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Talks

Rational degenerations of m-curves and totally positive Grassmannians

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In this talk I shall connect two areas of mathematics: the theory of totally positive Grassmannians and the rational degenerations of m-curves using the theory of the KP-2 equation. Thanks to recent papers by Kodama and Williams [3, 4] the relation between the class of line soliton solutions of KP-2 equation and the totally non-negative part of real finite dimensional Grassmannians is well established. On the other hand such soliton solutions may also obtained from the limit of regular real finite gap solutions of KP-2. Dubrovin and Natanzon [2] proved in 1988 that the algebro-geometric data of the regular real quasi-periodic solutions are associated to m-curves. We show how to associate to any point in the totally positive part of $Gr(N, M)$ the algebro-geometric data a la Krichever [5] for the corresponding line soliton solution, i.e. the rational degeneration of a regular m-curve of genus $g = N(M - N)$ and the divisor of poles of the associated KP wave-function. The results presented are in collaboration with P.G. Grinevich [1].

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Some tropical analogues of integrable equations in (2+1) dimensions

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We introduce some tropical analogues of integrable lattice equations, starting from tropicalization of compatibility or consistency conditions. Examples connected with KP hierarchy and Darboux equations are considered.

Parabolic similaritons in optical fibres

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Recent developments in nonlinear optics have brought to the fore of intensive research an interesting class of pulses with a parabolic intensity profile and a linear instantaneous frequency shift or chirp [1, 2]. Parabolic pulses propagate in optical fibres with normal group-velocity dispersion in a self-similar manner, holding certain relations (scaling) between pulse power, duration and chirp parameter, and can tolerate strong nonlinearity without distortion or wave breaking. These solutions, which have been dubbed similaritons, were demonstrated theoretically and experimentally in fiber amplifiers in 2000 [3]. Similaritons in fiber amplifiers are, along with solitons in passive fibres, the most well-known classes of nonlinear attractors for pulse propagation in optical fibre [3, 4], so they take on major fundamental importance. The unique properties of parabolic similaritons have stimulated numerous applications in nonlinear optics, ranging from ultrashort high-power pulse generation to highly coherent continuum sources and to optical nonlinear processing of telecommunication signals.

In this talk, we review the physics underlying the generation of parabolic similaritons as well as recent results obtained in a wide range of experimental configurations.

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Transformations from traveling wave solutions to non-traveling wave solutions of evolution equations

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Studying properties of evolution equations arising in different physical contexts commonly starts from assuming the traveling wave (TW) solution form which reduces the problem to an ordinary differential equation (ODE). A variety of direct methods for finding such solutions have been designed but usually there is no algorithmic way to proceed further from this stage. In the present study, a method, which allows constructing non-traveling wave solutions of an evolution equation from known traveling wave solutions, is developed and applied to some types of equations. The method represents a generalization of a direct method for defining solitary wave solutions of evolution equations developed and applied to the higher order KdV-type equations in [1], [2] which allowed identifying new types of solutions, such as the Generalized Kaup–Kupershmidt (GKK) solitons [1] and static solitons [2]. In the present study, the modified procedure of the method defines transformations from TW solutions of an evolution equation to more general solutions of the same equation.

The procedure, as applied to an evolution equation in (x, t) variables, is based on the Ansatz for solution in new variables with one of them being the traveling wave argument $\xi(x, t)$ and the second variable being a 'potential' $p(x, t) = \int u(x, t)dx$ where $u(x, t)$ is the unknown solution. Having the arbitrary functions of ξ contained in the Ansatz defined, the solution can be determined by solving the first order quasilinear partial differential equation for $p(x, t)$. The desired transformations are obtained if some specific forms of the Ansatz are assigned which results in that the functions of ξ contained in the Ansatz are expressed through solutions of ODEs for TW solutions of the evolution equation. Transformations to non-TW solutions are provided by the solutions of the quasilinear equation for $p(x, t)$.

The transformations can be naturally used for finding new solutions of a given equation. Having the TW solutions (for example, solitary wave solutions) defined in an explicit form, more general non-TW solutions can be also explicitly determined. However, the transformations are of interest in themselves as they can give insight into some general properties of the equations. Even in the widely studied case of the Korteweg– de Vries (KdV) equation, a new type of transformations, which convert one-soliton solutions into two-soliton solutions, is found. Such transformations can be used for construction of many-parameter families of evolution equations possessing two-soliton solutions, in a way similar

to that used in [3] for constructing families of equations possessing one-soliton solutions. It allows to classify equations admitting two-soliton solutions, which may, to some extent, be considered as candidates for integrable equations. Another example is the dual Sawada–Kotera (SK) and Kaup–Kupershmidt (KK) equations. Transformations from traveling wave solutions of the SK equations to non-traveling wave solutions of the KK equation and, vise versa, transformations from traveling wave equations of the KK equations to non-traveling wave solutions of the SK equation can be defined. Such transformations are found also for the mixed scaling weight KdV-KK and KdV-SK equations, with the KdV flow included, which naturally arise in physical problems as a result of extending an asymptotic expansion to higher orders.

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Finite-dimensional representations of shift operators, remarkable matrices and matrix functional equations

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In this talk I plan to report—to the extent time will permit—quite recent results. (i) The identification of $(N \times N)$-matrices providing *finite-dimensional representations* of two types of "shift" operators, $\check{\delta}(x)$ respectively $\hat{\delta}(y)$, acting as follows on functions $f(z)$ of the variable z, $\check{\delta}(x)\ \ f(z) = f(x\ z)$ respectively $\hat{\delta}(y)\ \ f(z) = f(z+y)$; representations which are *exact*—in a sense that shall be explained—in the functional space spanned by polynomials of degree less than (the *arbitrary* positive integer) N. [1] (ii). The identification of $(N \times N)$-matrices which are explicitly expressed in terms of N arbitrary numbers or in terms of the N zeros of named polynomials of degree N and which feature *remarkable properties*, such as eigenvalues which are *explicitly known* and have *Diophantine* characteristics. [1] (iii). The identification of *matrix functional equations*, such as, for instance, $\mathbf{G}(y)\ \ \mathbf{F}(x) = \mathbf{F}(x)\ \ \mathbf{G}(x\ y)$—where, here and below, $\mathbf{F}(x)$ respectively $\mathbf{G}(y)$ are $(N \times N)$-matrix-valued functions of the scalar variables x respectively y—and of a class of *nontrivial* solutions of these functional equations [2]; and likewise the pair of matrix functional equations $\mathbf{G}(x)\ \ \mathbf{F}(y) = \mathbf{G}(xy)$ and $\mathbf{G}(x)\ \ \mathbf{G}(y) = \mathbf{F}(y/x)$, which feature only the altogether trivial solutions $F(x) = G(y) = 0$ and $F(x) = G(y) = 1$ in the scalar $(N = 1)$ case, but possess nontrivial solutions in the matrix $(N > 1)$ case [3].

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Bihamiltonian cohomology and deformations of Poisson pencils of hydrodynamic type

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We review our recent results [1, 2, 3], in collaboration with H. Posthuma and S. Shadrin, on the computation of the bihamiltonian cohomologies of Poisson pencils of hydrodynamic type.

First we recall the necessary background and the main motivation for our work. We consider the problem of the deformation of the compatible Poisson brackets for the class of bihamiltonian integrable hierarchies of PDEs which admit hydrodynamic limit. In particular we consider the KdV case, the general scalar case and the generic case of semisimple Poisson pencils with n dependent variables. This deformation problem is governed by certain bihamiltonian cohomology groups.

Then we show how to compute the bihamiltonian cohomology groups in some of the cases considered above. In particular we show the vanishing of the third bihamiltonian cohomology in the generic n-dimensional case by introducing certain filtrations on a related differential complex from which the vanishing of the first page of the associated spectral sequence follows. It follows that the extension of an infinitesimal deformation to a full dispersive deformation is unobstructed.

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Poisson cohomology of scalar multidimensional Dubrovin–Novikov brackets

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This contribution presents some results in the classification of dispersive deformations of the multidimensional Dubrovin–Novikov (DN) Poisson brackets [1]. DN brackets are Poisson structures on the space of local functionals, where the densities are functions from a D–dimensional to a N–dimensional manifold. They were introduced as a generalization of the Poisson brackets of hydrodynamic type, as suitable structure to define $D+1$ dimensional Hamiltonian PDEs [2].

In analogy with the Poisson geometry of finite dimensional manifolds, we can regard DN brackets as being defined by a Poisson bivector on an infinite dimensional manifold; moreover, we can use such a bivector to define the Lichnerowicz cochain complex of (local) p-vectors. For the case $D = 1$, Ezra Getzler proved that all the positive cohomology groups of the complex vanish [3]. In particular, that means that all the dispersive deformations of the brackets are trivial, i.e. they are generated by a Miura transformation of the undeformed bracket.

The triviality, up to the first order, of some cohomology groups for $D = N = 2$ has recently been proved by direct computation [4]. In this contribution, we focus on the scalar brackets ($N = 1$) and prove, using methods from homological algebra, that the cohomology groups are in general non trivial, deriving in particular a direct formula for the dimension of the cohomolgy groups when $D = 2$.

