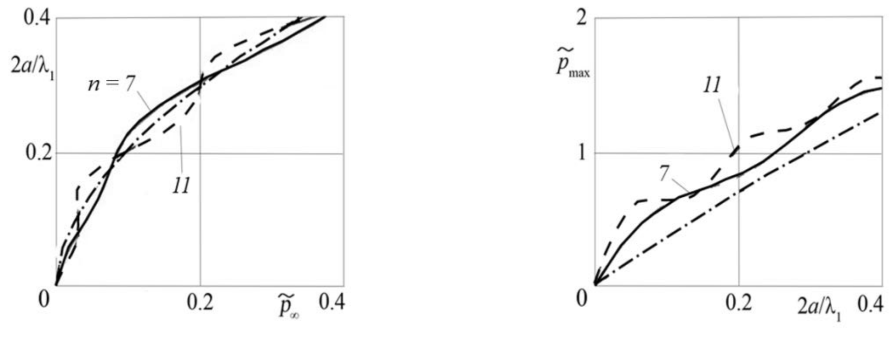

Figure 1: Dimensionless contact pressure versus contact length at different n in comparison with single cosine profile (dash-dot line): (a) nominal pressure, (b) maximum pressure.

The results illustrate, that nominal and maximum pressures have oscillations, correspondent to second harmonics of profile, which influence the stress-strain state of the contact stronger than contact area dependence.

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References

[1] I.Y. Schtaierman. *Contact Problem of Theory of Elasticity*, Moscow, Gostekhizdat, 1949.

[2] I. Goryacheva. *Contact Mechanics in Tribology* , Kluwer, 1997.

[3] J.M. Block and L.M. Keer, Periodic contact problems in plane elasticity, *Journal of Mechanics of materials and structures*., 3:1207–37, 2008.

[4] I.Y. Tsukanov, Effects of shape and scale in mechanics of elastic interaction of regular wavy surfaces *Proc. IMech Part J: J Engineering Tribology* 231:332-40, 2017.

SESSION 9

An error analysis of Nitsche's method for contact problems

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Summary: We present an error analysis of Nitsche's method for contact problems. By reinterpretation of the method as a stabilised mixed method, we derive optimal a priori and a posteriori estimates.

Introduction

The method of Nitsche [1] has been shown to be a very successful method for approximating contact problems, cf. the recent survey [2]. The mathematical analysis presented has, however, not been entirely satisfactory. For the a priori error analysis the assumption that the solution is in the Sobolev space H^s, with $s > 3/2$, has been needed. The a posteriori analysis has been made under a non-rigorous saturation assumption.

The new error analysis

In our paper [3] we made the observation that there is a close connection with Nitsche's method and a stabilised mixed finite element method, and we advocated the use of the former since it has the advantage that it directly yields a method with an optimally conditioned, symmetric, and positively stiffness matrix. The error analysis is also very straight-forward (but, as mentioned, not optimal).

In our work we readress the error analysis of Nitsche's method. In an earlier paper [4] we developed a framework for analysing stabilised mixed finite element methods for the model membrane problem. This we now apply for the linear elastic contact problem. We first reformulate the method as a variational inequality in mixed form with the contact force as a Lagrange multiplier. For this we prove the stability, with the right noms, for both the continuous and discrete problem. The continuous stability yields the a posteriori estimates, whereas the a priori is a consequence of the stability of the discrete problem.

References

[1] J. Nitsche. Über ein Variationsprinzip zur Lösung von Dirichlet-Problemen bei Verwendung von Teilräumen, die keinen Randbedingungen unterworfen sind. *Abh. Math. Sem. Univ. Hamburg*, 36:9–15, 1971. Collection of articles dedicated to Lothar Collatz on his sixtieth birthday.

[2] Franz Chouly, Mathieu Fabre, Patrick Hild, Rabii Mlika, Jérôme Pousin, and Yves Renard. An overview of recent results on Nitsche's method for contact problems. In 121, editor, *Proceedings of the UCL Workshop 2016 on Geometrically Unfitted Finite Element Methods and Applications*, Lect. Notes Comput. Sci. Eng. Springer, 2017.

[3] Rolf Stenberg. On some techniques for approximating boundary conditions in the finite element method. *Journal of Computational and Applied Mathematics*, 63(1-3):139–148, 1995.

[4] Tom Gustafsson, Rolf Stenberg, and Juha Videman. Mixed and stabilized finite element methods for the obstacle problem. *SIAM Journal on Numerical Analysis*, 55(6):2718–2744, 2017.

Adhesive contacts of complex shape: Simulation by Boundary Element Method with mesh-dependent detachment criterion

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Summary: We describe an efficient method of simulation of adhesive contacts of arbitrary complex shapes in the half-space approximation and under assumption of short range adhesive interactions (JKR limit). We used the mesh-dependent detachment criterion introduced 2015 by Pohrt and Popov. Validation is provided by comparison with known analytical solutions and experiments.

Introduction

In the last ten years, the FFT-based Boundary Element Method has taken the position of the most efficient numerical technique for simulation of contacts of bodies having complex shapes (as e.g. contacts of fractally rough surfaces.) However, it took another several years to generalize this technique to adhesive contacts. The first such formulation of "adhesive BEM" was suggested 2015 by Pohrt and Popov [1] and was followed in a rapid sequence 2016 by a very similar work [2] as well as 2017 by another formulation based on the energetic balance [3]. All above works assume infinitely small range of action of adhesive forces (which corresponds to the JKR-approximation of adhesion theory, [4]. The method formulated in [1] was tested and applied to a large variety of different types of shapes in [5] and was generalized to contacts of functionally gradient materials in [6].

The main idea of the detachment criteria is very simple: It is just the direct application of the energy balance principle first used by Griffith for describing propagation of cracks [7]: It is assumed that a computational element detaches if the elastic energy released du to detachment is equal to the work of adhesion of the element. Surprisingly, this simple principle occurs to be extremely robust and efficient and leads to a numerical method which is completely tolerant to the shape of the contact boundary or the orientation of the computational net [5].

Simulation

In Figure 1 and Figure 2, an example of sequences of contact configuration and the corresponding force-distance dependency are shown for the adhesive contact of a flat, oddly shaped 2D cylinder. The configurations are obtained by boundary-elements-simulation according to [1]. Normalization of the pull off-distance d is

$$d_{\text{dimless}} = \frac{d}{\sqrt{L\gamma_{12}/E^*}}, \tag{1}$$

where d [mm] is measured from neutral position (surface stresses vanish), L is the edge length of the square computational domain [mm], γ_{12} is the separation energy of the two surfaces [N/mm] and E^* [N/mm^2] is the reduced modulus of elasticity, obtained from the two bodies' elastic moduli E and poisons ratios ν of contacting bodies:

$$E^* = \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right)^{-1}. \tag{2}$$

The adhesive Force F [N] is normalized as follows

$$F_{\text{dimless}} = \frac{F}{\sqrt{L^3 \gamma_{12} E^*}} . \qquad (3)$$

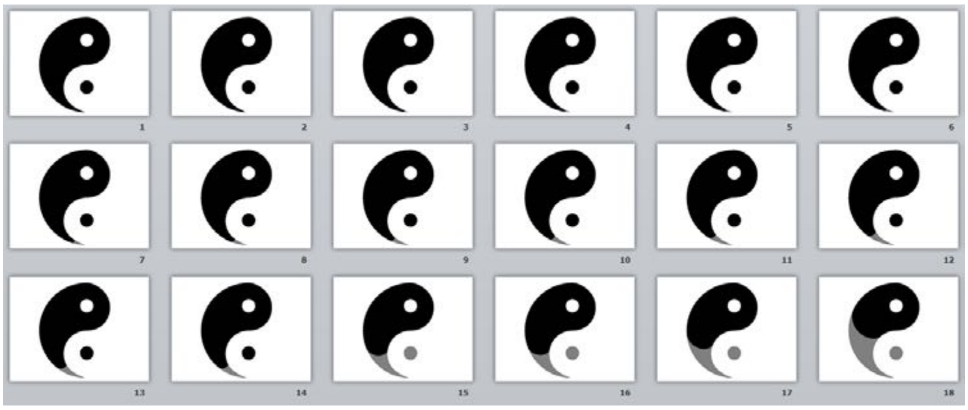

Figure 1: Consecutive phases of detachment of a flat-ended stamp in form of “Yin and Yang”. The last shown configuration is the last stable configuration. After that, the stamp instantaneously detaches completely.

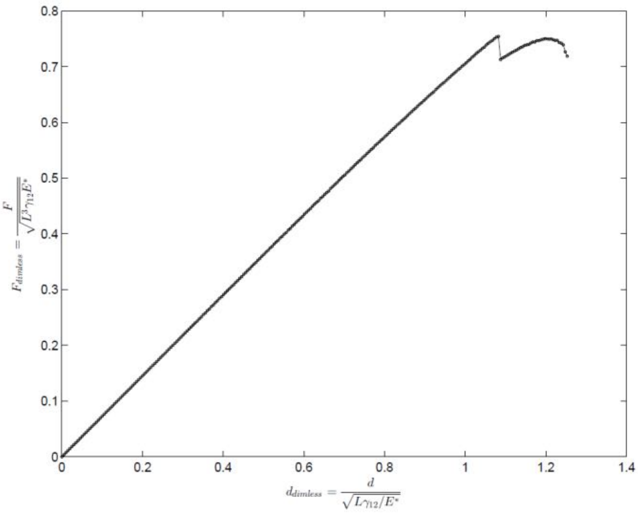

Figure 2: Dependence of the normalized normal force on the normalized distance.

In the presentation, a large variety of indenter shapes will be discussed, [8].

References

[1] R. Pohrt, V.L. Popov, Adhesive contact simulation of elastic solids using local mesh-dependent detachment criterion in Boundary Elements Method, Facta Universitatis, Series: Mechanical Engineering, 13: 3-10, 2015.

[2] S. Hulikal, K. Bhattacharya, N. Lapusta, A threshold-force model for adhesion and mode I fracture, arXiv:1606.03166

[3] V. Rey, G. Anciaux, J.-F. Molinari, Normal adhesive contact on rough surfaces: efficient algorithm for FFT-based BEM resolution, *Comput. Mech.* DOI: 10.1007/s00466-017-1392-5 , 2017.

[4] K.L. Johnson, K. Kendall, A.D. Roberts, Surface energy and the contact of elastic solids, *Proc. R. Soc. London A* 324: 301–313, 1971.

[5] V.L. Popov, R. Pohrt, Q. Li, Strength of adhesive contacts: Influence of contact geometry and material gradients, *Friction*, 5: 308-325, 2017.

[6] Q. Li, V.L. Popov, Boundary element method for normal non-adhesive and adhesive contacts of power-law graded elastic materials, *Computational Mechanics*, 2016, DOI 10.1007/s00466-017-1461-9.

