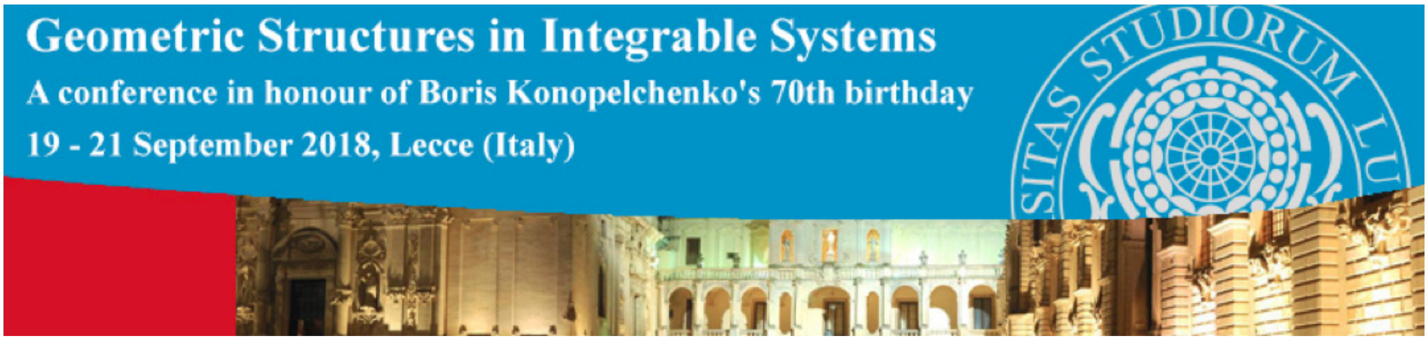

Abstracts booklet

(last updated: September 14, 2018)

UNIVERSITÀ DEL SALENTO

Dipartimento di Matematica e Fisica
"Ennio De Giorgi"

Dressing in the complex plane and geometry

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Abstract. I would like to review some old and new results concerning different modifications of dressing schemes in the complex plane and related integrable systems in geometry.

Self-phase modulation in optical fibres

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Abstract. Self-phase modulation (SPM), one of those very fascinating effects discovered in the early days of nonlinear optics [1], refers to the phenomenon by which an intense optical beam propagating in a Kerr medium (e.g., an optical fibre) induces through the nonlinearity of the medium a modulation of its phase that is proportional to its own intensity profile. For an input pulsed beam, the induced time-dependent phase change is associated with a modification of the optical spectrum, which depends on the initial frequency modulation (chirp) of the pulse electric field. If the pulse is initially Fourier-transform-limited or up-chirped, SPM leads to spectral broadening, whereas an initially down-chirped pulse is spectrally compressed by the effects of SPM [2]. For strong SPM, the optical spectrum can exhibit strong oscillations.

In this work, firstly we present a simple theoretical approach to predict the main features of optical spectra affected by SPM, which is based on regarding the optical spectrum modification as an interference effect [3]-[4]. The typical oscillatory character of the spectrum indeed originates from strong excursions of the instantaneous frequency, so that in general there are contributions from different times to the Fourier integral for a given frequency component. Depending on the exact frequency, these contributions may constructively add up or cancel each other. A two-wave interference model is found sufficient to describe the SPM-broadened spectra of initially transform-limited or up-chirped pulses, whereas a third wave should be included in the model for initially down-chirped pulses. Simple but fully tractable analytical formulae are derived, which accurately predict the positions of the extreme values of the spectra. The latter provide a more plausible measure of the spectrum extent than the approximate or root-mean-square expressions for the spectral bandwidth that are commonly used.

Secondly, we discuss a simple technique to suppress undesirable SPM of optical pulses in fibre-optic systems, which is based on using an electro-optic phase modulator to impart the opposite phase to the pulses and, thus, emulates the use of a material with a negative nonlinear index of refraction. We present a proof-of-principle experiment demonstrating that for input pulses with standard intensity profiles, such as Gaussian or hyperbolic secant pulses, the use of a simple sinusoidal drive signal for the phase modulator with appropriate amplitude and frequency is sufficient to reduce the nonlinear spectrum broadening to a large degree, and to significantly enhance the spectral quality of the pulses while their temporal duration remains unaffected [5]. Further, we present an in-depth characterisation of the SPM mitigation by a sinusoidally time varying phase based on analytical results and numerical simulation of the governing equation [6]. We assess the effects of the initial pulse shape and duration on the effectiveness of the technique, and we highlight the differences between pre- and post-propagation compensation schemes.