First, we show that the Poisson–Lichnerowicz complex of a scalar D-dimensional bracket is isomorphic to the one of the bracket

$$\{u(x^1,\dots,x^D),u(y^1,\dots,y^D)\} = \frac{\partial}{\partial x^D}\delta(x-y),$$

then we study the cohomology of that complex on the densities of local p-vectors. We denote $H^p_d(\hat{\mathcal{A}})$ the homogeneous componentes of the cohomology groups, where p is the grading in the cochain complex and d the order of the differential polynomials that are elements of $\hat{\mathcal{A}}$. Finally, by an exact sequence argument we get $H^p_d(\hat{\mathcal{F}})$, namely the cohomology groups on the space of p-vector, which is defined by the formal integration

$$\hat{\mathcal{F}} = \frac{\hat{\mathcal{A}}}{\partial_{x^1}\hat{\mathcal{A}} + \partial_{x^2}\hat{\mathcal{A}} + \cdots + \partial_{x^D}\hat{\mathcal{A}}}.$$

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Hurwitz numbers, Belyi pairs, Grothendieck dessins d'enfant, and matrix models

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Belyi pairs are functions mapping Riemann surfaces of genus g on the complex projective line with branchings at a fixed number of points (at three points for the case of original Belyi pairs and Grothendieck's dessins d'enfant corresponding to these pairs).

We begin by construct the matrix model describing the case of three branching points ([1]) passing then to more general models describing the case of n branching points ([2]). All these models are tau functions of the KP hierarchy and upon some constraints on their generating functions their solutions can be attained using the topological recursion technique.

Namely, we show that the case of n branching points is described by the matrix model represented by an integral over a chain of Hermitian matrices

$$\int \prod_{i=1}^{n-1} DH_i \, e^{-N\, tr[V(H_1)+(\beta-\gamma)\log H_2+\gamma H_1 H_2^{-1}+\gamma H_2 H_3^{-1}+\cdots+\gamma H_{n-2}H_{n-1}^{-1}+U(H_{n-1})]},$$

whose free energy is the generating function for Hurwitz numbers with ramification profiles specified at the first and the last, nth, ramification points and encoded in the two potentials $V(H_1)$ and $U(H_{n-1})$, the weight also segregate the third ramification point whose profile μ contributes the factor $\beta^{l(\mu_2)}$ whereas the remaining $n-3$ points come with the same constants $\gamma^{l(\mu_k)}$, $k=3,\ldots,n-2$.

These new matrix chains manifest braid-group symmetries and conjecturally can be related to the effective Yang–MIlls theories in the strong-coupling limit (or to special quantum Toda chains).

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Binary Darboux Transformations in Bidifferential Calculus

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We investigate the '*Miura equation*'

$$\bar{\mathrm{d}}g + (\mathrm{d}\phi)\, g = 0, \tag{1}$$

where d and $\bar{\mathrm{d}}$ are derivations acting from an associative algebra $\mathcal{A}$ into $\mathcal{A}$-bimodule Ω^1. For (1) there exists a universal (for any choice of d and $\bar{\mathrm{d}}$) solution generating method [1, 2], which is an abstract version of Binary Darboux Transformations.

If derivations d and $\bar{\mathrm{d}}$ extend to maps $\mathcal{A} \stackrel{\mathrm{d},\bar{\mathrm{d}}}{\to} \Omega^1 \stackrel{\mathrm{d},\bar{\mathrm{d}}}{\to} \Omega^2$, with another $\mathcal{A}$-bimodule Ω^2 such that $\mathrm{d}^2 = \bar{\mathrm{d}}^2 = \mathrm{d}\bar{\mathrm{d}} + \bar{\mathrm{d}}\mathrm{d} = 0$, then (1) implies

$$\mathrm{d}\bar{\mathrm{d}}\phi + \mathrm{d}\phi\, \mathrm{d}\phi = 0, \qquad \mathrm{d}\,[(\bar{\mathrm{d}}g)\, g^{-1}] = 0. \tag{2}$$

Depending on the choice of bidifferential calculi (d, $\bar{\mathrm{d}}$, $\mathcal{A}$, Ω^1, Ω^2), from (2) one recovers differential and difference integrable systems, including two versions of the self-dual Yang-Mills equation, matrix two dimensional Toda lattice, matrix Hirota-Miwa (Hirota bilinear difference equation), (2+1)-dimensional NLS, KP and Davey-Stewartson equations. Elaborating the main theorem from [2] we construct families of solutions of the latter systems. In case of the KP-I equation and the DS system we obtain multiple pole lumps [3,4] and soliton and dromion solutions [5,6] respectively. We also address the respective matrix counterparts.

This talk is based on joint work with A. Dimakis and F. Müller-Hoissen.

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Metrisability of projective structures and integrability of particular 2nd order ODEs

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Consider the set of affine torsion-free connections on an n-dimensional simply-connected smooth orientable manifold M. We define the following equivalence relation: two connections Γ and $\hat{\Gamma}$ are *projectively equivalent* if they share the same umparametrised geodesics. A *projective structure* is an equivalence class of such equivalence relation. Therefore, a projective structure can be defined from each umparametrised geodesic equation on a manifold, once local coordinates are chosen.

We will be particularly interested in the 2-dimensional problem [1], whose main ideas generalise almost immediately to higher dimensions. By choosing local coordinates (x, y), every umparametrised geodesic equation is of the form

$$y'' = A_3(x, y)y'^3 + A_2(x, y)y'^2 + A_1(x, y)y' + A_0(x, y),$$

where we have considered x as the independent variable, or $y = y(x)$. The projective structure can be read off from the coefficient functions $A_i(x, y)$.

In this talk, we shall explain how one can find first integrals of a second order ODE of the above form by solving a relatively simple problem of metrisability of the corresponding projective structure. To illustrate this, recall that the existence of a Killing vector imply the existence of conserved quantities along geodesics, which will immediately give rise to a first integral of the umparametrised equation under the Hamiltonian picture. However, the existence a Killing vector does not cover the general case. In fact, we will show that whenever there are more than one metric giving rise to the same umparametrised geodesic equation, we can construct a first integral. The presence of Killing vector corresponds to a particular case: the existence of degenerate solutions of the metrisability problem, i.e., to the existence of a symmetric 2-form with vanishing determinant, which does not define a metric.

We illustrate the efficiency of this technique by studying the integrability of the six Painlevé equations, which are ODEs of the above form depending on 4 parameters, usually denoted by $\alpha, \dots, \delta$. It turns out that their corresponding projective structures are metrisable for well-known values of these parameters: those for which the Painlevé equations are fully integrable.

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Quartics, sextics, and beyond

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I will review the 19th century classical invariant theory of Cayley and Sylvester, and present a solution of an outstanding open problem concerning binary sextics. This is joint work with Roger Penrose.

New developments in the Hamiltonian reduction approach to integrable many-body systems

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The aim of this talk is to review our recent results about the construction of classical integrable systems of Calogero-Moser-Sutherland type in the setting of Hamiltonian reduction. The basic idea behind this approach is that many interesting systems can be viewed as "shadows" of canonical free systems having rich symmetries on higher dimensional phase spaces. The master phase spaces include, e.g., cotangent bundles of finite-dimensional Lie algebras and Lie groups together with their Poisson-Lie symmetric deformations and affine analogues. Here we shall focus on generalizations of the derivation of the trigonometric Sutherland system due to Kazhdan-Kostant and Sternberg [1].

We shall first explain how a large family of generalized spin Sutherland systems can be obtained from two different parent systems, namely, from free motion either on a finite-dimensional compact simple Lie group or on an infinite-dimensional current algebra [2]. We shall then recall the derivation of the hyperbolic and trigonometric BC_n Sutherland systems by means of reduction of free motion on the groups $SU(n, n)$ and $SU(2n)$, respectively. Finally, novel results on Ruijsenaars type deformations of these systems will be presented [3, 4].

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On the integrability in Grassmann geometries: integrable systems associated with fourfolds in $\mathbf{Gr}(3,5)$

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The talk is based on joint work with B Doubrov, B Kruglikov and V Novikov.

We investigate a class of dispersionless integrable systems in 3D associated with fourfolds in the Grassmannian $\mathbf{Gr}(3,5)$, revealing a remarkable correspondence with Einstein-Weyl geometry and the theory of $\mathbf{GL}(2,R)$ structures. Generalisations to higher dimensions are also discussed.

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Towards a classification of the solutions to the NLS equation.

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Abstract

The solutions to the one dimensional focusing nonlinear Schrödinger equation (NLS) are given in terms of wronskians [1]; a degenerate representation gives solutions in terms of determinants of order $2N$ for any nonnegative integer N [2]. They can be written as a product of an exponential depending on t by a quotient of two polynomials of degree $N(N+1)$ in x and t depending on $2N-2$ parameters [3]. It is remarkable to stress that when all the parameters of this representation are equal to 0, we recover the famous Peregrine breathers P_N; these solutions appear as deformations of P_N breathers whose the maximum of the modulus is equal to $2N+1$ [4]. An attempt of classification of the solutions is given and several conjectures about the structure of the solutions are also formulated.