[7] A.A. Griffith, The Phenomena of Rupture and Flow in Solids, *Philosophical Transactions of the Royal Society of London, A* 221: 163–198, 1921.

[8] R. Pohrt, V.L. Popov, Force-distance-dependencies for the adhesive contact of cylinders, 2016, http://dx.doi.org/10.14279/depositonce-4957

Discontinuous Galerkin modeling of wave propagation in a solid with micro-cracks in frictional contact

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The discontinuous Galerkin (DG) method is used to investigate the dynamics of (damaged) materials with micro-cracks in frictional unilateral contact. The explicit leapfrog scheme is used for the time discretization while the nonlinear conditions on the micro-cracks are treated by using a specific flux choice and an augmented Lagrangian technique.

Introduction

The derivation of accurate relationships between the micro-structure (pores, micro-cracks) and overall elastic properties of brittle materials (rocks, ceramics,..) is an ongoing problem in material science, geophysics, and solid mechanics. The large majority of numerical schemes that treat the wave propagation in materials with micro-fractures are using the finite-difference method. Some of them take the cracks as secondary point sources and others use penny-shaped weak inclusions to model the micro-cracks. In contrast, for the "explicit interface" approaches the fracture is assumed to have a vanishing width across which tractions are continuous, but displacements and velocities are allowed to have jumps.

The aim of this paper is to use the discontinuous Galerkin (DG) method to investigate the dynamics of (damaged) materials with a nonlinear micro-structure (micro-cracks in frictional contact). In the classical finite element technique, inner boundary conditions require a geometrical treatment, hence the computational effort became very important for a large number of micro-cracks. In contrast, in the DG method the inner boundary conditions are modeled by the flux choice without any additional computational cost even for many micro-cracks.

Numerical Approach

The general framework of the numerical scheme used here is based on the second order numerical scheme proposed by Etienne et al. [1]: the explicit leapfrog scheme in time and a centered flux choice for the inner element faces. The nonlinear conditions on the micro-cracks will be treated as special flux choices, while the resulting nonlinear equations at each time step are solved by using an augmented Lagrangian technique.

Testing the numerical schemes

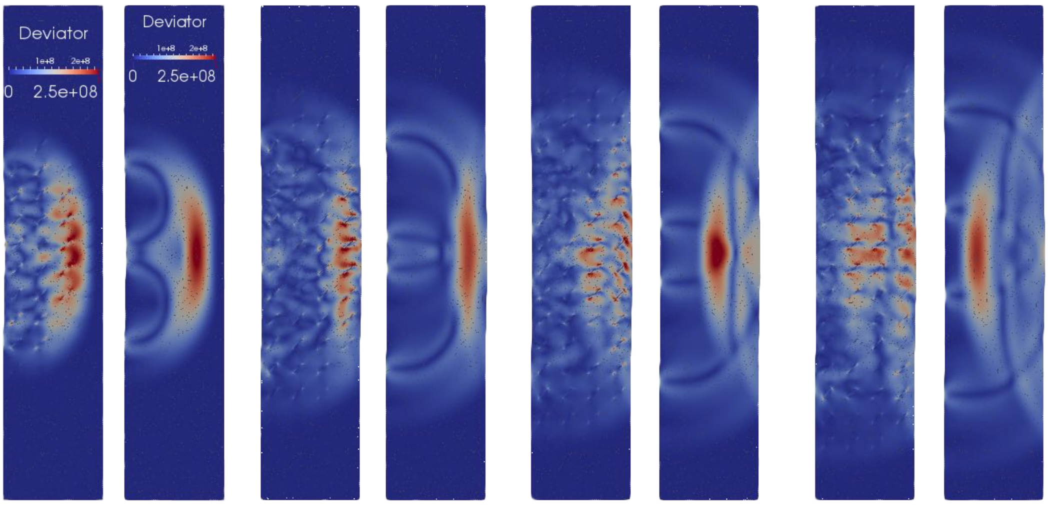
In order to test the algorithms, we considered two in-plane problems for which we can construct an exact solution. In both problems, we have compared the results of our numerical schemes with the analytical solutions. For the frictionless unilateral contact problem the Uzawa and compliance methods give accurate results. Generally, the Uzawa scheme is more accurate but it requires a higher computational cost. For frictional contact we remark a very good approximation illustrating the accuracy of the proposed numerical scheme.

Effective wave velocity

This technique was used to compare the effective wave velocity in a damaged material obtained by direct DG computation and by the analytical formula, deduced form the effective elasticity of a cracked solid theory (NIC, DS and SCS approaches). Even if the stress field loses its homogeneity (unloading zones around the micro cracks and high stress concentration on the crack tips, compressive waves that propagates in the opposite direction, etc) the pulse has an over-all front wave at each moment. This is an important point which allowed us to compute the over-all wave speed. We found that, the over-all wave speed is slower than the theoretical speed and the difference is very important for large values of the crack density parameter. If the wave length is of order of the crack length, the speed wave is strongly dependent on wavelength, but for a large wavelength the speed wave depends only on the crack density parameter.

Blast impact on a cracked material

The aim of this chapter is to illustrate how the DG method can be used to investigate more complex wave propagation phenomena. We have analyzed the wave generated by a blast in a cracked material (81 vertical, horizontal or inclined frictional or frictionless micro-cracks). We found that the cracks' orientation affects the wave propagation and their scattering. The friction phenomena between the faces of the micro-cracks are affecting the wave propagation only for the mode II behavior but if the waves activate principally the mode I, the role played by the friction is negligible.



(a)

Figure 1: Micro-cracks oriented at $\theta = \pi/4$. Comparison between the propagation of the blast wave in a cracked material with friction (left) and an undamaged one (right). Four snapshots of the stress deviator (color scale in Pa) at $t = 0.5T, 0.7T, 0.8T$ and $t = 0.9T$.

References

[1] Etienne, V and Chaljub, E and Virieux, J and Glinsky, N. An hp-adaptive discontinuous Galerkin finite-element method for 3-D elastic wave modelling, *Geophysical Journal International*, 183:941–962, 12010.

The effect of plasticity on the shakedown of coupled frictional contacts

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Summary: This contribution considers the issue of the shakedown of coupled contacts with frictional interfaces and elastic-plastic material. Our purpose is to show the effect of plasticity on the contact sensitivity to initial conditions. We adopted a metric to measure the degree of coupling of the system, due both to a material mismatch and a domain mismatch. We used an efficient optimisation algorithm to compute the upper shakedown limit, and performed a series of analyses to assess the effect of different initial conditions on the system response.

Introduction

Frictional shakedown has attracted the attention of researchers, as it shares a strong analogy to the phenomenon first observed in the theory of plasticity. However, due to the fact that the most common frictional laws, such as Coulomb's, are pressure-dependent, the flow rule is generally non-associated, and limit analyses theorems do not hold, save some particular cases. Specifically, these occur when the contact is *uncoupled*, that is, the normal pressure is not affected by frictional slips [1, 2]. When coupling is present, the system response depends on the initial conditions, and the well-known Melan theorem provides a necessary but no longer sufficient condition for the shakedown. In such a case, there is a *conditional region*, delimited by a lower bound below which the shakedown is guaranteed, and an upper bound, above which the shakedown is impossible [3]. Flicek et al. [4, 5] studied the response of coupled and uncoupled contacts and investigated the contact sensitivity to initial conditions, in the case of an elastic material. When plastic zones develop in an uncoupled system, it has been noticed that they might help to discourage frictional slip, possibly being beneficial to the shakedown [6].

The purpose of the present contribution is to get an insight into the effects of plasticity on the frictional shakedown of coupled systems. Specifically, we try to answer to a series of questions: how does coupling affect the shakedown when plastic zones develop in the material? What is the effect of the plastic zones on the size of the conditional region, that is, do they increase or shrink its size? All the calculations are applied to a simple model, consisting of a thin plate containing a frictional crack, which enables us to explore several sources of system coupling.

Formulation

Shakedown in elastic-plastic systems with friction

In an elastic-plastic system with frictional contact interfaces, the shakedown condition occurs (i) when no plastic flow occurs and (ii) when each point along the contact remains in a state of stick. According to the theorem formulated in [2], the necessary condition for the shakedown is the existence of a safe *residual* state such that both the yield condition in the stress space and the slip condition in the space of the contact forces are satisfied at any time instant. The residual contact forces are comprised of two terms, one depending directly on the frictional slips w and the other being a consequence of the plastic strains p. In a discrete formulation, we may write such forces as:

$$\mathbf{r}^R = \mathbf{r}^R(\mathbf{w}) + \mathbf{r}^R(\mathbf{p}) = \kappa\mathbf{w} + \kappa\mathbf{A}\mathbf{p} \tag{1}$$

where κ is a symmetric contact stiffness matrix and $\mathbf{A}$ is a rectangular matrix. It is evident that the effect of the plastic strains is to add extra slips along the interface.

Coupled systems

In general, coupling can be ascribed either to a *material* mismatch or a *domain* mismatch. The former occurs in the situations where the contact interface separates two materials with different properties, while the latter results from the two bodies being of a different shape. Both types are considered separately in our study. The degree of coupling is quantified adopting the same metric used in Flicek et al. [5]. From a matrix partition, we can highlight the normal-tangential component accounting for coupling, and thus define the following ratios:

$$\xi_w = \frac{\|\kappa_{tn}\|}{\|\kappa_{tt}\|}, \qquad \xi_p = \frac{\|(\kappa\mathbf{A})_{tn}\|}{\|(\kappa\mathbf{A})_{tt}\|} \tag{2}$$

where t and n denote the tangential and normal directions on the contact, respectively.

Shakedown conditional region

The shakedown conditional region is comprised between the two limits defined above: a lower limit λ_1, and an upper limit, λ_2. This is the same notation used in the paper by Ahn et al. [3], although here the shakedown with respect to the plastic strains is also considered. In our work, the shakedown limit λ_2 is computed with a constrained optimization, using an algorithm that includes both the frictional constraint on the contact interfaces and the yield constraint in the material [6]. As no efficient algorithm is available for the computation of λ_1, we explore the effect of the initial conditions by running a series of incremental analyses, in order to show that the coupled system may not shake down, even if the applied loads are lower than the limit λ_2. In this way, a rough assessment of the size of the conditional region is obtained.

References

[1] A. Klarbring, M. Ciavarella, and J. R. Barber. Shakedown in elastic contact problems with Coulomb friction. *Int. J. Solids Struct.*, 44(25-26):8355–8365, 2007.

[2] A. Klarbring, J. R. Barber, A. Spagnoli, and M. Terzano. Shakedown of discrete systems involving plasticity and friction. *Eur. J. Mech. A/Solids*, 64:160–164, 2017.