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Polynomials with multiple zeros and solvable dynamical systems including models in the plane with polynomial interactions

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Abstract. The interplay among the time-evolution of the *coefficients* $y_m(t)$ and the *zeros* $x_n(t)$ of a generic time-dependent (monic) polynomial provides a convenient tool to identify certain classes of *algebraically solvable* dynamical systems. Recently this tool has been extended to the case of *nongeneric* polynomials characterized by the presence, for *all* time, of a *single double* zero; and subsequently significant progress has been made to extend this finding to the case of polynomials featuring a *single* zero of *arbitrary* multiplicity. In this talk we report an approach suitable to deal with the most general case, i. e. that of a *nongeneric* time-dependent polynomial with an *arbitrary* number of *zeros* each of which features, for all time, an *arbitrary* (time-independent) *multiplicity*. We then focus on the special case of a polynomial featuring only 2 different zeros (of *arbitrary* multiplicity) and, by using a recently introduced additional twist of this approach, we thereby identify new classes of *algebraically solvable* dynamical systems of the following type:

$$\dot{x}_n = P^{(n)}(x_1, x_2) \ , \quad n = 1, 2 \ ,$$

with $P^{(n)}(x_1, x_2)$ two *polynomials* in the two variables $x_1(t)$ and $x_2(t)$.

This is joint work with Farrin Payandeh.

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New periodic solutions of some two-dimensional integrable nonlinear equations via dibar-dressing method

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On integrability of dispersionless Hirota type equations

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Abstract. I will give a review of integrability of dispersionless Hirota type equations with the emphasis on two recent results in 3D and 4D, namely (1) A generic integrable dispersionless Hirota type equation in 3D is defined by an even genus 3 theta constant; (2) An integrable dispersionless Hirota type equation in 4D is necessarily of Monge-Ampere type.
The talk is based on previous work with F. Cléry, B. Doubrov, L. Hadjikos, K. Khusnutdinova, B. Kruglikov, V. Novikov, M. Pavlov.

NLS rogue waves: from finite-gap solutions to elementary approximation

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Abstract. One of the basic model for rogue waves generation in the focusing Nonlinear Schrodinger equation with special Cauchy data. We show that if the initial data is a small spatially periodic perturbation of the unstable constant background, and the number of unstable modes is not too large, then the problem admits a series of simplifications:
1. Generic periodic problem is approximated by the finite-gap one, where the number of gaps is equal to the number N of the unstable modes multiplied by two.
2. For all parameters of the finite-gap solutions very elementary explicit expressions (up to ϵ^2 corrections) in terms of the Cauchy data are provided.
3. The theta-series are well-approximated by finite sums of exponents (for different times different approximations are used), and at each time interval the solution is approximated by Akhmediev $n(t)$-soliton solutions ($n \leq N$ for all t).

On long weakly-nonlinear surface and internal waves with curvilinear fronts

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Abstract. In the first part (joint work with C. Klein, V.B. Matveev and A.O. Smirnov), I will discuss a version of the Kadomtsev-Petviashvili (KP) equation for surface gravity waves related to elliptic-cylindrical geometry. We showed that there exist non-trivial transformations

between different versions of the KP equation associated with the physical problem formulation and used them to obtain new classes of approximate solutions for the Euler equations with free surface and rigid bottom boundary conditions.
In the second part (joint work with Xizheng Zhang), I will consider long weakly-nonlinear ring waves in a stratified fluid in the presence of a depth-dependent parallel shear flow (e.g., oceanic internal waves generated in narrow straits and river-sea interaction zones). We showed that there exists a linear modal decomposition in the set of Euler equations describing the waves, and used it to derive a 2D cylindrical Korteweg - de Vries (cKdV)-type amplitude equation and to study the waves with curvilinear fronts.