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1

Complete integrability of Nonlocal Nonlinear Schrödinger equation

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We start with the generic AKNS system

$$L\psi \equiv i\frac{d\psi}{dx} + (q(x,t) - \lambda\sigma_3)\,\psi(x,t,\lambda) = 0, \qquad q(x,t) = \begin{pmatrix} 0 & q_+ \\ q_- & 0 \end{pmatrix}, \qquad (1)$$

whose potential $q(x,t)$ belongs to the class of smooth functions vanishing fast enough for $x \to \pm\infty$. By generic here we mean that the complex-valued functions $q_+(x,t)$ and $q_-(x,t)$ are independent. Using L as a Lax operator we can integrate a system of two equations for $q_+(x,t)$ and $q_-(x,t)$ generalizing the famous NLS equation (GNLS). After the reduction $q_+(x,t) = q_-^*(x,t)$, this system reduces to the NLS equation; applying different 'nonlocal' reduction $q_+(x,t) = \epsilon q_-^*(-x,t) = u(x,t)$, $\epsilon^2 = 1$ we obtain the nonlocal NLS [1]:

$$i\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2} + \epsilon u^2(x,t)u^*(-x,t) = 0. \qquad (2)$$

which also finds physical applications.

We prove that the 'squared solutions' of (2) form complete set of functions thus generalizing the results of [2, 3], see also [4]. Then, using the expansions of $q(x,t)$ and $\sigma_3 q_t(x,t)$ over the 'squared solutions' we extend the interpretation of the inverse scattering method as a generalized Fourier transform also to the nonlinear evolution equations related to L. Next, following [3] we introduce a symplectic basis, which also satisfies the completeness relation and denote by $\delta\eta(\lambda)$ and $\delta\kappa(\lambda,t)$ the expansion coefficients of $\sigma_3\delta q_t$ over it. If we consider the special class of variations $\sigma_3\delta q(x) \simeq \sigma_3 q_t \delta t$ then the expansion coefficients $\delta\eta(\lambda) \simeq \eta_t\delta t$ and $\delta\kappa(\lambda,t) \simeq \kappa_t\delta t$. If $q(x,t)$ is a solution to the GNLS system we get:

$$\frac{\partial\eta(\lambda)}{\partial t} = 0, \qquad \frac{\partial\kappa(\lambda,t)}{\partial t} = 2\lambda^2,$$

i.e. the variables $\eta(\lambda)$ and $\kappa(\lambda,t)$ may ne understood as the action-angle variables for the generalized NLS system.

Finally, if we apply the local involution $q_+(x,t) = q_-^*(x,t) = u(x,t)$ we recover the well known action-angle variables of the NLS equation. If we apply the nonlocal involution $q_+(x,t) = q_-^*(-x,t) = u(x,t)$ we obtain the action-angle variables of the nonlocal NLS (2) in terms of the scattering data of L.

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On Integrable Discretisations for Grassmann-Extended NLS Equations: Darboux Transformations and Yang-Baxter Maps

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Integrable discretisations for a class of coupled nonlinear Schrödinger (NLS) type of equations are presented. The class corresponds to a Lax operator with entries in a Grassmann algebra. Elementary Darboux transformations are constructed. As a result, Grassmann generalisations of the Toda lattice and the NLS dressing chain are obtained. The compatibility (Bianchi commutativity) of these Darboux transformations leads to integrable Grassmann generalisations of the difference Toda and NLS equations. The resulting discrete systems will have Lax pairs provided by the set of two consistent Darboux transformations.

Finally, Yang-Baxter maps for the Grassmann-extended NLS equation will be presentes. In particular, we present ten-dimensional maps which can be restricted to eight-dimensional Yang-Baxter maps on invariant leaves, related to the Grassmann-extended NLS and DNLS equations. Their Liouville integrability will be briefly discussed.

Generalized symmetries in the soliton surfaces approach

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In this talk, some features of generalized symmetries of integrable systems will be discussed in order to construct the Fokas-Gelfand formula for the immersion of two-dimensional soliton surfaces in Lie algebras. The sufficient conditions for the applicability of this formula will be established. Further, a criterion for the selection of generalized symmetries suitable for their use in the Fokas-Gelfand immersion formula will be provided. Finally, some examples of their application will be included.

On quad equations consistent on the cube

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Since its introduction [5] the consistency around a cube have been used as a powerful tool for identifying integrable lattice equations defined on the square and finding new ones [1, 2, 3]. The definitive step in the classifications of the systems consistent on the cube have been done in [4]. In this work we review the basis of the consistency around the cube, give a complete account on how we can embed a quad equation defined on the elementary square into a lattice non-autonomous system [8].
Some examples are explicitly discussed using their algebraic entropy [6, 7] properties.

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The method of lambda-brackets in the theory of integrable Hamiltonian PDE

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Successes of the method of lambda-brackets include:

1. Classification of scalar Hamiltonian operators (=Poisson structures)
2. Computation of variational Poisson cohomology
3. General classical Hamiltonian reduction for PDE and generalized Drinfeld-Sokolov hierarches
4. Progress in Adler-Gelfand-Dickey theory
5. The theory of non-local Poisson structures
6. General Dirac reduction theory in the infinite-dimensional case
7. Progress in non-commutative Hamiltonian PDE.

I will explain some of this work, joint with A. De Sole, D. Valeri, and M. Wakimoto.

On 2+1 - dimensional cKdV-type equation for internal ring waves on a shear flow

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Oceanic waves registered by satellite observations often have curvilinear fronts and propagate over various currents. We study long linear and weakly - nonlinear ring waves in a stratified fluid in the presence of a depth-dependent horizontal shear flow, generalising the results obtained in [1, 2, 3]. It is shown that despite the clashing geometries of the waves and the shear flow, there exists a linear modal decomposition (different from the known decomposition in Cartesian geometry), which can be used to describe distortion of the wavefronts of surface and internal waves, and systematically derive a 2+1 - dimensional cylindrical Korteweg - de Vries - type equation for the amplitudes of the waves [4]. The general theory is applied to the case of the waves in a two-layer fluid with a piecewise - constant shear flow, with an emphasis on the effect of the shear flow on the geometry of the wavefronts. The distortion of the wavefronts is described by the singular solution (envelope of the general solution) of the nonlinear first-order differential equation, constituting generalisation of the dispersion relation in this curvilinear geometry. There exists a striking difference in the shape of the wavefronts of surface and interfacial waves propagating over the same shear flow.

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Regular triangulations of point sets and solitons in two dimensions

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We give an explicit connection between two-dimensional patterns generated by soliton solutions (also obtained by hyperplane arrangements in a tropical limit of the τ-function) and triangulations (subdivisions) of point sets determined by polygons inscribed in conic curves. Two dimensional integrable systems admitting those soliton solutions include the KP equation (for parabola), two-dimensional Toda lattice (for hyperbola) and the Davey-Stewartson systems (for ellipse).

The talk will be based on the thesis of my student, Jihui Huang, which will be completed soon.

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Symmetry reductions of Lax integrable 3D systems

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We give a complete description of symmetry reductions for the following 3D Lax integrable (i.e., admitting a ZCR with a non-removable parameter) equations:

the Pavlov equation	$u_{yy} = u_{tx} + u_y u_{xx} - u_x u_{xy},$	(1)
the 3D rdDym equation	$u_{ty} = u_x u_{xy} - u_y u_{xx},$	(2)
the universal hierarchy equation	$u_{yy} = u_z u_{xy} - u_y u_{xz}$	(3)

(see [1] and references therein). The result comprised more than 30 equations, but the majority of them were either exactly solvable or linearized by the generalized Legendre transformations. Nevertheless, there were 10 'interesting' reductions, among which two well known equations, i.e., the Liouville[1] and Gibbons-Tsarev equations. The rest nine can be divided in two groups by their symmetry properties: five equations admit infinite-dimensional Lie algebras of contact symmetries (with functional parameters) and four others possess finite-dimensional symmetry algebras. The integrability properties of these four equations were studied in [2] and the main results are as follows.

Equation (1) admits the covering

$$q_t = (q^2 - q u_x - u_y) q_x, \qquad q_y = (q - u_x) q_x.$$

The symmetry $\varphi_1 = u_t - 2x u_x - y u_y + 3u$ lifts to this covering and the reduction leads to the equation

$$v_{\eta\eta} = (v_\eta + 2\xi) v_{\xi\xi} - (v_\xi - \eta) v_{\xi\eta} - v_\xi \tag{4}$$

and the covering

$$w_\xi = \frac{-w}{w^2 - (v_\xi + \eta) w + \eta v_\xi - v_\eta - 2\xi}, \quad w_\eta = \frac{-w(w - v_\xi)}{w^2 - (v_\xi + \eta) w + \eta v_\xi - v_\eta - 2\xi}.$$

The reduction with respect to the symmetry $\varphi_2 = u_t - y u_x + 2x$ leads to the equation

$$v_{\eta\eta} = (v_\eta + \eta) v_{\xi\xi} - v_\xi v_{\xi\eta} - 2 \tag{5}$$

[1] Which is also linearizable by a well known differential substitution and is not considered below.

with the covering

$$w_\xi = \frac{-1}{w^2 - v_\xi w - v_\eta - \eta}, \quad w_\eta = \frac{v_\xi - w}{w^2 - v_\xi w - v_\eta - \eta}$$

By the change of variable $v \mapsto v - \eta^2/2$ Equation (5) reduces to the Gibbons-Tsarev equation, while the covering becomes the well known nonlinear Lax pair of this equation.