[3] Y. J. Ahn and J. R. Barber. Response of frictional receding contact problems to cyclic loading. *Int. J. Mech. Sci.*, 50(10-11):1519–1525, 2008.

[4] R. C. Flicek, D. A. Hills, J. R. Barber, and D. Dini. Determination of the shakedown limit for large, discrete frictional systems. *Eur. J. Mech. A/Solids*, 49:242–250, 2015.

[5] R. C. Flicek, M. R. W. Brake, and D. A. Hills. Predicting a contact's sensitivity to initial conditions using metrics of frictional coupling. *Tribol. Int.*, 108:95–110, 2017.

[6] A. Spagnoli, M. Terzano, J. R. Barber, and A. Klarbring. Non-linear programming in shakedown analysis with plasticity and friction. *J. Mech. Phys. Solids*, 104:71–83, 2017.

Nitsche's methods for elasto-plastic contact small deformation problems : theorical and numerical aspects

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Frictional, elasto-plastic multi-body contact problems play an important role in mechanical engineering. The nonlinearities caused by geometric contact and frictional constraints combined with the nonlinearity in the material law result in challenging numerical problems in forms of variational inequalities and therefore efficient solving methods are needed. This presentation will deal with so called Nitsche's methods. The Nitsche method originally proposed in aims at treating the boundary or interface conditions in a weak sense, according to the Neumann boundary operator associated to the partial differential equation and in a consistent formulation. It differs in this aspect from standard penalization techniques which are generally non-consistent. Moreover, no additional unknown (Lagrange multiplier) is needed and no discrete inf–sup condition must be fulfilled, contrarily to mixed methods. A first application to contact mechanics was presented in [1] and a mathematical analysis for linearized kinematics has been published in [2], [3]. Unlike any other method, Nitsche's method is at the same time variationally consistent and therefore optimally convergent. Additionally, more advanced variants require an adjoint term including the stresses computed from the test function to obtain, depending on a parameter, either a symmetric variant or a skew symmetric variant, which is stable for any positive penalty parameter see [4].

Very recently, we start to use Nitsche's method to elasto-plastic contact problems (with Von Misès flow rule). In this note, we describe the use of Nitsche's method to prescribe a contact (with or without Coulomb friction condition) between two elasto-plastic bodies in the small deformations framework. This corresponds to a weak integral contact condition which as some similarity with the ones which use Lagrange multipliers. The here proposed approximation strategy has been implemented in the open-source finite element library GetFEM++ [6] for small and large elastic or hyperelastic [5] contact with or without friction, and now extended to plastic behavior.

Some numerical examples, such as the elasto-plastic Hertz contact in Figure 1, will demonstrate the efficiency and accuracy of the presented extensions to Nitsche's method for contact problems.

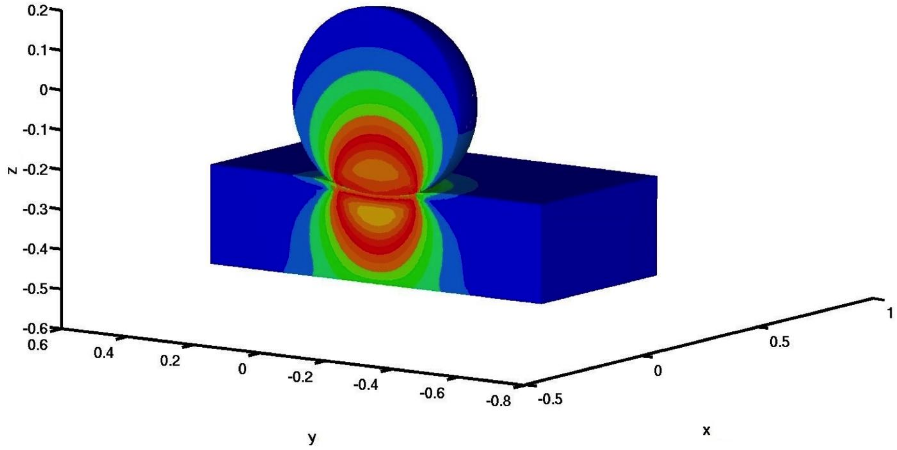

Figure 1 : 3D numerical Hertz contact with contour plot of Von Mises stress.

[1] P. Wriggers and G. Zavarise,A formulation for frictionless contact problems using a weak form introduced by Nitsche, Comput. Mech., 41, 2008.

[2] F. Chouly and P. Hild. A Nitsche-based method for unilateral contact problems : numerical analysis. SIAM J. Numer. Anal., 51:2(2013), pp. 1295–1307.

[3] F. Chouly, P. Hild and Y. Renard. Symmetric and non-symmetric variants of Nitsche's method for contact problems in elasticity: theory and numerical experiments. Math. Comp., 84:1089--1112, 2015.

[4] F. Chouly, M. Fabre, P. Hild, J. Pousin, Y. Renard, An overview of recent results on Nitsche's method for contact problems , Proceedings of the UCL Workshop 2016 on Geometrically Unfitted Finite Element Methods and Applications in Lect. Notes Comput. Sci. Eng, Vol. 121, Springer, 2017.

[5] F. Chouly, R. Mlika, Y. Renard. An unbiased Nitsche's formulation of large deformation frictional contact and self-contact. to appear in Comp. Meth. Appl. Mech. Engng., 2017.

[6] J. Pommier, Y. Renard. GetFEM++, an open source generic C++ library for finite element methods. http://download.gna.org/getfem/html/homepage/.

SESSION 10

A thermo-mechanical contact formulation for liquid and solid membranes

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Summary: We present an FE membrane formulation for solid-solid and solid-liquid thermo-mechanical contact. Mechanical contact with and without friction on surfaces and on lines is considered as well as thermal surface contact. The capability of the formulation is illustrated by several challenging numerical examples.

Introduction

A geometrically exact FE membrane formulation that is suitable for solids and liquids is presented in [1, 2]. The formulation accounts for enclosed volume, surface area and mechanical contact constraints. It is based on curvilinear coordinates and applicable to classical Lagrangian interpolation, as well as to isogeometric interpolation of higher continuity. This results in a general, robust and efficient contact formulation that avoids the use of local Cartesian coordinate systems and the transformation of derivatives.

Mechanical contact

At the contact surface the usual impenetrability constraint

$$g_{\mathrm{n}} > 0 \tag{1}$$

in normal direction is observed for both solid and liquid membranes. For solid-solid contact, tangential contact forces can also be enforced at the contact surface. In the case of frictional surface contact, the distinction between sticking and sliding is based on the slip criterion

$$f_{\mathrm{s}} \begin{cases} < 0 & \text{sticking ,} \\ = 0 & \text{sliding .} \end{cases} \tag{2}$$

During sticking, the contact traction is defined by the sticking constraint

$$g_{\mathrm{t}}^{\alpha} = 0 \quad \alpha = 1, 2, \tag{3}$$

while it is characterized by a constitutive law during sliding. During solid-liquid contact, sharp contact angles can be formed at the contact line. Under hydrostatic conditions, tangential contact forces are only transferred at this line. In the case of frictionless line contact (constant contact angle), the contact conditions along the contact line can be enforced by simply applying a line load. In the case of frictional line contact (varying contact angle), three states can be distinguished: contact line pinning, contact line advancing and contact line receding. The three states are characterized by the sticking constraint (3) together with the contact angle range

$$\theta_{\mathrm{r}} \leq \theta_{\mathrm{c}} \leq \theta_{\mathrm{a}}, \tag{4}$$

where θ_{a} and θ_{r} are the limit values during advancing and receding [3].
A large range of applications is captured by the given formulation. These include

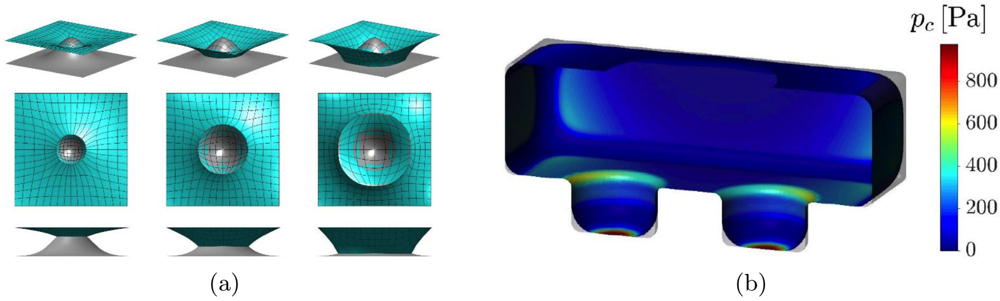

Figure 1: (a) contact between a liquid sheet and a rigid asperity for varying contact angles [2], (b) inflation of a rubber membrane in a bounded cavity: contact pressure

balloon-cushion contact, droplet-substrate contact, droplet-droplet contact, droplet pinning, droplet sliding and cavity wetting among others [1–3]. Moreover, the membrane formulation is coupled with volumetric FE analysis [4] or alternatively with boundary element analysis [5] to consider the fluid flow within the enclosed media.

Thermal contact

The presented membrane formulation is also suitable to model thermal contact between two bodies. The heat flux across an interface is defined by means of its heat transfer coefficient h_{tc}

$$q_{\mathrm{c}} = \boldsymbol{q}_{\mathrm{c}} \cdot \boldsymbol{n} = h_{\mathrm{tc}}\, \Delta T. \tag{5}$$

The value of h_{tc} strongly depends on the mechanical contact state of the interacting bodies. For single-scale contact problems, it can directly be determined from

$$h_{\mathrm{tc}} = \begin{cases} \infty \rightarrow \Delta T = 0 & \text{mech. contact}\,, \\ 0 & \text{no mech. contact}\,. \end{cases} \tag{6}$$

For multi-scale contact problems, the local h_{tc} is determined from the macroscopic contact state and the microscopic surface geometry. Coupling mechanical and thermal contact results in a formulation that accounts for thermo-mechanical contact at interfaces. Thermo-mechanical contact plays a major role in many industrial applications like injection molding or additive manufacturing.

Acknowledgement

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References

[1] R.A. Sauer, T.X. Duong and C.J. Corbett, A computational formulation for constrained solid and liquid membranes considering isogeometric finite elements, *Comput. Methods. Appl. Mech. Eng*, 271:48–68, 2014.

[2] R.A. Sauer, Stabilized finite element formulations for liquid membranes and their application to droplet contact, *Int. J. Numer. Meth. Fluids*, 75:519–545, 2014.

[3] R.A. Sauer, A frictional sliding algorithm for liquid droplets, *Comput. Mech.*, 58:937–956, 2016.