Rational solutions and solitons for the KP hierarchy

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Abstract. It is well known that the Schur polynomials satisfy the Hirota bilinear equations of the KP hierarchy, and that each Schur polynomial can be parametrized by a unique Young diagram. We also know that the KP solitons (exponential solutions) can be parametrized by certain decomposition of the Grassmannians. In the talk, I will explain the connection between the rational solutions and the KP solitons in terms of the Young diagrams. More explicitly, I will show how one gets a rational solution from a KP soliton. I will also discuss a connection between quasi-periodic solutions (theta or sigma functions) and the KP solitons.

Quasi-Lagrangian integration for the hydrodynamic type systems

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Abstract. Mixed Lagrangian-Eulerian description is considered to integrate some systems of the hydrodynamic type including 2D Boussinesq equations for convection of ideal fluids and 2D Prandtl inviscid equation. This description for two-dimensional flows is very familiar to that developed for the 2D Benney equations by Zakharov [1] and for 2D ideal MHD [2]. The density ρ for the Boussinesq equations and the velocity component u parallel to the wall for the Prandtl equation, in the case of the constant pressure, play the role of Lagrangian coordinate and the second - Eulerian - variable coincides with the Cartesian coordinate x. In new variables the continuity equation transforms into a linear relation between streamfunction and the function $y = y(x, \rho.t)$ which yields the isoline of the constant density, $\rho = \text{const}$. For the inviscid Prandtl equation u stands instead of ρ. In its turn, the linear relation can be resolved by introducing the generating function θ. As a result, the Boussinesq system reduces to the only equation for θ. Derivative of the generating function with respect to x gives the mapping $y = y(x, \rho.t)$. Appearance of a singularity of the breaking type are defined from

vanishing of the mapping Jacobian, $J = y_\rho \to 0$. For the Boussinesq system we discuss two possible scenario of the singularity formation.
For inviscid Prandtl equation the generating function is easily obtained if the pressure at large distance from the wall tends to constant. In this case, a solution can be found in the implicit form by applying initial-boundary conditions. The found solution describes appearance of a singularity on the boundary which is probably connected with a separation point.
At the end of the talk we discuss the problem of integration for both 2D and 3D Euler equations by means of the so-called vortex line representation. Such description is applied to all hydrodynamic equations with the frozenness property. We show how this method allows one to study the three-dimensional integrable hydrodynamics [3] with the Hamiltonian $H = \int |\omega| d\mathbf{r}$ where ω is the vorticity. For this model we demonstrate appearance of breaking for continuously distributed vortex lines.

The work was supported by the Russian Science Foundation grant 14-22-00174 .

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Bi-Flat F-manifolds, complex reflection groups and integrable systems of conservation laws

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Abstract. We show that bi-flat F -manifolds can be interpreted as natural geometrical structures encoding the almost duality for Frobenius manifolds without metric. Using this framework, we extend Dubrovin's duality between orbit spaces of Coxeter groups and Veselov's v-systems, to the orbit spaces of exceptional well-generated complex reflection groups of rank 2 and 3. We finally discuss some applications to integrable systems of conservation laws.

The talk is based on joint works with A. Arsie.

In the wake of Jacobi and Levi Civita

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Abstract. In 1843 Jacobi solved the geodesic flow on a triaxial ellipsoid by means of the elliptic coordinates in R3. Seventy years later Tullio Levi Civita explained analytically the reasons for the success of the method of Jacobi by introducing his well-known "separability conditions" . In my talk I will reconsider the work of Jacobi and Levi Civita. The purpose is to present a geometric interpretation of their ideas which may serve as a bridge with the modern theory of integrable systems.

Triangle geometry of spin states and nonlinear superposition of probabilities describing these states

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Abstract. On the spin-1/2 example, we demonstrate that quantum states can be described by standard probability distributions, which contain the same information that the wave function and the density matrix do. Within the framework of these approach, called for spin-1/2 the quantum suprematism representation, the probability distributions are illustrated by simplex or triangle geometry or by the *Triada of Malevich's Squares* associated with the triangles and new quantum relations for areas of the squares are obtained. The superposition principle for spin states and quantum interference phenomenon are expressed as an explicit new nonlinear addition rule for the probability distributions describing the quantum states and illustrated as the addition of two triadas of Malevich's squares. We discuss some analogy of the triangle geometry of spin-1/2 states related to the $O(3)$ group and the pyramid geometry related to the hydrogen-atom dynamical symmetry $O(4,2)$. The results are reviewed in [1-5].