Equation (2) admits the covering

$$q_t = (u_x + q)q_x, \qquad q_y = -\frac{u_y q_x}{q}.$$

The symmetry $\varphi = u_t - xu_x - u_y + 2u$ can be prolonged to a symmetry of the covering and as the result of φ-reduction we obtain the equation

$$v_{\eta\eta} = (v_\xi - \xi)v_{\xi\eta} - v_\eta(v_{\xi\xi} - 2) \tag{6}$$

with the covering

$$w_\xi = \frac{-w^2}{w^2 + (v_\xi - \xi)w + v_\eta}, \quad w_\eta = \frac{v_\eta w}{w^2 + (v_\xi - \xi)w + v_\eta}.$$

Finally, Equation (3) admits the covering

$$q_z = \frac{(qu_z - u_y)q_x}{q^2}, \qquad q_y = \frac{u_y q_x}{q}$$

and the reduction with respect to the symmetry $\varphi = u_z + u_x + yu_y + u$ prolonged to the covering leads to the equation

$$v_{\eta\eta} = v_\eta v_{\xi\xi} - (v_\xi + v)v_{\xi\eta} + v_\xi v_\psi \tag{7}$$

with the covering

$$w_\xi = \frac{-w^3}{w^2 - (v_\xi + v)w - v_\eta}, \quad w_\eta = \frac{-v_\eta w^2}{w^2 - (v_\xi + v)w - v_\eta}.$$

Equations (4)–(6) are pair-wise inequivalent with respect to contact transformation.

Using the standard reversal procedure, i.e., passing from a one-dimensional covering

$$w_\xi = X(\xi, \eta, v, v_\xi, v_\eta, w), \qquad w_\eta = Y(\xi, \eta, v, v_\xi, v_\eta, w)$$

to the infinite-dimensional covering

$$\psi_\xi = -X(\xi, \eta, v, v_\xi, v_\eta, \lambda)\psi_\lambda, \qquad \psi_\eta = -Y(\xi, \eta, v, v_\xi, v_\eta, \lambda)\psi_\lambda, \tag{8}$$

and expanding (8) in formal Laurent series in λ, we constructed infinite hierarchies of nonlocal conservation laws for Equations (4)–(6).

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Self-Duality and Symmetric Deformations of Integrable PDE

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It is widely known that many integrable dispersionless PDE in 3D and 4D can be obtained as reductions of Einstein-Weyl (EW) and Self-Duality (SD) equations respectively for conformal structures in 3D and 4D; these latter are in turn integrable by the twistor theory methods [3, 5]. In the joint work with Eugene Ferapontov [2] we made this observation into a sequence of theorems for some classes of differential equations.

We will discuss these results and some of their generalizations. The EW and SD equations have Lax pair formulations, which were made explicit in a recent paper by Maciej Dunajski, Eugene Ferapontov and the author [1]. We will briefly discuss some of the forms of these master-equations.

The EW/SD check is very algorithmic and thus is useful in verifying integrability of dispersionless PDE. In particular, given an ansatz for deformation of a known (model) integrable equation, we can classify integrable PDE among them. Deformations of the second starting from the heavenly-type equations in 4D were recently classified in a joint work with Oleg Morozov [4]. These provide some (new) infinite families of integrable PDE (with interesting moduli spaces). We will report on these and further results.

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Tetrahedron equation and generalized quantum groups

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In quantum integrable systems in three dimension (3D), an important role is played by the Zamolodchikov tetrahedron equation and the Isaev-Kulish 3D reflection equation:

$$R_{356}R_{246}R_{145}R_{123} = R_{123}R_{145}R_{246}R_{356},$$
$$R_{456}R_{489}K_{3579}R_{269}R_{258}K_{1678}K_{1234} = K_{1234}K_{1678}R_{258}R_{269}K_{3579}R_{489}R_{456}.$$

They are equalities among linear operators acting on the tensor product of 6 and 9 vector spaces respectively, and the indices specify the components on which the operators R and K act nontrivially. They serve as 3D analogue of the Yang-Baxter and the reflection equations postulating certain factorization conditions on straight strings which undergo the scattering R and the reflection K by a boundary plane.

I shall survey the recent developments on these systems of equations and related quantum group theoretical aspects. They include the quantized algebra of functions $A_q(g)$ on classical simple Lie algebra g, classification of irreducible A_q-modules (Soibelman 1991), intertwiners of the A_q-modules as a solution to the tetrahedron equation (Kapranov-Voevodsky 1994), the first nontrivial solutions to the 3D reflection equation [1], identification of the intertwiners with transition matrices of the PBW bases of the positive part of the quantized universal enveloping algebra $U_q(g)$ for general g [2], reduction of the tetrahedron equation to the Yang-Baxter equation producing quantum R matrices for families of generalized quantum groups extrapolating a number of quantum affine algebras and their q-oscillator representations [3].

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Fermi-Pasta-Ulam recurrence and modulation instability

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The phenomenon of recurrence in nonlinear systems with many degrees of freedom was first observed in numerical experiment by Fermi, Pasta and Ulam in 1954. The idea of Fermi was to ascertain how randomization due to the nonlinear interaction leads to the energy equipartition between large number of degrees of freedom in the mechanical chain. The length of the chain achieved $N = 64$ oscillators with quadratic nonlinearity and long-wave initial conditions were used. Instead of the energy equipartition numerics showed after some definite time recurrence to the initial data accompanied by a quasi-periodic energy exchange between several initially exited modes. Since that time this problem became known as the Fermi-Pasta-Ulam (FPU) problem and was one of the most attractive subjects for numerous investigations. Later, mainly by efforts of N. Zabusky, these results were repeated by means of more powerful computers. Besides, there were observed many other peculiarities in this problem (for details see the original papers of Zabusky & Kruskal (1965), Deem & Zabusky (1967)). It was a time of forerunner of the era of integrability for nonlinear systems.

Since the discovery of the IST for the KDV by Gardner, Greene, Kruskal & Miura (1967), and later for the NLS by Zakharov & Shabat (1972), many aspects of the FPU recurrence became more clear. In 1971 Zakharov and Faddeev proved that the KDV equation, which, in particular, can be obtained from the FPU system in the continuous limit for waves propagated in one direction, represents completely integrable Hamiltonian system. Later, in 1974 Zakharov demonstrated that the Boussinesq equation which can be considered as the direct continuous limit for the FPU system also belongs to the systems integrable by the IST. According to Zakharov (1974) the long-time randomization for the FPU system can be explained by the "distance" of that system to the nearest fully integrable one. In this case its dynamics will follow in accordance with the nearest integrable system up to the moment when the deviation from the integrable trajectory can change up to the order of 1 and this time can be taken as an estimate randomization time.

In this paper we give qualitative arguments to explain the FPU analog for the NLS. Analytically there are known a lot of exact solutions (Kuznetsov (1977), Peregrine (1983), Akhmediev, Eleonsky & Kulagin (1985), Zakharov & Gelash (2012, 2013), etc.) which show the recurrence of the condensate solution after interaction with solitons. After leaving solitons the condensate recovers with the same amplitude but has a different phase. This is the analog of the FPU recurrence for the NLS.

The NLS equation, as well known, has a simplest stationary solution in the form of the so-called cnoidal wave. This solution can be expressed in terms of the elliptic Weierstrass function which can be represented as the infinite periodic lattice of solitons (see, e.g. [1]). When the lattice period tends to infinity this solution yields the one-soliton solution of the NLS. In another limit the cnoidal wave transforms into the condensate solution with $\psi = \text{const}$. The condensate is unstable relative to the modulation instability, also known as the Benjamin-Feir instability. The same statement about instability is also valid for the cnoidal wave [2]. The growth rate in this case can be found exactly by means of the dressing procedure and expressed in terms of Weierstrass σ and ζ functions. When the distance between solitons becomes large enough the maximal growth rate turns out to be exponentially small so that in the limit of infinite period it gives stability for one-soliton solution. However, the linear theory can not provide the FSU recurrence. It can be understood within a nonlinear theory only.

As known, the phase space of the NLS as an integrable Hamiltonian system consists of discrete number of solitons and non-soliton (continuous) part. According to Zakharov and Shabat the interaction between solitons is elastic and pairing. For scattering of two solitons it results in changes only of two soliton parameters, i.e. coordinates of center of mass and phases. The cnoidal wave is of the form of the soliton lattice. Therefore any soliton from the lattice after interaction with a soliton propagating along the cnoidal wave will undergo the same shift for its center of mass and phase. This means that after scattering of the propagating soliton with the lattice the cnoidal wave will restore its previous form, up to definite spatial and phase shifts. Evidently, the same statement will be valid for condensate as the partial solution of the cnoidal wave. The interaction of condensate with any soliton after its propagation will restore amplitude of the condensate but its phase will be different. Scattering of a soliton with the nonsoliton part also remains the soliton form with a change of center of mass of the soliton and its phase. Thus, the cnoidal wave undergoing by the modulation instability, at the nonlinear stage, should recover its form getting some phase and spatial shifts. This is the qualitative explanation of the FSU recurrence for the cnoidal wave and for the condensate, in particular. It is necessary to underline that the same phenomenon takes place for the KDV cnoidal wave that was found by Kuznetsov & Mikhailov in 1974 [1].