[4] R.A. Sauer and T. Luginsland, A monolithic fluid-structure interaction formulation for solid and liquid membranes including free-surface contact, *arXiv*: 1710.02128, 2017.

[5] M. Harmel and R.A. Sauer, Boundary element and finite analysis for the efficient simulation of fluid-structure interaction and its application to mold filling processes *PAMM*, in press

Numerical Approaches to dry and lubricated contact mechanics between linearly viscoelastic rough solids

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In this work, we review a variety of numerical methodologies that we have developed in the last years to deal with the contact mechanics between linearly viscoelastic solids. Precisely, we have focused on the development of Boundary Element approaches, where the surface displacements and the pressure distribution have been related by means of integral equations. In the case of lubricated condition, the solid problem has been coupled with the Reynolds equations, which have been numerically solved by employing a finite difference scheme.

Soft matter mechanics has an intrinsically high level of complexity: this is due to the strongly time-dependent and usually nonlinear constitutive stress-strain relations governing its response. Further intricacy is added when soft bodies are brought into contact and the problem is exacerbated by the geometry of the intimately mating surfaces. Over the past two decades, the continuously growing technological relevance of engineering applications, involving polymeric materials is requiring specific efforts to shed light on these issues. Indeed, the smart design of engineering elements, like e.g. tires, seals, dampers, is a key point in current applied mechanics research: a more efficient design of automotive tires or an improved sealing action for mechanical seals could have a prominent impact in everyday life by providing significant energy savings and improved wear resistance. However, these optimization efforts strictly require an accurate comprehension of the viscoelastic properties of mechanical problems.

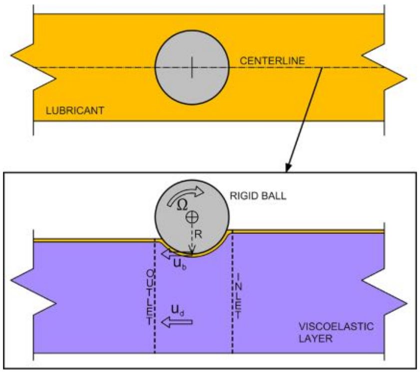

Figure 1: schematic of the problem studied in the lubricated case. A rigid sphere rolls over a linearly viscoelastic layer.

We have developed a variety of methodologies aimed at implementing a rigorous mathematical characterisation of soft contact mechanics, *i.e.* the interactions of the "soft body" with its environment. Specific efforts have been addressed to understand how the surface roughness, whose spectrum covers several orders of magnitude, and the material mechanical properties influence the contact in terms of stresses, strains and, ultimately, friction. Particular attention has been paid to deal with viscoelastic materials, which exacerbate the problem complexity due to the time-dependant behaviour: Boundary Elements models to simulate both steady-state and reciprocating conditions have been introduced and are capable of predicting interfacial strains, stresses and dissipation [1-3]. Recently, a new methodology, coupling the BE solid solver and a Finite Difference scheme, has been introduced to include interfacial lubrication [4-6] (see Figure 1).

Results reported in Figure 2 show how the contact solution in the case of viscoelastic contacts differs from the classic elasto-hydrodynamic solution.

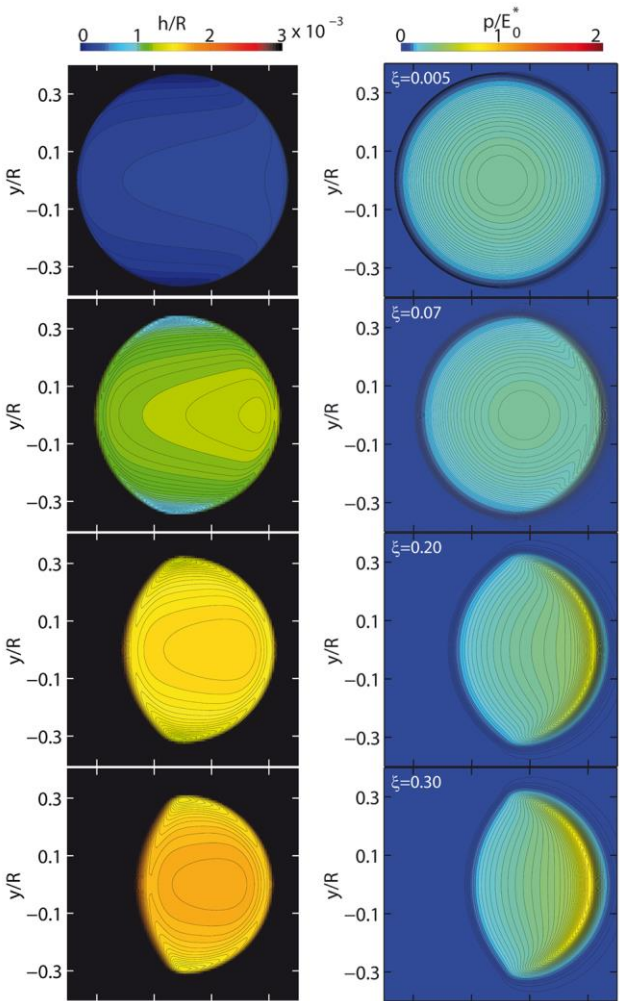

Figure 2: Contour plots of the film thickness (left) and the pressure distribution (right) for different values of the dimensionless speed.

References

[1] G. Carbone, C. Putignano, A novel methodology to predict sliding/rolling friction in viscoelastic materials: theory and experiments., *Journal of the Mechanics and Physics of Solids*, 61, 8, 1822-1834, (2013).

[2] C. Putignano, G. Carbone, D. Dini, Rough viscoelastic sliding contact: theory and experiments , *Physycal Review E*, 93, 043003, (2016).

[3] G. Carbone, C. Putignano. Rough viscoelastic sliding contact: theory and experiments. *Physical Review E*, 89, 032408, (2014).

[4] C. Putignano, D. Dini, Soft matter lubrication: does solid viscoelasticity matter?, *ACS Applied Materials & Interfaces*, 9 (48), 42287-42295, (2017).

[5] Venner C.H. , Lubrecht A.A., Multilevel Methods in Lubrication, *Elsevier Tribology Series*, Ed. D. Dowson et al., V. 37, (2000).

[6] Stupkiewicz S., Lengiewicz J., Sadowski P., Kucharski S., Finite deformation effects in soft elastohydrodynamic lubrication problems, *Tribology International*, 511-522, 93, (2016).

Patterned and hierarchical surfaces for fluid and granular lubrication

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Summary: In this work we present numerical simulations of fluid and granular lubrication between hierarchical surfaces. We show how surface patterns can be precisely engineered to control the macroscopic tribological properties of lubricated contacts.

Introduction

The precise control of friction coefficients can be of extreme interest in many engineering applications. Recent studies have demonstrated that by realizing surfaces with 2D or 3D textures, it is possible to modify the frictional properties of both dry [1] and lubricated [2] contacts. Similarly, hierarchical surface structures [3] and functionally graded materials [4] observed in Nature have stimulated the development of artificial bio-inspired solutions, with outstanding frictional and adhesive properties.
Here we extend the concept of surface hierarchy to fluid and granular lubrication problems, providing useful insights for the design of smart tribo-materials and innovative solutions for lubricated contacts.

Hydrodynamic lubrication

We consider the case of a plane slider bearing [5], whose geometry is modified with a patterned profile presenting several hierarchical levels. By employing finite-volume Computational Fluid Dynamics simulations, we show the dependence of the load-carrying capacity and the frictional force on the surface structure. The results are also compared with the approximate solution given by the Reynolds' equation.
One of the main findings is that it is possible to tune the macroscopic coefficient of fricton of the bearing in order to minimize or maximize the frictional force. In addition, the precise design of the surface patterns can be used to control the emergence of cavitation, which can heavily affect the performance of the bearing (see, e.g., Ref. [6]).

Granular lubrication

In recent years, textured surfaces have been used also in presence of solid (or granular) lubricants, with the main objective of reducing wear and thus increasing the life of tribo-contacts [7–9]. The granular lubrication is investigated by means of the Discrete Element Method [10], considering visco-elastic particles confined between rigid surfaces. We find the general trend that surface patterns and hierarchies reduce the global coefficient of friction. However, the complex microscale (i.e. particle-particle) interactions, as well as the particle size distribution and the presence of cohesive forces, can also have a significant effect on the frictional response of the system.

References

[1] G. Costagliola, F. Bosia and N.M. Pugno, Static and dynamic friction of hierarchical surfaces, *Phys. Rev. E*, 94:063003, 2016.

[2] D. Gropper, L. Wang and T.J. Harvey, Hydrodynamic lubrication of textured surfaces: A review of modeling techniques and key findings, *Trib. Int.*, 94:509–529, 2016.

[3] M. Nosonovsky and B. Bhushan, Multiscale friction mechanisms and hierarchical surfaces in nano- and bio-tribology, *Mater. Sci. Eng. R*, 58:162–193, 2007.

[4] S. Suresh, Graded materials for resistance to contact deformation and damage, *Science*, 292:2447–2451, 2001.

[5] B.J. Hamrock, S.R. Schmid and B.O. Jacobson, *Fundamentals of Fluid Film Lubrication*, 2nd ed., Marcel Dekker, 2004.

[6] J. Zhang and Y. Meng, Direct observation of cavitation phenomenon and hydrodynamic lubrication analysis of textured surfaces, *Tribol. Lett.*, 46:147–158, 2012.

[7] L. Rapoport, A. Moshkovich, V. Perfilyev, I. Lapsker, G. Halperin, Y. Itovich and I. Etsion, Friction and wear of MoS_2 films on laser textured steel surfaces, *Surf. Coat. Tech.*, 202:3332-3340, 2008.

[8] N. Kawasegi, H. Sugimori, H. Morimoto, N. Morita and I. Hori, Development of cutting tools with microscale and nanoscale textures to improve frictional behavior, *Precis. Eng.*, 33:248–254, 2009.

[9] T. Hu, L. Hu and Q. Ding, Effective solution for the tribological problems of Ti-6Al-4V: Combination of laser surface texturing and solid lubricant film, *Surf. Coat. Tech.*, 206:5060-5066, 2012.

[10] P.A. Cundall and O.D.L. Strack, A discrete numerical model for granular assemblies, *Géotechnique*, 29:47–65, 1979.

Trapped fluid in contact interface

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Summary: We studied the mechanical and hydrological behavior of contact interfaces with trapped fluid in 2D and 3D and elaborated a monolithic finite-element scheme for this problem. We report the reduction of friction under increasing load, emergence of stress singularities at contact edges and higher interface transmissivity due to fluid entrapment.