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An Assessment of Lagrangian and Hamiltonian formalisms, classical and quantum

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Abstract. By starting with experimental data (trajectories) we consider how evolutionary equations, described by Lagrangians or Hamiltonians ,are arrived at. We argue that a natural construction provides us with implicit differential equations instead of explicit differential equations ,as provided by the Hamiltonian description.The transition from the implicit to the explicit equations shows very clearly how possible bi-Hamiltonian descriptions are arrived at.The geometrical view of the Lagrangian description shows that the Lagrangian describes implicit differential equations on the phase-space,and in many cases we get bi-Langrangian descriptions. By assuming that the classical description should arise from a suitable limit of the quantum description ,we analyze the corresponding ambiguity in the description of quantum systems,both in the Hilbert space formulation and the C*-Algebra formulation.

Dbar in the Sky

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Abstract. The study of dbar equations is one of the main subjects of Boris research. In particular he discovered a dispersionless version of dbar equations which is nicely related to the theory of quasiconformal mappings. I was very fortunate to collaborate with Boris in that work. Recently, I have become aware that similar tools are being used in the study of gravitational lensing: the deflection of light by distributions of matter.

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Recursion, Hamiltonian and preHamiltonian operators for differential-difference equations

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Abstract. We introduce preHamiltonian pairs of difference operators and study their connections with Nijenhuis and Hamiltonian operators [1]-[2].
We begin with a rigorous setup of the problem in terms of the Euclidean domain of difference operators and the corresponding skew field $\mathcal{Q}$ of *rational* (pseudo–difference) operators over a difference field F with a zero characteristic ground field (of constants) $\mathrm{k} \subset \mathrm{F}$ and the principal ideal ring $\mathcal{M}_n(\mathcal{Q})$ of matrix rational (pseudo–difference) operators. In particular, we give a criteria for a rational operator to be weakly non–local. A difference operator A is called preHamiltonian if its image is a Lie subalgebra with respect to the Lie bracket of evolutionary vector fields on F:

$$\forall a, b \in \mathrm{F},\ \exists c \in \mathrm{F}, \quad \text{such that} \quad [A(a), A(b)] = A(c).$$

Two preHamiltonian operators form a preHamiltonian pair if any k–linear combination of them is preHamiltonian. Then we show that preHamiltonian pairs naturally lead to Nijenhuis operators. Moreover, every Nijenhuis operator can be represented in terms of a preHamiltonian pair. This provides a systematic method to check whether a rational operator is Nijenhuis.
We show that a pseudo–difference Hamiltonian operator can be represented as a ratio AB^{-1} of two difference operators, where A is preHamiltonian and B satisfies certain identity. We show that a skew-symmetric difference operator is Hamiltonian if and only if it is preHamiltonian and satisfies simply verifiable conditions on its coefficients. We show that if H is a rational Hamiltonian operator, then to find a second Hamiltonian operator K compatible with H is the same as to find a preHamiltonian pair A and B such that $AB^{-1}H$ is skew-symmetric.
As an application, we construct a preHamiltonian pair, a Nijenhuis (recursion) and Hamiltonian operators for the differential-difference equation recently discovered by Adler & Postnikov. The Nijenhuis operator obtained is not weakly non-local. We prove that it generates an infinite hierarchy of local commuting symmetries. We also illustrate our theory on the well known examples including Volterra, modified Volterra, Toda, Ablowitz-Ladik, Kaup-Newell and Narita-Itoh-Bogayavlensky differential-difference equations.

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Hydrodynamic theory of phase transitions

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Abstract. A phase transition denotes a drastic change of state of a thermodynamic system due to a continuous change of parameters. Based on the theory of nonlinear conservation laws

and shock waves we describe an approach to phase transitions that relies on the solution of Maxwell relations. This theory provides an exact mathematical description of discontinuities of order parameters and phase transitions via nonlinear integrable PDEs. It allows to classify universal classes of equations of state and interpret the occurrence of critical points in terms of the dynamics of nonlinear shock wave fronts. The approach is shown at work in the case of mean field magnetic and fluid models, nematic liquid crystals and random graphs.