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Domain shape dependence of semiclassical corrections to energy

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As was shown in various works [1, 2], a number of dimensions accounted for in the classical system has notable impact on quantum corrections to the energy of its solutions. In this contribution stationary solutions of one-dimensional Sine-Gordon and ϕ^4 systems are embedded in a multidimensional theory with explicitly finite domain in the added dimensions

$$S(\psi, T) = Ta^{d-2}G \int_0^1 \int_D \left(\frac{1}{2c^2} \left(\frac{\partial \psi}{\partial t} \right)^2 - \frac{1}{2} \sum_{n=1}^{d} \left(\frac{\partial \psi}{\partial x_n} \right)^2 - V(\psi) \right) \prod_{n=1}^{d} dx_n dt \tag{1}$$

with T and a as scaling parameters for time and space variables respectively, G carrying all relevant physical constants, D as the spatial domain, c as linear wave propagation speed in the dimensionless variables used, d being the total number of spatial dimensions included, $V(\psi) = m^2(1 - cos^2(\psi))$ in case of Sine-Gordon and $V(\psi) = -m^2\psi^2 + \frac{m^2}{v^2}\psi^4$ in case of ϕ^4 system. Energy corrections are calculated using zeta-function regularisation [1]

$$\Delta E = -\frac{\hbar}{iT} \lim_{s \to 0_+} \frac{\partial}{\partial s} \frac{1}{\Gamma(s)} \int_0^\infty \tau^{s-1} \int_{[0,1]\times D} (g_L(\tau, \overrightarrow{x}, \overrightarrow{x}) - g_{L_0}(\tau, \overrightarrow{x}, \overrightarrow{x}))d\overrightarrow{x}\, d\tau, \tag{2}$$

where $\overrightarrow{x}$ covers all variables of the classical system including time whereas g_L and g_{L_0} are Green functions solving

$$\left(\frac{\partial}{\partial \tau} - A \left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta + \frac{d^2 V}{d\psi^2}(\varphi) \right) \right) g(\tau, \overrightarrow{x}, \overrightarrow{x}_0) = \delta(\tau)\delta(\overrightarrow{x} - \overrightarrow{x}_0) \tag{3}$$

with A as an imaginary constant resulting from the derivation of semiclassical corrections and φ being the classical solution considered (in case of g_L) or the vacuum solution (in case of g_{L_0}).

Most publications on the subject used continuum approximation for the spectral problems in the dimensions added to the given system (see [1, 2, 3]), which meant that the specific geometries of researched physical situations were largely unaccounted for. In this contribution a more in-depth treatment of the problem is sought and energy corrections are analytically calculated for a class of specific geometries with emphasis on the effects of shape and scale of the physical system on the corrections.

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Discretizing the Liouville equation

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The main purpose of this presentation is to show how structure reflected in partial differential equations can be preserved in a discrete world and reflected in difference schemes.

The Liouville equation is the simplest periodic reduction of the integrable two dimensional Toda lattice (two continuous and one discrete variable) and is known to be linearizable by a transformation to the wave equation.

Three different structure preserving discretizations of the Liouville equation are presented here and then used to solve specific boundary value problems. The results are compared with exact solutions satisfying the same boundary conditions. All three discretizations are on four point lattices. One preserves linearizability of the equation, another the infinite dimensional symmetry group as higher symmetries, the third preserves the maximal finite dimensional subgroup of the symmetry group as point symmetries. A 9-point invariant scheme that gives a better approximation of the equation, but worse numerical results for solutions is presented and discussed.

Higher Spin Lifshitz Theories and the KdV-Hierarchy

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In this work [1] three dimensional higher spin theories in Chern-Simons formulation with Lifshitz symmetry of scaling exponent z and the gauge algebra $SL(N, R)$ are investigated. We show that an explicit map exists for all z and N mapping the Lifshitz Chern-Simons theory to the (n, m) element of the KdV hierarchy. Furthermore we show that the map and hence the conserved charges are independent of z. We derive these result from the Drinfeld-Sokolov formalism of integrable systems.

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Haantjes manifolds and Veselov systems

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Haantjes manifolds are a mild generalization of bihamiltonian manifolds. They are of interest because they are the natural setting where to define a Lenard complex. A Lenard complex on a Haantjes manifold is a triple (X, θ, K_j), where$X$ is a vector field on the manifold M, θ is a 1-form, and K_j are commuting tensor fields of type $(1, 1)$, in number equal to the dimension of the manifold.By acting on X and θ, the recursion operators K_j define a Lenard chain of vector fields $X_j = K_j X$ and a Lenard square of 1-forms $\theta_{jm} = K_j K_m \theta$. The triple (X, θ, K_j) is a Lenard complex if the vector fields X_j commute in pairs and define a basis on TM,and if the 1-forms θ_{jm} are exact. Lenard complexes are ubiquitous in the theory of integrable systems. In the talk I plan to exhibit the Lenard complex associated with Veselov systems.

Separable quantizations of Stäckel systems

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In this talk I will addresses the issue of separable and integrable quantizations of commuting sets of quadratic in momenta Hamiltonian of the form

$$H(x,p) = \frac{1}{2}A^{ij}(x)p_i p_j + V(x) \tag{1}$$

The Hamilton operator (quantum Hamiltonian) $\widehat{H} = -\frac{\hbar^2}{2}\nabla_i A^{ij}\nabla_j + V(x)$ acting on the Hilbert space $L^2\left(Q, |\det g|^{1/2}\, dx\right)$ of square integrable (in the measure $\omega_g = |\det g|^{1/2}\, dx$) complex functions on Q is called a *minimal quantization* of the Hamiltonian (1) *in the metric* g that also defines the operators ∇_i of the asociated Levi-Civita connection.

In the standard approach to the quantization of (1) one assumes that $g = A^{-1}$ (as it has been done in the classical works [1] and [2] devoted to the problem of separability of classical Hamilton-Jacobi equation associated with (1)) i.e. the tensor A is taken as a contravariant metric generating the connection ∇_i. This is a natural assumption, but it leads to severe limitations on the process of quantization of (1).

I this talk I first explain the notion of minimal quantization and its relation to the more general quantization theory developed in [3, 4, 5]. Then I demonstrate that many Hamiltonian systems of the form (1) - that can not be separably quantized in the classical approach of Robertson and Eisenhardt - can be separably quantized if we extend the class of admissible quantizations through a suitable choice of Riemann space adapted to the Poisson geometry of the system. Actually, in this talk I will demonstrate that for every quadratic in momenta Stäckel system (defined on an n dimensional Poisson manifold) for which its Stäckel matrix consists of monomials in position coordinates there exist infinitely many minimal quantizations - parametrized by n arbitrary functions - that turn this system into a quantum separable Stäckel system. I also explain the origin of so called quantum correction terms, observed - but not explained - in [6] and [7]

The results presented in this talk can be to some extent found in [8].

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Cluster varieties and integrable systems

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I describe a class of integrable systems on Poisson submanifolds of the (generally - affine) Poisson-Lie groups, enumerated by cyclically irreducible elements the co-extended affine Weyl groups. Their phase spaces admit cluster coordinates and the integrals of motion are cluster functions. This class of integrable systems coincides with the constructed by Goncharov and Kenyon out of dimer models on a two-dimensional torus and classified by the Newton polygons. Particular examples include the well-known relativistic Toda chains, the system of "pentagram map" and their many generalizations.

Gravipulsons

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We search for self-gravitating oscillating scalar lumps (pulsons) in the field model

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} &= \kappa \left[\phi_{,\mu}\phi_{,\nu} - \left(\frac{1}{2}\phi_{,\alpha}\phi^{,\alpha} - U(\phi) \right) g_{\mu\nu} \right], \\ \phi^{;\alpha}_{;\alpha} + U'(\phi) &= 0 \end{aligned}$$

with the potential

$$U(\phi) = \frac{m^2}{2}\phi^2 \left(1 - \ln\frac{\phi^2}{\sigma^2} \right).$$

With the use of the Krylov-Bogoliubov type asymptotic expansion in gravitation constant the pulson solutions of this Einstein-Klein-Gordon system are obtained in the Schwarzschild coordinates. They are expressed in terms of solutions of the singular Hill's equation. The masses of the obtained pulsons are calculated. The initial conditions are found under which the pulson solutions become periodic. These conditions are then used in direct numerical integration of the Einstein-Klein-Gordon system. It is shown that they do evolve into a very long-lived periodic pulson. Stability of the self-gravitating pulsons and their possible astrophysical applications are briefly discussed.

Differential-difference and finite-difference integrable systems associated with Kac-Moody algebras

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We consider Lax operators for two–dimensional "periodic" Toda type systems corresponding to classical series of Kac-Moody algebras and $G_2^{(1)}$ [1]. For these Lax operators we construct systematically elementary Darboux transformations and integrable differential-difference systems (Bäcklund transformations). Conditions of Bianchi permutability for Bäcklund transformations, or, more precisely, the commutativity conditions for the Darboux transformations lead to systems of integrable partial difference equations. Thus, with every classical Kac-Moody Lie algebra and $G_2^{(1)}$ we associate an integrable Toda type system, a pair of differential-difference systems and a partial difference system. These differential-difference systems represent Bäcklund transformations for the Toda type system and serve as (non-local) symmetries for the partial-difference system of equations.