Introduction

Natural and industrial surfaces always possess roughness under certain magnification, and the contact between solid bodies occurs on separate patches corresponding to asperities of contacting surfaces. Lubrication is an efficient mechanism for friction and wear reduction, however, if the applied external load is high enough or if sliding velocities are small, asperities of both surfaces get in direct contact. At the same time, the lubricating fluid may be trapped in valleys surrounded by contact patches [see Fig. 1(a)], which has a strong effect on the contact properties. The trapped fluid opposes the growth of the real contact area, while the applied external load is shared between the contacting asperities and the pressurized fluid, which provides an additional load-carrying capacity. As a result, the global coefficient of friction is decreasing with the increasing external load. Moreover, under high load the trapped fluid escapes the trap, which leads to the depletion of the contact area and thus to a further reduction of the frictional resistance.

The effect of lubricant entrapment is important in such engineering applications, as cold metal forming and rolling, in biological sciences (reduction of friction between cartilage surfaces in human joints) and in geophysical studies, for modeling basal sliding of glaciers and landslides, caused by an elevation of the fluid pressure in pores inside the rock. Moreover, the fluid entrapment is of importance for static sealing applications.

First, we studied the contact problem between a solid with a regular wavy surface and a rigid flat, taking into account fluid trapped in the interface. Second, we addressed the sealing problem, i.e. thin fluid flow through the rough contact interface under increasing external load, and investigated the effect of the fluid entrapment.

Trapped fluid in a contact interface with a regular waviness

We studied in detail the mechanical contact between a deformable body with a wavy surface and a rigid flat taking into account pressurized fluid trapped in the interface, see Fig. 1(b), in plane strain formulation. A finite element model was developed for a general problem of trapped fluid for frictionless and frictional contact, compressible and incompressible fluid models, elastic and elasto-plastic material models, see [1].

We showed that in case of incompressible fluid, the real contact area and the global coefficient of friction decrease monotonically with the increasing external pressure. Ultimately the contact opens and the fluid occupies the entire interface resulting in vanishing

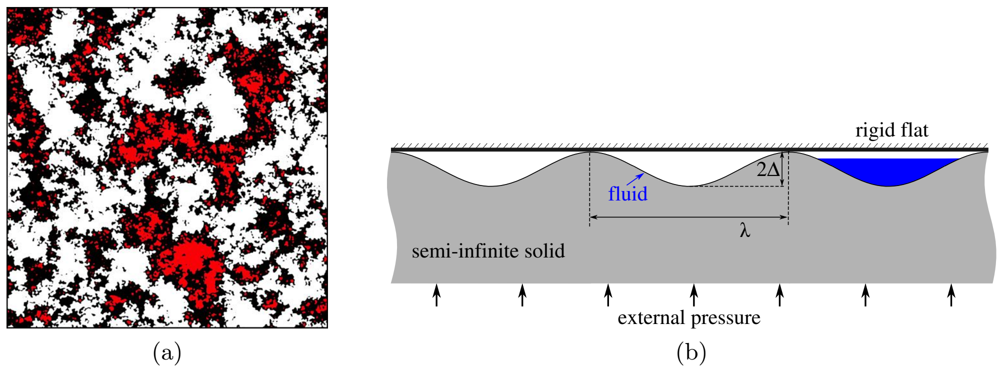

Figure 1: (a) Morphology of the contact interface between two elastic half-spaces with rough surfaces: black shows the real contact area, white is out of contact, red highlights the "trapped" area, delimited by non-simply connected contact patches. (b) Sketch of the trapped fluid problem in 2D formulation.

of static friction. In case of compressible fluid with pressure-dependent bulk modulus, we demonstrated a non-monotonous behavior of the global coefficient of friction due to a competition between non-linear evolution of the contact area and that of the fluid pressure. In case of elastic-perfectly plastic materials, we also observed fluid permeation into the contact interface, which results in a drop of the global coefficient of friction. Finally, we discovered the emergence of singularity-like peaks in the frictional tractions near the contact edges, accommodating the surface shear stress redistribution resulting from the reduction of the contact area. We proposed an approximate analytical formulation of this process based on the similarity with the interfacial crack propagation mechanism.

We generalized the trapped fluid problem to consider 3D contact interface between a bi-sinusoidal surface and rigid flat, filled with fluid. The evolution of the shape of the trapped zone under the increasing external load was also investigated.

Influence of the fluid entrapment on the interfacial fluid flow

The evolution of the real contact area and free volume distribution under increasing external load determines such contact properties as friction, wear, adhesion, but also controls heat and mass transport in and through contact interfaces. Here we investigated the influence of fluid entrapment on the properties of realistic contact interfaces. We solved the problem of thin creeping fluid flow through the free volume formed by a solid with a representative surface roughness brought in contact with a rigid flat, see Fig. 1(a).

We developed an original monolithic approach for strong coupling of the mechanical contact with interfacial fluid flow [2], enabled automatic detection of non-simply connected contact patches and introduced the behavior of the pressurized fluid trapped in these zones into the finite-element framework. We showed that under high loads the account for the trapped fluid results in a significantly higher transmissivity of the rough contact interface than simpler models, which neglect the effect of hydrostatic pressure on deformation of the solid, especially close to the percolation limit.

References

[1] A.G. Shvarts and V.A. Yastrebov, Trapped fluid in contact interface, submitted, 2017. Preprint: arxiv.org/abs/1712.03954

[2] A.G. Shvarts and V.A. Yastrebov, Fluid flow across a wavy channel brought in contact, submitted, 2017. Preprint: arxiv.org/abs/1712.06312

Investigating Methods to Influence Particle Damper Nonlinearities using Coupled SPH-DEM Simulations

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Summary: One of the main disadvantages of particle dampers is that their performance is highly sensitive to vibration amplitude changes. In this work various options to control these undesirable nonlinearities, such as introducing a liquid or using complex container geometries, are investigated. In order to quantify the influence of such changes and to better understand the various dissipation mechanisms, coupled SPH-DEM simulations are performed and validated using experimental data.

In recent times, due to their relatively simple design and largely temperature independent behavior, particle dampers (PD) have increasingly found applications in attenuating vibrations in complex mechanical systems [1]. Since the effectiveness of a PD depends on the motion of arbitrary particles relative to each other and to its boundaries, so called particle methods, such as the Discrete Element Method (DEM), are preferred to adequately model the complex dynamics of a PD [2]. In this work, such an approach will be used to model PDs. In order to validate the simulation models a physical experiment is set up. This experiment consists of a PD, in this case a cylindrical acrylic container (diameter $D = 50$ mm and variable length $L = 20 - 100$ mm) filled with spherical steel or glass particles, which is mounted on a vertical leaf-spring (effective stiffness $k_{\text{eff}} = 300$ N/m, effective oscillation frequency $\omega_{\text{eff}} = 5.5$ Hz). The leaf-spring is displaced from its equilibrium position and its free oscillation behavior is analyzed for various particle sizes, fill-ratios and enclosure geometries. The insights gained during these experiments were utilized to identify and validate the discrete element models.

In agreement with the results in [3], initial observations on the amplitude decay suggest the existence of two distinctively different damping phases. The first phase (collisional phase) where the amplitude decays rapidly, and the second phase (non-collisional phase) where the amplitude decays slowly compared to the first phase. In general, the agreement between experiment and simulation for the vertical leaf-spring oscillator were found to be excellent, considering the model assumptions made (no air-resistance, only one principal oscillation direction). Simulation results suggests that during the first phase the energy dissipated due to collisions is quantitatively larger in comparison to that dissipated due to inter-particle friction. Moreover, for a given enclosure volume it was found that there exists an optimal fill-ratio at which maximum damping performance is achieved.

Visual observations reveal that the high damping during the collisional phase can be attributed to the collective movement of the particles against the enclosure wall. On the other hand, lower damping during the non-collisional phase is due to the absence of relative movement between particles. In addition to these two phases, the particle motion exhibits a transition phase (directly after the collisional phase), where the particle layers slide (in a viscous sense) over each other before loosing their kinetic energy and coming to an abrupt halt. Even though significant efforts have been made to understand the dynamics during the initial collisional phase [1], very little efforts have been taken to understand or influence the intermediate transition phase. It is expected, that deliberately extending this transition phase, even just by a small amount, can help reduce the undesirably high amplitude dependency of PDs. This transition phase (or viscous phase) could be influenced by deliberately introducing a liquid or by using complex enclosure geometries in the PD. In this work both of these options were investigated.

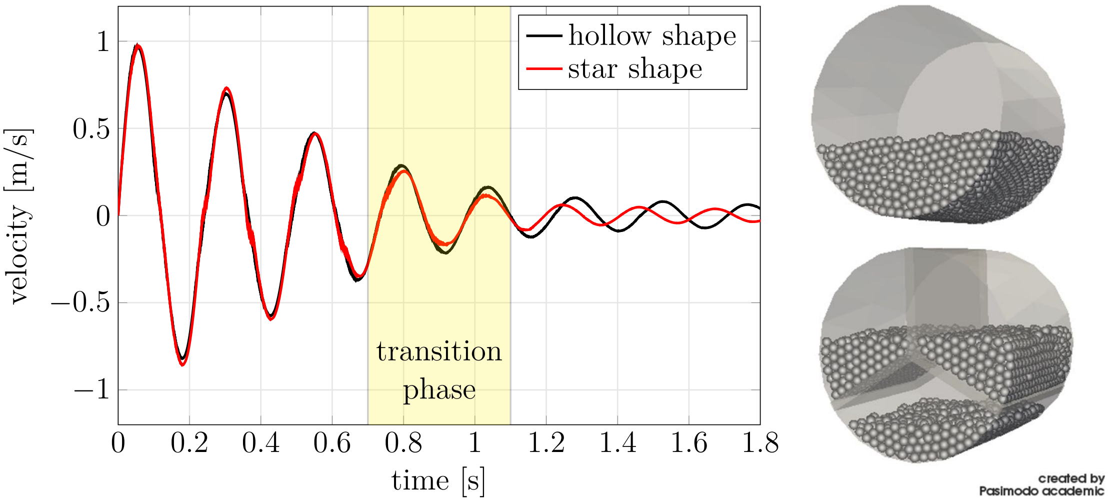

Figure 1: Simulation results comparing the damping performance of two different container geometries (left). Star shaped internal geometry (bottom-right) and hollow geometry (top-right).

The simulation results in Figure 1 show that complex internal geometries indeed have a profound impact on the amplitude behavior of PDs. Especially during the transition phase the complex star shaped container geometry seems to perform better, by keeping the particles in relative motion for a longer time span, compared to the simple hollow shape. Moreover, during the transition phase the particle layers exhibit a sliding motion. This suggests that inter-particle friction might play a more prominent role during this phase. As a next step introducing liquid in the particle mix, the interface friction can be influenced, thereby influencing the amount of energy dissipated during the transition phase. In order to describe the fluid motion simulatively, the Smoothed Particle Hydrodynamics (SPH) method is used. By using a coupled SPH-DEM approach as in [4], it is shown that complex interactions, especially during the viscous phase, between particle-fillings, liquid, and enclosure geometry can be sufficiently well modeled. In order to gain further insights, numerical studies for various particle and liquid mixtures were carried out.