Tropical limit of matrix Boussinesq solitons

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Abstract. Soliton solutions of the $m \times n$ matrix "good" potential Boussinesq equation

$$\phi_{tt} - 4\beta\phi_{xx} + \frac{1}{3}\phi_{xxxx} + 2\left(\phi_x K \phi_x\right)_x - 2\left(\phi_x K \phi_t - \phi_t K \phi_x\right) = 0\,,$$

where K is a constant $n \times m$ matrix and $\beta > 0$, are generated via a binary Darboux transformation. The dependent variable of the matrix Boussinesq equation is $u = 2\phi_x$.
The "tropical limit" of a soliton solution consists of a graph in two-dimensional space-time, together with values of the dependent variable along its segments. This associates with the wave solution a classical point particle picture, in which free "particles" (with internal degrees of freedom) interact at definite events (points in space-time). Whereas in case of matrix KdV solitons at such an event the matrix data are related by a Yang-Baxter map, such a map appears to be insufficient in the Boussinesq case to describe all possible soliton interactions in the tropical limit.
If K is a vector ($n = 1$ or $m = 1$), then the above equation is equivalent to a simple extension of the scalar Boussinesq equation. Whereas in the more general matrix case a Yang-Baxter map is nonlinear, in the vector case it becomes linear and is thus given by an R-matrix. The possibility to include this particularly simple but still insightful case is our main reason for considering the above K-modified equation.

This talk is based on joint work, *arXiv:1805.09711 [nlin.SI]*, with A. Dimakis and X.-M. Chen.

Lagrangian multiforms and quantum variational principle

F.W. Nijhoff

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Abstract. In 2009 Lobb and Nijhoff introduced the notion of Lagrangian multiforms, as a variational description of the key integrability notion of multidimensional consistency (MDC). We discuss some recent advances in this novel theory and also show how this theory is lifted to the quantum level in terms of quantum mechanical propagators. Some new results on universal Lagrangian structures will be mentioned as well. (This work is partly in collaboration with Sarah Lobb, Steven King, Duncan Sleigh and Vincent Caudrelier).

Integrability in 3D

V. Novikov

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Abstract. We consider the problem of detecting and classifying integrable partial differential (and difference) equations in 3D. Our approach is based on the observation that dispersionless limits of integrable systems in 3D possess infinitely many multi-phase solutions coming from the so-called hydrodynamic reductions. We consider a novel perturbative approach to the classification problem of dispersive equations. Based on the method of hydrodynamic reductions, we first classify integrable quasilinear systems which may (potentially) occur as dispersionless limits of soliton equations in 3D. To reconstruct dispersive deformations, we require that all hydrodynamic reductions of the dispersionless limit are inherited by the corresponding dispersive counterpart. This procedure leads to a complete list of integrable third and fifth order equations, which generalize the examples of Kadomtsev-Petviashvili, Veselov-Novikov and Harry Dym equations as well as integrable Davey-Stewartson type equations, some of which are apparently new. We also consider the problem of dispersive deformations on the Lax representation level and thus show that our approach allows starting from the dispersionless Lax representations to construct the fully dispersive Lax pairs representing the fully dispersive integrable systems.

We extend this approach to the fully discrete case. Based on the method of deformations of hydrodynamic reductions, we classify discrete 3D integrable Hirota-type equations within various particularly interesting subclasses as well as a number of classification results of scalar differential-difference integrable equations including that of the intermediate long wave and Toda type.