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Random partitions and the quantum Benjamin-Ono hierarchy

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The Cauchy identity for Jack symmetric functions defines an ensemble $M_V(\beta, L)$ of random partitions λ. This ensemble appears in the dynamics of the quantum Benjamin-Ono fluid in (1+1)-dimensions at coupling β with L-periodic initial profile V. For both formal and analytic V, at fixed $\beta > 0$, we obtain an all-order $1/L$ expansion of the linear statistics of $M_V(\beta, L)$ in a diffusive scaling limit. This result has the same form as Chekhov-Eynard's all-order $1/N$ refined topological expansion of the connected correlators of the log-gas on the line in a potential V at inverse temperature β [1]. Our derivation relies on the infinite hierarchy of conserved currents of this system exhibited by Nazarov-Sklyanin in collective field variables [2]. As an application, we prove a law of large numbers and central limit theorem in this limit: (i) the random interface λ concentrates on Okounkov's limit shape ω_V [3] and (ii) macroscopic fluctuations around ω_V occur at order $1/L$ and converge to a Gaussian process Φ_V whose covariance depends universally on V. The exact calculation of these asymptotics in terms of V hinges on an analytic continuation made possible by the inversion formula for Toeplitz operators on the circle with symbol V due to Krein and Calderón-Spitzer-Widom [4, 5].

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Towards an algebraic framework of hydrodynamic integrability

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Hirota–type PDEs (i.e., of he form $F(u_{ij}) = 0$) are naturally understood as hypersurfaces in Lagrangian Grassmannians. The latter possess a very interesting geometry, which I will quickly review. In particular, I will focus on the notion of rank–one vectors and rank–one submanifolds, which constitute an essential tool to define hydrodynamic integrability. Then I will show that the Plücker embedding space is equipped with a (conformal) symplectic structure or a pseudo–Riemmannian metric of neutral signature, depending on the parity of the number n of independent variable, and that the Lagrangian Grassmannian is isotropic with respect to such a structure. The moduli space of Hirota–type PDEs fulfilling some additional property, like the linearisability, correspond to a singular stratum of the dual variety of the Lagrangian Grassmannian.

In the sub–class of non–degenerate Monge–Ampère equations, the linearisability coincides with integrability, if $n = 3$, whereas, if $n = 4$, integrability corresponds to the singular locus of the dual variety. For $n \geq 4$, Ferapontov conjectured that any non–degenerate integrable PDE of Hirota–type must be of Monge–Ampère type [1], and I will formulate such a conjecture in the proposed algebraic framework.

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Lagrangian Multiform Theory: recent advances and future perspectives

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Lagrangian multiform theory was initiated in 2009 with the paper [1] by Sarah Lobb and the speaker, where a novel proposal was formulated on how the fundamental integrability property of *multidimensional consistency* (MDC) can be integrated into the variational approach to integrable systems. The idea is based on the key discovery that well-chosen Lagrangians for both continuous as well as discrete integrable systems, when embedded in a multidimensional space of independent variables, obey a special relation, called the *closure* property, which suggests that these Lagrangians are in fact (difference or differential) forms with the condition that they are closed on solutions of the equations of the motion. (Note that the Lagrangians in this case are not volume forms as in the conventional theory, and that the closure only holds "on-shell", i.e. for solutions of the Euler-Lagrange equations and not as a trivial identity). On the basis of this observation a new point of view on variational calculus was developed which brings into the picture not only variations with respect to the dependent variables, but also variations with respect to the geometry in the space of independent variables. The validity and universality of this new approach was verified for many classes of examples, discrete as well as continuous and for 1D, 2D and 3D systems, cf. [2]-[7]. In the simplest case of 1D systems, i.e. the case of commuting flows of ordinary differential and ordinary difference equations, the basic principles of the new variational calculus were laid down in [6, 8, 9] cf. also [10] for the 2D case. Further contributions to the theory were made by other researchers as well, cf. e.g. [11]-[14], adding to the growing body of research in this novel direction.

In the talk I will give a brief overview of the theory, highlight the main ideas and give some explicit examples and some new results. I will also point to what in my view are the directions of travel. In fact, where this new variational theory is essentially different from any conventional theory of variational calculus is that rather than the Lagragians being chosen on the basis of secondary considerations (e.g. physical principles, symmetry arguments, etc.) in the multiform theory *the Lagrangians themselves should be considered as solutions of the variational equations*. I will try and discuss the possible ramifications of this idea for fundamental physics and in particular quantum mechanics.

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Multilinear Baker-Hirota operators with application to nonlinear differential equations

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1 Introduction

Nonlinear differential equations with soliton solutions can be written in a bilinear form in terms of Hirota D operator:

$$D_x^m D_t^n = (\partial_x - \partial_{x'})^m (\partial_t - \partial_{t'})^n F(x) \cdot F(x')|_{x'=x,t'=t} \tag{1}$$

For an example, the KdV equation

$$u_t + 6uu_x + u_{xxx} = 0 \tag{2}$$

transforms to

$$D_x(D_x^3 + D_t)F \cdot F = 0, \tag{3}$$

under $u = 2\partial_{xx} \log F$ substitution. This representation allows to apply certain explicit methods to constuct multisoliton solutions. In fact, every bilinear equation has at least two-soliton solution. Conditions for three-soliton solutions are studied in J.Hietarinta's papers.

2 Baker-Hirota operators

Here we study multilinear generalization of (1), also referred as Baker-Hirota (B-H) operator [1, 2]:

$$H_{n,1} = S \circ \mathcal{D}(\underbrace{F \cdot F \cdot \ldots \cdot F}_{n \text{ times}}) = \left(\partial_1 F \cdot F \cdot \ldots \cdot F + \gamma F \cdot \partial_2 F \cdot \ldots \cdot F + \cdot + \gamma^{n-1} F \cdot F \cdot \ldots \cdot \partial_n F\right)|_{x_i'=x} \tag{4}$$

where γ – nth root of unity, S is a symmetrization operator. For given n there are exact $(n-1)$ symmetrical B-H operators, formed over γ^k permutations. For example, the trilinear case [1] is given by

$$H_{3,1} = \partial_1 + e^{\frac{2}{3}i\pi}\partial_2 + e^{\frac{4}{3}i\pi}\partial_3 \tag{5}$$

$$H_{3,2} = \partial_1 + e^{\frac{4}{3}i\pi}\partial_2 + e^{\frac{2}{3}i\pi}\partial_3 \tag{6}$$

For odd i bilinear operator $D^i(F \cdot F)$ is identically zero. Similar condition hold for higher order multilinear operators. Denote $[i_1, i_2, \ldots i_{n-1}] =$

$H_{n,1}^{i_1} H_{n,2}^{i_2} \dots H_{n,n-1}^{i_{n-1}}$. We found that for any prime n

$$\sum_{k=1}^{n-1} \binom{n-2}{k} i_k \bmod n \neq 0 \tag{7}$$

implies $[i_1, i_2, \dots i_{n-1}] \equiv 0$, so non-zero B-H operators form a lattice in $n-1$ dimensions.

3 Multilinear forms of nonlinear equations

We discuss multilinear forms of Lax, Sawada-Kotera hierarchies and some other KdV-like equations. The following results are obtained by direct computation. Note here that we do not consider additional parameters to B-H, i.e. all forms are represented in $1+1$ variables.

Trilinear form for KdV equation:

$$\left(T_x^4 T_x^* + T_x^2 T_t\right) F \cdot F \cdot F = 0. \tag{8}$$

The equation of fifth order from Lax hierarchy has trilinear form

$$(20 T_x^3 T_x^{*3} + 7 T_x^6 + 27 T_x^* T_t) F \cdot F \cdot F = 0 \tag{9}$$

and 2 quadrilinear forms. One of these form is

$$\left(G2_x^1 G3_x^6 + 5 G1_x^1 G2_x^3 G3_x^3 + 12 G1_x^2 G2_t^1\right) F \cdot F \cdot F \cdot F = 0, \tag{10}$$

where $G1$, $G2$, $G3$ – quadrilinear operators.

Lax equation of seventh order has a pentalinear form:

$$4368 P^{[0,3,1,4]} + 2310 P^{[0,6,0,2]} - 648 P^{[1,7,0,0]} - 44284 P^{[0,2,3,3]} + 625 u_t = 0 \tag{11}$$

Fifth order equation from Sawada-Kotera hierarchy has bilinear and trilinear forms. We also found multilinear forms for *new* and *tail* equations with two-soliton solution.