References

[1] Heckel, M.; Sack, A.; Kollmer J.E.; Pöschel, T.: Granular Dampers for the Reduction of Vibrations of an Oscillatory Saw, *Physica A: Statistical Mechanics and its Applications*, Vol. 391, pp. 4442–4447, 2012.

[2] Wong, C.X.; Daniel, M.C.; Rongong, J.A.: Energy Dissipation Prediction of Particle Dampers, *Journal of Sound and Vibration*, Vol. 319, pp. 91–118, 2009.

[3] Marhadi, K.S.; Kinra, V.K.: Particle Impact Damping: Effect of Mass Ratio, Material, and Shape, *Journal of Sound and Vibration*, Vol. 283, pp. 433–448, 2005.

[4] Gnanasambandham, C.; Eberhard, P.: Predicting the Influence of an Added Liquid in a Particle Damper using Coupled SPH and Discrete Element Method, *PAMM*, 2017. in press

SESSION 11

Random response of a sliding block with Coulomb friction

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Summary: the random response of a sliding block with Coulomb friction subjected to a stationary zero-mean Gaussian white noise is investigated. The Fokker-Planck-Kolmogorov equation in the transition probability density function is solved analytically.

Introduction

The dynamics of a rigid body sliding on a rough surface and excited by a stochastic agency has attracted the attention of many scholars. If this is a Gaussian white noise stochastic process, from a theoretical point of view, the statistical characterization of the motion can be obtained by solving the associated Fokker-Planck-Kolmogorov equation (FPK) in the transition probability density function (PDF) of the response. However, in the presence of a nonlinear damping term the FPK equation is not analytically solvable unless the damping term is a function of the mechanical energy of the system.

Here, we consider the case of a rigid block that slides on a surface with Coulomb (dry) friction in the absence of a restoring force and of any other damping mechanism. The second order motion differential equation is transformed into a first order one. In this way, the FPK equation is analytically solved. The evolution of the transient PDF of the velocity is shown.

Position of the problem

The motion equation of a rigid body sliding on a rough surface with Coulomb friction and no restoring force is written as

$$m\ddot{X}(t) = -\mu g m \,\text{signum}(\dot{X}) + \sigma W(t), \tag{1}$$

where m is the mass of the body, μ is the friction coefficient, g is the gravity acceleration, σ is a positive parameter representing the strength of the excitation, signum(•) is the sign function, and $W(t)$ is a stationary zero-mean Gaussian white noise process having autocorrelation function $E[W(t)W(t+\tau)] = \delta(\tau)$.

Keeping into account that $V(t) = \dot{X}(t)$, with the aid of the transformation $V = \sigma Y / m$, Eq. (1) is recast as

$$\dot{Y}(t) = -\kappa \,\text{signum}(Y) + W(t), \tag{2}$$

where $\kappa = \mu g m / \sigma$. The Fokker-Planck-Kolmogorov equation associated with Eq. (2) is

$$\frac{\partial p}{\partial t} = \kappa \frac{\partial}{\partial y}\left[\text{signum}(y)\, p\right] + \frac{1}{2}\frac{\partial^2 p}{\partial y^2}, \tag{3}$$

where $p = p_Y\left(y,t \,\middle|\, y_0,t_0\right)$ is the transition probability density function of $Y(t)$. The initial condition is $p_Y(y_0,t_0) = \delta(y - y_0)$, while the boundary conditions impose vanishing probability flux at $\pm\infty$.

The steady-state probability density function is found as

$$p_{Y,st}(y) = \kappa \exp\left(-2\kappa |y|\right). \tag{4}$$

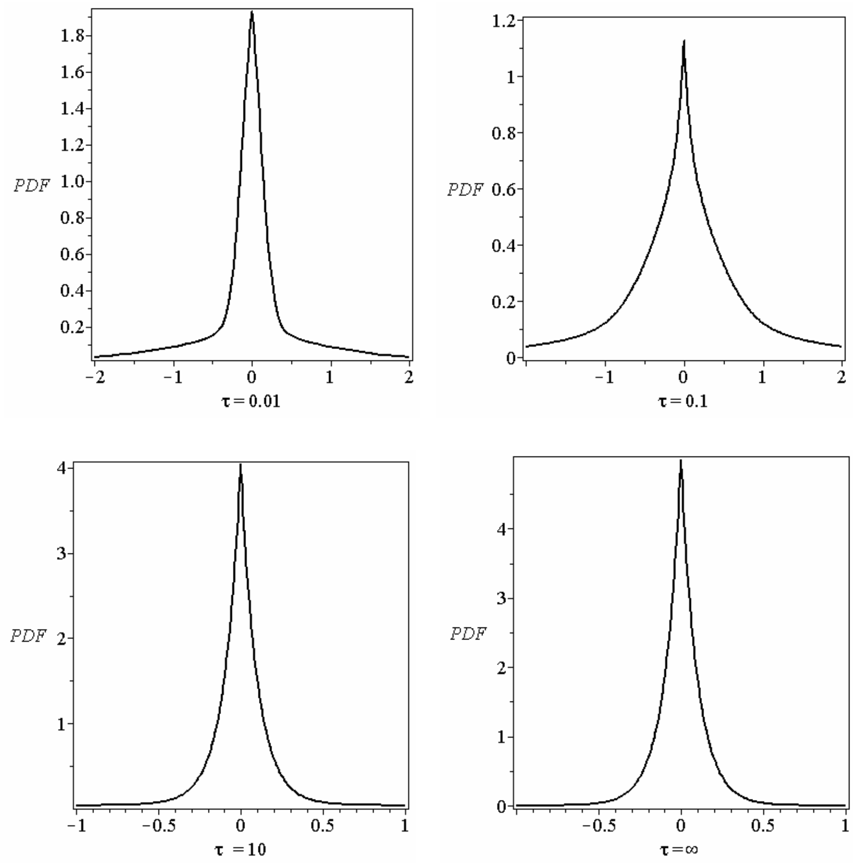

Figure 1: plots of the transition PDF for different times $\left(\tau = 2\kappa^2 m^2 t\right)$.

In order to determine the transition probability density function, the ansatz of separate variables is made, that is $p_Y\left(y,t\,|\,y_0,t_0\right) = C\,T(t)\,\varphi(x)$, which gives rise to the Sturm-Liouville problem

$$\varphi''(x) + 2\kappa\varphi'(x) + \lambda\varphi(x) = 0\,. \tag{5}$$

The expression of the transition probability density function is not given because of space limitation.

Results and conclusions

The transition probability density function is plotted in Fig. 1 for the following values of the parameters: $\mu = 0.05, m = 1, \sigma = 0.01$. These values yield $\kappa = 5$. In the plots the abscissa is $2\kappa m^2 y$ and the ordinate, the time, is $\tau = 2\kappa^2 m^2 t$. y_0 is zero.

Because of the sign function for all times the probability density function has a cusp in the origin. As the initial probability density function is a Dirac delta, that is a spike, for short times it is peaked but with pronounced tails (first plot). For intermediate times the largest value diminishes (second plot), before increasing again. In the steady state, the probability mass is concentrated around the origin.

Knowing the transition probability density function allows computing the time evolution of the mean square value, the correlation function, and the power spectral density: these quantities are not reported because of space limitation.

Evolution of real contact area under shear and the value of static friction of soft materials

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Summary: Direct measurements on dry rough contacts involving soft materials show that the real contact area significantly decreases under shear. All data obey a single reduction law enabling excellent predictions of the static friction force. The reduction rate of individual contacts obeys a scaling law valid from micrometric to millimetric junctions.

Introduction

The frictional properties of a rough contact interface are controlled by its area of real contact. In particular, the real contact area is proportional to the normal load, slowly increases at rest through aging, and drops at slip inception. Accounting for these three dependencies together has been a major success in the science of friction because it provides a consistent picture of the physical mechanisms underlying the ubiquitous state-and-rate friction law [1], which is obeyed by multicontacts in a variety of materials.

However, a series of experimental observations suggest that the picture may not be fully comprehensive yet. These observations, made on smooth contacts, have repeatedly indicated that the area of apparent contact depends on the value of the tangential load applied to the interface. In particular, the area of apparent contact decreases when smooth elastomer-based sphere/plane contacts are increasingly sheared [2,3]. It is therefore tempting to hypothesize that not only smooth but also rough interfaces have a dependence of their contact area on the tangential load. To test this hypothesis, we carried out experiments to monitor, in multicontacts involving elastomers or human fingertips, the evolution of the area of real contact when the tangential load is increased from 0 to macroscopic sliding [4].

Methods summary

A slider made of a flat, smooth bare glass plate is placed in frictional contact onto a rough elastomer block of polydimethylsiloxane (PDMS). The slider is driven horizontally by a motorized linear stage moving at constant velocity. The normal load is applied using dead weights and the tangential force is monitored as the slider is driven toward macroscopic sliding. Noninvasive, in situ contact imaging is performed simultaneously and allows to accurately measure the evolution of the area of real contact as the interface is increasingly sheared and starts to slide macroscopically.

Results summary

The area of real contact at the static friction peak is found to be proportional to the corresponding static friction force. This proportionality defines a frictional shear strength of the interface which varies when various coatings are applied on the glass surface.

Well before static friction, the area of real contact is found to decrease significantly, up to 30%, as the tangential load is increased towards macroscopic sliding. The decrease is found to be quadratic as a function of the tangential load, for all contacts tested.

The two above-mentioned laws allow one to predict the value of the static friction force, which is found in excellent agreement with the measurements.

Those results, obtained using PDMS, are found to also apply to human fingertip contacts.

We then compared the area-reduction rate of individual micro-junctions within rough contacts with that of smooth sphere/plane contacts. We found that it follows a well-defined scaling law, from micrometer-sized micro-contacts to millimeter-sized sphere/plane contacts.

Discussion

The reduction of apparent contact area under shear observed in the literature was interpreted either as a result of viscoelasticity or adhesion. In our experiments on the real contact area [4], we could rule out viscoelasticity, leaving open an adhesion-based origin.

While the adhesive friction models from the literature treat the case of axisymmetric smooth contacts [2,3], both assumptions are broken in our experiments.

Our results indicate that, as soon as shear is applied to a rough contact, its area of real contact can vary significantly. This calls for improved rough contact models incorporating shear stimuli in addition to the classical normal loading conditions.

References

[1] T. Baumberger and C. Caroli, Solid friction from stick–slip down to pinning and aging, *Adv. Phys.*, 55:279–348, 2006.