Hurwitz numbers and matrix integrals labeled with ribbon graphs

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Abstract. We consider products of complex random matrices from independent complex Ginibre ensembles. The products include complex random matrices $Z_i, Z_i^\dagger$, $i = 1, \ldots, n$, and $2n$ sources (these are the complex matrices C_i, C_i^*, $i = 1, \ldots, n$, which play the role of parameters). We consider collections of products $X_1, \ldots, X_F$, constainted by the property, that each of the matrices of the set $\{Z_i C_i, Z_i^\dagger C_i^*, i = 1, \ldots, n\}$ is included only once on the product $X = X_1 \cdots X_F$. It can be represented graphically as a collection of F polygons with a total number of edges $2n$, and the polygon with number a encodes the order of the matrices in X_a. The matrices Z_i and $Z_i^\dagger$ are distributed along the edges of this collection of polygons, and the sources are distributed at their vertices. The calculation of the expected values involves pairing the matrices Z_i and $Z_i^\dagger$. There is a standard procedure for constructing a $2D$ surface by paiwise gluing edges of polygons, this procedure results to a *ribbon graph* embedded in

the surface Σ_{E^*} of some Euler characteristic E^* (this graph also known as *embedded graph* or *fatgraph*). We propose a matrix model that generates spectral correlation functions for matrices X_a, $a = 1, \ldots, F$ in the Ginibre ensembles, which we call the matrix integral, labeled network chord diagram. We show that the spectral correlation functions generate Hurwitz numbers H_{E^*} that enumerate nonequivalent branched coverings of Σ_{E^*}. The role of sources is the generation of ramification profiles in branch points which are assigned to the vertices of the ribbon graph drawn on the base surface Σ_{E^*}. The role of coupling constants of our model is to generate ramification profiles in F additional branch points assigned to the faces of the ribbon graph (the faces of the 'triangulated' Σ_{E^*}). The Hurwitz numbers for Klein surfaces can also be obtained by a small modification of the model. To do this, we pair any of the source matrices (in that case presenting a hole on Σ_{E^*}) with the tau function, which we call Mobius one. The presented matrix models generate Hurwitz numbers for any given Euler characteristic of the base surface E^* and for any given set of ramification profiles.

Parabolic regularization of the gradient catastrophes for the Burgers-Hopf equation and Jordan chain

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Abstract. Non-standard parabolic regularization of gradient catastrophes for the Burgers-Hopf equation is proposed. It is based on the analysis of all (generic and higher order) gradient catastrophes and their step by step regularization by embedding the Burgers-Hopf equation into multi-component parabolic systems of quasilinear PDEs with the most degenerate Jordan block. Probabilistic realization of such procedure is presented. The complete regularization of the Burgers-Hopf equation is achieved by embedding it into the infinite parabolic Jordan chain. It is shown that the Burgers equation is a particular reduction of the Jordan chain. Gradient catastrophes for the parabolic Jordan systems are also studied.

Integrable Dispersive Effects in the Shallow Water with Vorticity

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Abstract. We consider a new hydrodynamic reduction (the so called waterbag reduction) of the Benney system, describing one-dimensional propagation of long surface waves in the fluid of finite depth. We describe integrable dispersive extensions of this model. Such differential-difference systems generalise the Manakov Vector Nonlinear Schroedinger Equation.

Darboux system: Liouville reduction and explicit solution

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Steklov Mathematial Institute and NRU Higher Shool of Eonomics, Moscow
Landau Institute for Theoretial Physis, Moscow, and Karachay Cherkessia State University, Karachaevsk

Abstract. Class of solutions of the Darboux system in R^3 that obey condition of factorization of the associated second order linear problem is introduced. It is proved that this reduction enables (local) solvability of the Darboux system, explicit solution of this problem is given for two kinds of dependent variables. Explicit representations for Lame coefficients and solution of the linear problem is given as well.

Inverse scattering transform and solitons for square matrix nonlinear Schrodinger equations

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Abstract. In this talk we discuss the Inverse Scattering Transform (IST) and soliton solutions for a class of matrix nonlinear Schrodinger-type systems whose reductions include two equations which model certain hyperfine spin F=1 spinor Bose-Einstein condensates, and two novel equations which were recently shown to be integrable, and which have applications in non-linear optics and 4-component fermionic condensates. In addition, the general behavior of the soliton solutions for all four reductions is analyzed in detail, and some novel solutions are presented.