Equation	Bilinear	Trilinear	Quadrilinear	Pentalinear
Lax3	$D_x(D_x^3 + D_t)$	$2T_x^2 T_x^{*2} + 3T_x^* T_t$	$2G1_x^2 G3_x^2 + G1_x^2 G2_t^1$	YES
	-	$??T_x^4 T_x^* + ??T_x^* T_t$	YES	
Lax5	-	$20T_x^3 T_x^{*3} + 7T_x^6 + 27T_x^* T_t$	YES	
		-	$G2_x^1 G3_x^6 + 5G1_x^1 G2_x^3 G3_x^3 + 12G1_x^2 G2_t^1$	
Lax7	-	-	-	YES
	-	-	-	-
SK5	$D_x(D_x^5 + D_t)$	$10T_x^3 T_x^{*3} - T_x^6 + 9T_x^* T_t$	YES	
		$T_x^5 T_x^{*2} + T_x^2 T_t$	YES	
SK7		$14T_x^4 T_x^{*4} + 14T_x^7 T_x^* + 27T_x T_t$	$3G3_x^8 + 7G1_x^2 G3_x^6 - 42G1_x^2 G2_x^4 G3_x^2 + 64G1_x^2 G2_t^1$	
		-	$7G1_x^2 G2_x^3 G3_x^4 + G1_x^1 G2_x^1 G3_x^7 + 16G1_x^2 G2_t^1$	
new7	$D_x(D_x^7 + D_t)$	$35T_x^4 T_x^{*4} - 4T_x^7 T_x^* + 27T_x T_t^*$	YES	
		-	$7G2_x^4 G3_x^{*4} - G1_x G2_x^2 G3_x^5 + 12G1_x^2 G2_t^1$	
tail9	$D_x(D_x^9 + D_t)$	$7T_x^5 T_x^{*5} - 5T_x^8 T_x^{*2} + 9T_x T_t^*$	YES	

4 Conclusion

In recent study we obtain bases of high order Baker-Hirota operators. Using them we represent certain integrable nonlinear PDEs in multilinear form. Direct methods of constructing solutions are available in bilinear case, but for $n > 3$ it is unclear how to apply them. So the next step is to investigate symmetries for multilinear equations.

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On the classification of discrete Hirota-type equations in 3D

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In the series of recent publications [1-4] we proposed a novel approach to the classification of integrable differential/difference equations in 3D based on the requirement that hydrodynamic reductions of the corresponding dispersionless limit are 'inherited' by the full dispersive equation. In this paper we extend our approach to fully discrete equations. Our only constraint is that the initial ansatz possesses a non-degenerate dispersionless limit (this is the case for all known Hirota-type equations). Based on the method of deformations of hydrodynamic reductions, we classify 3D integrable equations of Hirota type within various particularly interesting subclasses.

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Ubiquitous symmetries

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We present an overview of some of our recent work:

(1) quantization of the isochronous Calogero's goldfish model [1], and its relationship with Darwin [3], [2], [6];

(2) classical superintegrable systems are indeed linear [7], [4];

(3) for Riccati and Abel chains nonlocal symmetries (esp. λ-symmetries [5]) come from Lie symmetries.

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Cubic-Quintic NLS Equations with Four-Dimensional Symmetry Algebras

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In this communication we aim at studying a class of cubic-quintic nonlinear Schrödinger equations, given in the form

$$iu_t + u_{xx} + g(x,t)|u|^2u + q(x,t)|u|^4u + h(x,t)u = 0 \tag{1}$$

in which the complex coefficients g, q and h will be assumed to have some specific forms so that the equation under consideration admits a four-dimensional Lie symmetry algebra. The motivation for this study comes from the work [1] on a general class of cubic-quintic nonlinear Schrodinger equations given as

$$iu_t + f(x,t)u_{xx} + k(x,t)\,u_x + g(x,t)|u|^2u + q(x,t)|u|^4u + h(x,t)u = 0. \tag{2}$$

We performed in [1] classification of this family of equations according to Lie symmetry algebras they can admit. There u is a complex-valued function, f is real-valued, and k, g, q, h are complex-valued functions. (1) appears as a canonical equation when classifying the family (2) with respect to Lie symmetries.

The purpose of this contribution is to share the results on solutions of reductions of the canonical PDEs of the form (1). There are four representative equations in this class, from which several first, second and third order reduced ODEs are obtained through well known algorithm of Lie (see [2]). Although the reduced equations do not appear in a manageable form due the quintic nonlinearity and though most of them do not have the Painleve property, in specific cases we were able to obtain exact analytical solutions for equations with variable coefficients, through truncation in Painleve series. What is more, one of those solutions is expected to expose a blow-up behaviour in finite time in an appropriate L_p space, due to the conformal invariance of the PDE itself, through the machinery similar to the one studied in [3].

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Integrable Multidimensional Quasilinear Systems of First Order and Their 1+1 Dimensional Dispersive Reductions

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We introduce some three-, four-, five-, six- etc. dimensional quasilinear systems of first order, which are integrable by the method of hydrodynamic reductions.

We show that these multidimensional systems also possess 1+1 dispersive reductions.

For instance, we construct such a four dimensional enveloping system for the KdV equation, a four dimensional enveloping system for the constant astigmatism equation and five dimensional enveloping system for the Boussinesq equation.

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Commutator identities on an associative algebra and non-Abelian integrable equations

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In [1] and [2] it was shown that integrable equations are associated with commutator identities on associative algebras. Here were develope this approach for non-Abelian case starting with a simple commutator (in the algebraic sense) identity. By means of a special dressing procedure we prove that this identity results in non-Abelian Hirota difference equation. We present a regular procedure for derivation of non-Abelian (differential-difference and difference) integrable equations as special limiting cases of the general procedure.

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Integrability in the systems with Dirac dispersion of low-energy excitations

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New classes of matter known as topological insulators and Weyl semimetals are characterized by linear dispersion of low-energy electron excitations on the surface and in the bulk, respectively. The electron states on the surface of these so-called Dirac materials have a fixed spin orientation for each momentum. The electron states in topological insulators are topologically protected by the time-reversal symmetry. A condition for the existence of Weyl semimetal is breaking of either inversion or time-reversal symmetry. The topological protection manifests itself as massless Dirac modes propagating along the edge and the surface of topological insulators or in the bulk of Weyl semimetals and on their surface in the form of Fermi arc states. Study of the properties of surface electron states being a hallmark of the topological nature of Weyl nodes enables one to clarify some features of the topological protection by a symmetry. This macroscopic exhibition of the topological order offers new application areas.

We have used the N-terminal scheme for studying the edge state transport in two-dimensional topological insulators. We found an universal non-local response in the integrable limit of the nonlinear ballistic transport approach. We have exactly calculated the density of surface states in Weyl semimetals and shown that it possesses a logarithmic singularity for $\varepsilon \to 0$ decreasing linearly for the intermediate energy ε of the surface electron states and approaching zero as $\sqrt{1-\varepsilon}$ for $\varepsilon \to 1$. This resembles the behavior of a set of two orthogonal one-dimensional Dirac metals embeded in two-dimensional space.

Quantization of Poisson structures on Painleve monodromy varieties

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We discuss quantum algebras related to cubics arising as monodromy data varieties for Painlevé equation. We describe some examples of non-commutative cubics unifying the "quantum Painlevé cubics" and cubic superpotentials for $3D$ (generalized) Sklyanin algebras. Such general potentials appear in a description of moduli spaces of vacuum states in $N = 4$ supersymmetric Yang-Mills field theory.

Recent results on integrable dispersionless PDEs in multidimensions

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We first review the formal aspects of the theory of integrable dispersionless PDEs in multidimensions (including, as distinguished examples, the dispersionless Kadomtsev - Petviashvili, the heavenly and the Boyer-Finley equations) arising as commutation condition of multidimensional vector fields, obtained in collaboration with S. V. Manakov: the IST formalism for solving the Cauchy problem, the construction of the longtime behavior of solutions and of exact implicit solutions, and the analytical aspects of multidimensional wave breaking [1] - [4]. We also present some recent results including, in particular, some rigorous aspects of such a theory, obtained in collaboration with P. Grinevich and D. Wu [5].

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Hamiltonian operators of Dubrovin-Novikov type in 2D

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First order Hamiltonian operators of differential-geometric type were introduced by Dubrovin and Novikov in 1983 [1], and thoroughly investigated by Mokhov [3]. In 2D, they are generated by a pair of compatible flat metrics g and $\tilde{g}$ which satisfy a set of additional constraints coming from the skew-symmetry condition and the Jacobi identity.

The aim of this talk is to present results we have recently obtained in a joint work with E.V. Ferapontov and P. Lorenzoni [2] concerning non-degenerate Hamiltonian structures of this type. We show that the skew-symmetry condition and the Jacobi identity are equivalent to the requirement that $\tilde{g}$ is a linear Killing tensor of g with zero Nijenhuis torsion. This allowed us to obtain a complete classification of n-component operators with $n \leq 4$ (for $n = 1, 2$ this was done before). For 2D operators the Darboux theorem does not hold: the operator may not be reducible to constant coefficient form. All interesting (non-constant) examples correspond to the case when the flat pencil $g, \tilde{g}$ is not semisimple, that is, the affinor $\tilde{g}g^{-1}$ has non-trivial Jordan block structure. In the case of a direct sum of Jordan blocks with distinct eigenvalues we obtain a complete classification of Hamiltonian operators for any number of components n, revealing a remarkable correspondence with the class of trivial Frobenius manifolds.