[2] A.R. Savkoor and G.A.D. Briggs, The effect of tangential force on the contact of elastic solids in adhesion, *Proc. R. Soc. Lond. A*, 356:103–114, 1977.

[3] J.F. Waters and P.R. Guduru, Mode-mixity-dependent adhesive contact of a sphere on a plane surface, *Proc. R. Soc. A*, 466:1303–1325, 2009.

[4] R. Sahli, G. Pallares, C. Ducottet, I.E. Ben Ali, S. Al Akhrass, M. Guibert and J. Scheibert, Evolution of real contact area under shear and the value of static friction of soft materials, *PNAS*, 115:471–476, 2018.

Experimental validation of three-points bending test with various contact approaches

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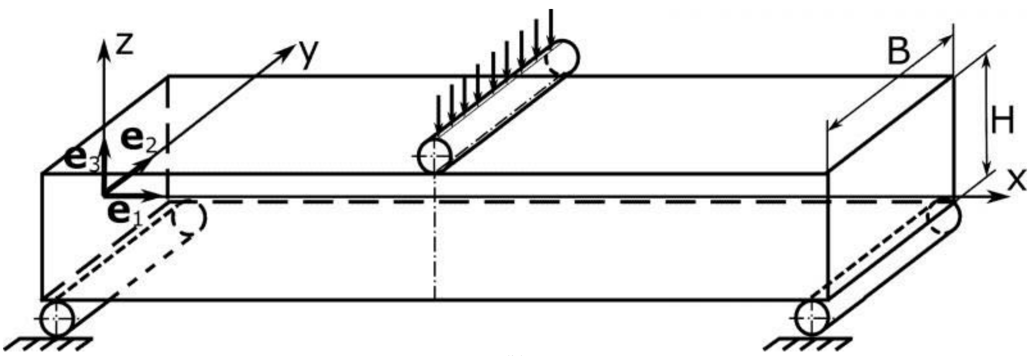
Summary: The three-point bending test is widely used standard test for multilayered structures. Due to geometrical sizes of cylindrical supports comparable to tested beam, this test can be a perfect example to validate various contact algorithms (Lagrange or penalty) as well as to answer the question if we need to discretize the supporting cylinders.

The widely used engineering validation of the three-point bending test does not include the contact modeling and assumes that only point forces are applied. In the real size test which necessary to validate the composite panel used in aircraft industry, the sizes of cylindrical supports, see Fig. 1. are in a range of the sizes of specimen. Two sets of specimens with width B=9 mm. and height H=9 mm are tested: the long with full length L=170 mm and the short ones with L=50 mm. The most important for contact is that the diameter of supporting as well as loading cylinders is d=10 mm. The geometry of contact is definitely influencing the results.



(b)

(a)

Figure 1: Geometry of the three-bending test. Diameter of loading (supporting) cylinders is *d*.

The three layers panel is made of carbon orthotropic composite HSE 180 REM with a softer core (E_1=131 GPa, E_2= E_3=5.9 GPa, ν_{12}= ν_{13}=0.29). Due to soft core the whole specimen is experiencing highly nonlinear physical behavior especially for shear stresses, even for small deflections. Identification of the full material model is a quite a cumbersome process, because it will require the modeling on a micro level with identification of the carbon filaments and filling. Another procedure, which is proper for engineering needs as well as very fast for validations: the directional modulus are measured in 1D standard extension test, while the shear modulus are G_{12}= G_{13} are identified via the three-points bending test. The measured result for the core is showing the softening behavior during loading until damage, see Fig. 2. Such an effect is visually illustrated by high deformation of the core. Long lasting investigations, both experimental, and theoretical ones [1-12] allow to develop the following strategy:

1) The material model can be assumed as physically non-linear with a softening core validated in the experiment. This allows to avoid the highly sophisticated multi-scalar models for the validations of micro parameters. The model exploiting anisotropy and tabulated via experiments softening behavior is fully sufficient for further needs.

2) The contact behavior between supporting and loading cylinders can be modeled as embedded Hertz stresses between the plate and supporting (loading) cylinders.

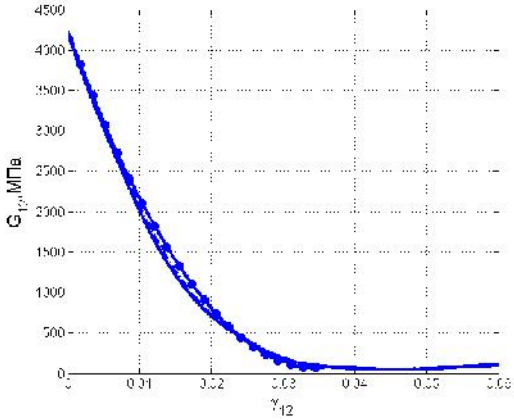


Figure 2: Experimental results: softening of the core for shear stresses.

The current investigation is aimed now to validate the contact behavior using the computational contact mechanics algorithms only. Namely the contact stresses are not imbedded as in 2), but contact is directly modeled via a various sets of contact algorithm using both Lagrange and penalty methods as well as specially developed for rigid surfaces [13], [14].

References

[1] V.N. Paimushin, V.I. Shalashilin, *Doklady Akademii Nauk* 2004, 396 (4), 492–495.

[2] V.N. Paimushin, V.I. Shalashilin, *Journal of Applied Mathematics and Mechanics* 2005, 69 (5), 773–791.

[3] V.N. Paimushin, V.I. Shalashilin, *Journal of Applied Mathematics and Mechanics* 2006, 70 (5), 91–101.

[5] V.N. Paimushin, *Mechanics of Solids* 2007, 42 (3), 437–446.

[6] V.N. Paimushin, *Journal of Applied Mathematics and Mechanics* 2007, 71 (5), 772–805.

[7] V.N. Paimushin, N.V. Polyakova, *Journal of Applied Mathematics and Mechanics* 2009, 73 (2), 220–236.

[8] V.N. Paimushin, N.V. Polyakova, *Journal of Applied Mathematics and Mechanics* 2011, 75 (6), 691–699.

[9] R.A. Kayumov, S.A. Lukankin, V.N. Paimushin, Kholmogorov S.A., *Uchenye Zapiski Kazanskogo Universiteta. Seriya Fiziko-Matematicheskie Nauki*, 2015, 157 (4), 112–132.

[10] RD 50-675-88, *Calculations and strength tests in engineering. Composite materials Methods for interlaminar shear tests*, Report, 1989.

[11] D790-03, *Standard Test Methods for Flexural Properties of Unreinforced and Reinforced Plastics and Electrical Insulating Materials*, Report, vol. 08.01, Plastics, ASTM International, West Conshohocken, PA, 2005.

[12] I.B. Badriev, V.V. Banderov, M.V. Makarov, V.N. Paimushin, *Applied Mathematical Sciences* 2015, 9 (78), 3887–3895.

[13] Konyukhov A, Schweizerhof K. *Computational Contact Mechanics. Geometrically Exact Theory for Arbitrary Shaped Bodies*. Springer; 2013

[14] Konyukhov A, Schweizerhof K. *On some aspects for contact with rigid surfaces: surface-to-rigid surface and curves-to-rigid surface algorithms*. Comput Meth Appl Mech Eng. 2015;283:74-105.

An experimental study of traction characteristics at wheel flange/rail gauge corner interface

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Summary: The twin-disk frictional test was carried out to investigate the mechanism of flange climb derailment. It was found that there is a peak value of the wheel/rail traction coefficient during running-in period. And the existence of subsurface plastic layer just below the contact position to the increase of traction characteristics was indicated.

Introduction

Since the frictional condition between railway wheel and rail has an important role in the transmission of driving force and braking force, it should be kept at a high level to secure the appropriate acceleration performance and braking distance. On the other hand, it is known that high traction coefficient in lateral direction at sharp curves increases the risk of wheel climb derailment occurring [1] and several derailments have occurred within relatively soon after the re-profiling of the wheel [2]. The aim of this work was to investigate traction characteristics of wheel flange/rail gauge corner interface during running-in period to understand the mechanism of the wheel climb derailment.

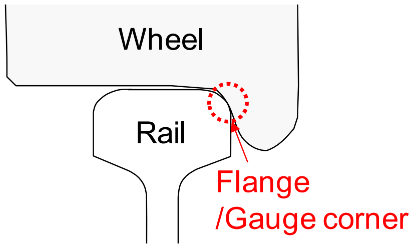

Figure 1: Wheel flange/rail gauge contact position

Methodology

Figure 2 shows a set of wheel disk and rail disks after attachment on the experimental apparatus. This apparatus can simulate the actual contact condition between wheel and rail using two servomotors. Furthermore, to simulate the wheel flange/rail gauge corner contact, the wheel disk was made to be conic in shape and the rail disk was made to be a “bowl” shape and the traction force in lateral direction was evaluated.

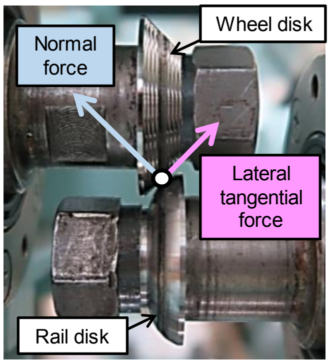

Figure 2: A set of wheel disk and rail disks after attachment on the experimental apparatus

Results

Figure 3 shows the change of traction coefficient in the lateral direction with running time. there is a peak value of the traction coefficient between wheel flange and rail gauge corner at running-in period.

Figures 4 shows the change of surface topography of wheel disk and rail disk for different stage. Here, the surface texture for “Stage II” were obtained by discontinuing a test after confirming the reproducibility of the change of traction coefficient (4th test in Figure 3).

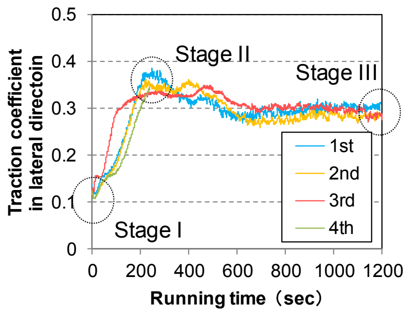

Figure 3: Change of traction coefficient in lateral direction with running time

It is clearly found that the surface deformation was increased with the progress of stage and the direction of plastic flow corresponded to that of lateral tangential force. And the roughness at Stage II kept the minimum value of Stage I.

Figures 5 shows the metallic structures measured by optical and scanning electron microscope. It is clearly found that the plastic deformation was increased with the progress of stages for all kinds of disks. Especially the subsurface plastic layer was found at Stage II. It is thought that the existence of this layer increases the proportion of bulk hardness to shear surface strength and the traction coefficient at Stage II.