Novel results on Superintegrable Systems

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Abstract. Having in mind the Taub-Nut system, we started by the very simple idea of investigating spectral problems solved in terms of polynomials that are orthogonal on a segment (a,b) rather than on the semi-line R_+ or on the full line. Of course the simplest example is provided by the Legendre polynomials, but we would like to have a model describing confinement in a curved space, and enjoying spherical, and then possibly hyperspherical, symmetry. According to our strategy, the model is exactly solvable by construction. The eigenfunctions are expressed in terms of the Ja- cobi orthogonal polynomials. The metric is singular for two values of the radial variables, the origin and a generic point $\xi > 0$.

Multi-Component Integrable Ermakov-Painlevé and Toda-Painlevé Systems

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Abstract. Novel integrable multi-component Ermakov-Painlevé and Toda-Painlevé systems are presented. The formalism incorporates recently introduced two-component Ermakov-Painlevé II and Ermakov-Painlevé IV systems as well as a multi-component Ermakov-Painlevé III system.

Exact rogue wave recurrence in integrable NLS type models. Regular behavior or blow up at finite time

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Abstract. We consider three basic focusing nonlinear Schroedinger (NLS) type models: the celebrated NLS equation, describing the amplitude modulation of weakly nonlinear quasi monochromatic waves, its integrable discretization, the Ablowitz-Ladik (AL) equation, and its integrable PT-symmetric analogue, and we show that a small periodic perturbation of the constant unstable background leads, under the hypothesis of a single unstable mode, to an exact recurrence of rogue waves (RWs), described in terms of elementary functions, whose free parameters, different at each appearance, are expressed in terms of the initial data through elementary functions. While the appearance of the RWs is described by regular functions for the NLS [1,2] and AL models, the so-called Akhmediev breather and its AL analogue, in the PT-NLS model, depending on the initial data, the RW appearance is described by two different exact solutions; the first one is regular in a certain range of parameters, while the other one gives always rise to blow up at finite time in points of space-time. Therefore, in the PT-NLS case, one obtains a space-time uniform description, in terms of elementary functions, of how a regular initial perturbation of the unstable background give rise to blow up at finite time [3]. These results are obtained here using elementary matched asymptotics techniques. Joint works with P. G. Grinevich and F. Coppini.

References

[1] P. G. Grinevich and P. M. Santini: "The finite gap method and the analytic description of the exact rogue wave recurrence in the periodic NLS Cauchy problem. 1", arXiv:1707.05659.

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[3] P. M. Santini, The first rogue wave appearance in the rogue wave periodic Cauchy problem for the PT-symmetric NLS: regular dynamics or blow up at finite time, Preprint 2018.

Integrable structure in discrete projective and Lie sphere differential geometries

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Abstract. Classical projective differential geometry and its Lie sphere geometric counterpart have proven to constitute a rich source of integrable systems. Based on a recently developed integrability-preserving discretisation procedure, we demonstrate that the same can be said about their natural discrete analogues. As a result, standard and novel discretisations of well-known and, perhaps, lesser-known integrable systems are obtained.

Recursion Operators for Multidimensional Integrable Systems

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Abstract. We present a construction of recursion operators for integrable multidimensional systems with isospectral Lax pairs written in terms of first-order scalar differential operators. Our approach is quite straightforward and can be readily applied using modern computer algebra software. The method is illustrated by several examples.

Geometry and Integrability in SL(2,Z) dynamics

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Abstract. Usual discrete dynamics can be considered as the action of the group of integers. What happens if we replace it by SL(2,Z)? I will discuss several important examples of SL(2,Z) actions and their deep relations with arithmetic and geometry.

The talk is based on joint work with K. Spalding.

Primitive potentials and bounded solutions of the KdV equation

(V. Zakharov and D. Zakharov)

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Abstract. We construct a broad class of bounded potentials of the one-dimensional Schrodinger operator that have the same spectral structure as periodic finite-gap potentials, but that are neither periodic nor quasi-periodic. Such potentials, which we call primitive, are non-uniquely parametrized by a pair of positive Holder continuous functions defined on the allowed bands. Primitive potentials are constructed as solutions of a system of singular integral equations, which can be efficiently solved numerically. Simulations show that these potentials can have a disordered structure. Primitive potentials generate a broad class of bounded non-vanishing solutions of the KdV hierarchy, and we interpret them as an example of integrable turbulence in the framework of the KdV equation.