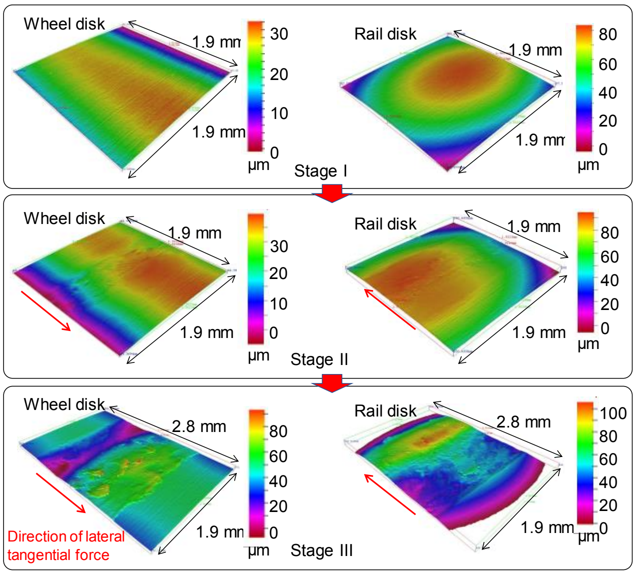

Figure 4: Change of surface topography of wheel disk and rail disk for different stage.

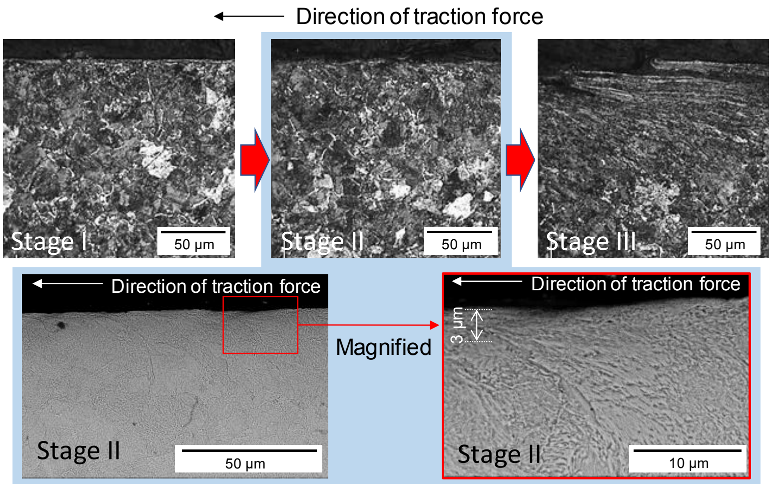

Figure 5: Metallic structures measured by optical and scanning electron microscope.

Conclusions

The traction characteristics of wheel flange/rail gauge corner interface was investigated. The test was carried out using twin-disk test machine. It was found that there is a peak value of the wheel/rail traction coefficient during running-in period. Surface conditions at the characteristic stages were also investigated. As a result, the subsurface plastic layer was found at Stage II. It is thought that the existence of this layer increases the proportion of bulk hardness to shear surface strength and the traction coefficient. These findings might inform rail service providers about optimal wheel profiling methods and surface treatments to control the frictional condition between wheel and rail and reduce the likelihood of flange climb derailment. The results of parametric tests and interface model will be presented to explain these phenomenon in the presentation.

References

[1] A. Matsumoto, et al., A new measuring method of wheel–rail contact forces and related considerations. *Wear*, 265(9), 1518-1525, 2008.

[2] A. Kataori, et al., The Influence of the Wheel/Rail Contact Point Condition on Friction Coefficient, Proceedings of 9th World Congress Railway Research, 2011.

Free vibration of functionally graded plates resting on elastic foundation

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University Hassiba Ben Bouali of Chlef.

Summary: the free vibration of functionally graded materials plates on an elastic foundation was investigated, based on the theory of FSDT with different shear factors. The position of the neutral surface for the FG plate is determined. The accuracy of the present solutions is verified by comparing the results obtained with the solutions found in literature.

Introduction

In recent years, functionally graded materials (FGMs) have gained considerable attention in many engineering applications. FGMs are considered as a potential structural material for future high-speed spacecraft and power generation industries.[1] Extensive studies have been conducted to analyze the behavior of structures in advanced composite materials. Vel and Batra [2], presented a three dimensional exact solution for free and forced vibrations of simply supported functionally graded rectangular plates. Huang et al. [3] investigated the benchmark solutions for thick FGPs resting on Winkler Pasternak elastic foundations using the 3D elasticity theory. Lu et al. [4], based on the 3D elasticity theory, studied the free vibration analysis of FG thick plates resting on elastic foundation.

Theoretical formulations

The material nonhomogeneous properties of FG plate P, as a function of thickness coordinate. The position of the neutral surface of the FG plate is determined to satisfy the first moment with respect to Young's modulus being zero as follows[5]

$$P(z) = P_M + (P_C - P_M)\left(\frac{z_{ns} + C}{h} + \frac{1}{2}\right)^k, \qquad C = \frac{\int_{h/2}^{h/2} E(z_{ms}) z_{ms}\, dz_{ms}}{\int_{h/2}^{h/2} E(z_{ms})\, dz_{ms}}$$

The governing equations of equilibrium can be derived by using the principle of virtual displacements. The equilibrium equations associated with the present FSDT are

$$\delta u_0: \quad \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_1 \ddot{u}_0 + I_2 \ddot{\varphi}_x$$

$$\delta v_0: \quad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_1 \ddot{v}_0 + I_2 \ddot{\varphi}_y$$

$$\delta \varphi_x: \quad \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_{xz} = I_2 \ddot{u}_0 + I_3 \ddot{\varphi}_x$$

$$\delta \varphi: \quad \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_{yz} = I_2 \ddot{v}_0 + I_3 \ddot{\varphi}_y$$

$$\delta w: \quad \frac{\partial Q_{xz}}{\partial x} + \frac{\partial Q_{yz}}{\partial y} - f_e = I_1 \ddot{w}$$

The displacement functions that satisfy the equations of boundary conditions are selected as the following Fourier series:

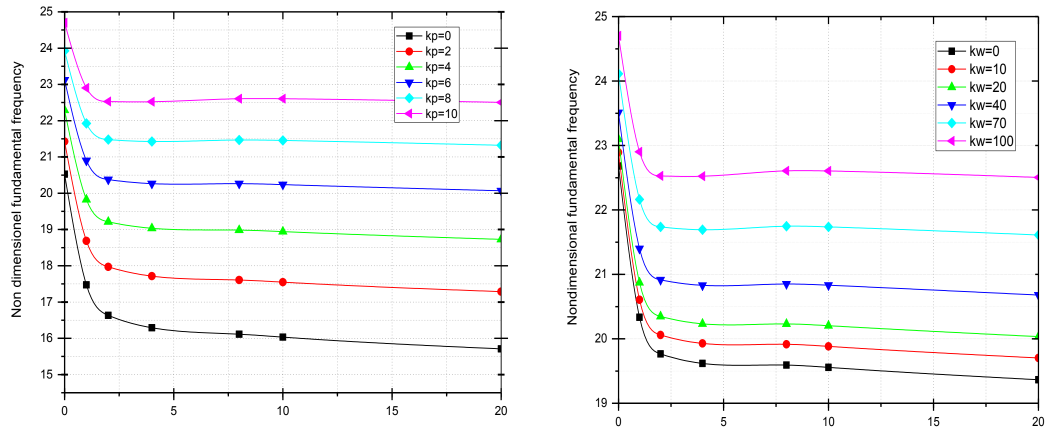
$$\begin{Bmatrix} u_0 \\ v_0 \\ \varphi_x \\ \varphi_y \\ w \end{Bmatrix} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn} \cos(\lambda x) \sin(\mu y) e^{i\omega t} \\ V_{mn} \sin(\lambda x) \cos(\mu y) e^{i\omega t} \\ W_{mn} \sin(\lambda x) \sin(\mu y) e^{i\omega t} \\ X_{mn} \sin(\lambda x) \sin(\mu y) e^{i\omega t} \\ Y_{mn} \sin(\lambda x) \sin(\mu y) e^{i\omega t} \end{Bmatrix}$$

Results

Example: The Poisson's ratio is fixed at m = 0.3. Comparisons are made with available solutions in literature. The material properties used in the present study are: (13)

Metal (Aluminium, Al): EM = 70 GPa ; ρc = 2702 kg/m3.

Ceramic (Alumina, Al2O3): EC = 380 GPa; ρm = 3800 kg/m3



(a) (b)

Figure 1: Soft-EHL problem: The effect of the power law index p on no dimensional fundamental frequency $\omega = a^2 \sqrt{\rho h / D}$ of FG plates on the elastic foundation: a Kp = 10; b Kw = 100.

Table 1: Formatting used in paragraph headings.

h/a	kw, kp	Akhavan et al[6].	Ait Atmane et al[7].	present
	0, 0	19.084	19.0658	19.7139
0.1	100, 10	25.6368	25.6236	26.1094
	1000, 100	57.3969	57.3923	57.6110
	0, 0	17.5055	17.4531	18.0425
0.2	100, 10	24.3074	24.2728	24.7004
	1000, 100	56.0359	56.0311	56.2259

References

[1] A.M. Zenkour, Generalized shear deformation theory for bending analysis of functionally graded plates, Applied Mathematical Modelling 30(1) (2006) 67-84.

[2] S.S. Vel, R.C. Batra, Three-dimensional exact solution for the vibration of functionally graded rectangular plates, Journal of Sound and Vibration 272(3) (2004) 703-730..

[3] Z.Y. Huang, C.F. Lü, W.Q. Chen, Benchmark solutions for functionally graded thick plates resting on Winkler–Pasternak elastic foundations, Composite Structures 85(2) (2008) 95-104.

[4] C.F. Lü, C.W. Lim, W.Q. Chen, Exact Solutions for Free Vibrations of Functionally Graded Thick Plates on Elastic Foundations, Mechanics of Advanced Materials and Structures 16(8) (2009) 576-584.

[5] D.-G. Zhang, Y.-H. Zhou, A theoretical analysis of FGM thin plates based on physical neutral surface, Computational Materials Science 44(2) (2008) 716-720.

plates, International Journal of Computational Methods 11(06) (2014) 1350082.

[6] H. Akhavan, S.H. Hashemi, H.R.D. Taher, A. Alibeigloo, S. Vahabi, Exact solutions for rectangular Mindlin plates under in-plane loads resting on Pasternak elastic foundation. Part II: Frequency analysis, Computational Materials Science 44(3) (2009) 951-961.

[7] H.A. Atmane, A. Tounsi, I. Mechab, Free vibration analysis of functionally graded plates resting on Winkler–Pasternak elastic foundations using a new shear deformation theory, International Journal of Mechanics and Materials in Design 6(2) (2010) 113-121.