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Separation of variables in the Clebsch model

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We study a problem of separation of variables in the classically integrable hamiltonian systems on the phase space of the dimension $2D = 4$. We formulate a necessary and sufficient condition for a pair of Poisson-commuting coordinates x_1, x_2 on this space to be the coordinates of separation. We apply the proposed approach to the case of Clebsch model on $e^*(3)$ and obtain separated coordinates x_1, x_2 and their conjugated momenta p_1, p_2 on the arbitrary coadjoint orbit of the Lie group $E(3)$. We discuss the relation of the obtained results with the Lax-pair based approach and with the "Neumann"-type coordinates of separation existing only on the special coadjoint orbits of $E(3)$.

On the monodromy of almost toric fibrations on the complex projective plane

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Let (M, ω) be a closed symplectic 4-manifold admitting an almost toric fibration in sense of Symington (see [1]). It is a Lagrangian torus fibration $\pi\colon M \mapsto B$ such that the fibers have only focus-focus or elliptic singularities. If there exists an almost toric fibration on M, then M is called an *almost toric manifold*.

In the work [1] the complete classification of the total spaces and bases of almost toric fibrations is obtained and the classification problem, up to fiber preserving symplectomorphism, for those fibrations also is formulated. In the case of CP^2 the possibilities for bases are described by the following statement.

There exist precisely four distinct bases for an almost toric fibration on the complex projective plane. The base is a 2-disk whose boundary has k corners ($k = 0, 1, 2, 3$) and $3 - k$ nodes.

The topological structure of the corresponding fibrations is mainly determined by the monodromy. We describe the monodromy in non-trivial cases of two and three nodes (see [2]).

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3D superintegrable systems with magnetic field

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We consider superintegrable systems, i.e. Hamiltonian systems that have more integrals of motion than degrees of freedom, in three spatial dimensions. Such Hamiltonian systems were considered and under some restrictions classified in [1, 2] for the case when the Hamiltonian is the sum of the kinetic energy in $\mathbb{R}^3$ and the scalar potential.

In [3] the structure of the gauge–invariant integrable and superintegrable systems involving vector potentials was considered in two spatial dimensions. Among other results it was shown there that under chosen assumptions imposed on the form of the potential, no superintegrable system with nonconstant magnetic field exists. Inspired by the approach used there we consider the Hamiltonian describing motion of 0–spin particle in three dimensions in a nonvanishing magnetic field, i.e. classically

$$H = \frac{1}{2}(\vec{P} + \vec{A})^2 + V(\vec{x}) \tag{1}$$

where $\vec{P}$ is the momentum, $\vec{A}$ is the vector potential and V is the scalar potential. The magnetic field $\vec{\Omega} = \mathrm{rot}\vec{A}$ is assumed to be nonvanishing so that the system is not gauge equivalent to a system with only the scalar potential. Quantum mechanically, observables are replaced by corresponding operators and the expression is totally symmetrized.

We suppose that at least three independent integrals of motion in addition to the Hamiltonian exist, all of them at most second order in momenta. We investigate the restrictions this imposes on the vector and scalar potential and on the magnetic field strength $\vec{\Omega}$. Next we consider how these conditions differ for the classical and quantum case, i.e. whether purely quantum superintegrable systems involving magnetic field without classical analogue exist. Last but not least we look for examples of 3D superintegrable systems with nonconstant magnetic field $\vec{\Omega}$.

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Complete integrability from Poisson-Nijenhuis structures on compact hermitian symmetric spaces

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We study a class of Poisson-Nijenhuis systems defined on compact hermitian symmetric spaces, where the Nijenhuis tensor is defined as the composition of Kirillov-Konstant-Souriau symplectic form with the so called Bruhat-Poisson structure. We determine its spectrum. In the case of Grassmannians the eigenvalues are the Gelfand-Tsetlin variables. We introduce the abelian algebra of collective hamiltonians defined by a chain of nested subalgebras and prove complete integrability. By construction, these models are integrable with respect to both Poisson structures. The eigenvalues of the Nijenhuis tensor are a choice of action variables. Our proof relies on an explicit formula for the contravariant connection defined on vector bundles that are Poisson with respect to the Bruhat-Poisson structure.

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The Haantjes Manifolds of Integrable and Separable Systems

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A general theory of finite-dimensional integrable systems is proposed, based on the geometry of Haantjes tensors [1]. Inspired by the very recent definition of Haantjes manifolds [2], we introduce the class of symplectic-Haantjes structures (or $\omega\mathcal{H}$ structures) and the notion of Lenard-Haantjes chains [3], as a generalization of the famous Lenard-Magri chains. Then, we prove that, under mild assumptions, the existence of a Haantjes structure is equivalent to the Liouville-Arnold integrability of each Hamiltonian system belonging to a Lenard-Haantjes chain [3]. Furthermore, we will revisit the theory of separation variables in symplectic-Nijenhuis manifolds (or $\omega\mathcal{N}$ manifolds) [4], under the new light thrown by the notion of Lenard-Haantjes chains, and will clarify the relation between $\omega\mathcal{H}$ and $\omega\mathcal{N}$ manifolds.

Applications of our approach to the study of some physically relevant systems will be presented.

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Vortex streets and solitons

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Classical von Karman vortex street is an example of periodic relative vortex equilibria, which are periodic configurations of vortices in the plane moving with constant velocity.

I will explain that it is simply related to the singularities in the complex domain of the classical soliton solution of the KdV equation and generalise this to multi-soliton case. The geometry of the corresponding relative vortex equilibria, which seem to be new, is still to be understood. I will discuss some qualitative results in this direction in the simplest cases.

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Dark-bright soliton solutions with nontrivial polarization interactions for the three-component defocusing nonlinear Schrödinger equation

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In this talk we present novel dark-bright soliton solutions for the three-component defocusing nonlinear Schrödinger equation with nonzero boundary conditions. The solutions are obtained within the framework of a recently developed inverse scattering transform for the underlying nonlinear integrable PDE, and unlike dark-bright solitons in the two component (Manakov) system in the same dispersion regime, their interactions display non-trivial polarization shift for the two bright components.

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1

Representations of $\mathrm{sl}(2, \mathrm{C})$ in the BGG category $\mathcal{O}$ and master symmetries

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In this talk, we show the indecomposable $\mathrm{sl}(2, \mathrm{C})$ modules in the Bernstein-Gelfand-Gelfand (BGG) category $\mathcal{O}$ naturally arise for homogeneous integrable nonlinear evolutionary systems. We then develop an approach to construct master symmetries for such integrable systems. This method enables us to compute the hierarchy of time-dependent symmetries. We finally illustrate the method using both classical and new examples. We compare our approach to the known existing methods used to construct master symmetries. For the new integrable equations such as a Benjamin-Ono type equation, a new integrable Davey-Stewartson type equation and two different versions of (2+1)-dimensional generalised Volterra Chains, we generate their conserved densities using their master symmetries.

Selberg integrals on Riemann surfaces

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A number of problems in physics and geometry yield a natural generalization of Selberg integral to Riemann surfaces. At large number of variables the integral can be evaluated as a series in powers of $1/N$. Each term in this series is a geometric invariant which carries an information about the Riemann surface.

Classical and Quantum Superintegrable Systems with N-th Order Integrals of Motion

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The general form of an integral of motion that is a polynomial of order N in the momenta is presented for a Hamiltonian system in two- dimensional Euclidean space. The classical and the quantum cases are treated separately, emphasizing both the similarities and the differences between the two. The main application will be to study superintegrable systems that allow one N-th order and one second order integral of motion. The connection with Nth order ODEs having the Painlevé property is discussed.

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Dispersionless DKP hierarchy and elliptic Loewner equation

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We show that the dispersionless versions of the DKP hierarchy (also known as the Pfaff lattice) and the Pfaff-Toda hierarchy admit suggestive reformulations through elliptic functions. We also consider one-variable reductions of the dispersionless DKP hierarchy and show that they are described by an elliptic version of the Loewner equation. With a particular choice of the driving function, the latter appears to be closely related to the Painleve VI equation with special choice of parameters.

Structure of asymptotic soliton webs in solutions of Kadomtsev-Petviashvili II equation

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Asymptotically in time, most multi-soliton solutions of the Kadomtsev-Petviashvili II equation self-organize in webs comprised of solitons and soliton-junctions. As distances between junctions grow, the memory of the structure of junctions in a connected pair ceases to affect the structure of either junction. As a result, every junction propagates at a constant velocity, which is determined by the wave numbers that go into its construction. A simple geometric consideration explains two characteristics of the webs. The first, and immediate, consequence is that asymptotic webs preserve their morphology as they expand in time. Another consequence, explains why, except in special cases, only 3-junctions (Y-shaped, involving three wave numbers) and 4-junctions (X-shaped, involving four wave numbers) can partake in the construction of an asymptotic soliton web.

Formation of curvature singularities on a fluid interface during the Kelvin-Helmholtz instability

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As is known, the interface between two fluids is unstable in the presence of a tangential velocity discontinuity. It was established by Moore [1] that, for the case of one fluid, the nonlinear stage of this instability, called the Kelvin-Helmholtz instability, is accompanied by developing singularities on the surface of the tangential discontinuity. For these singularities the discontinuity surface remains smooth, but its curvature becomes infinite in a finite time.

In the present work we consider the case of an interface — the boundary between two different fluids (i.e., Atwood numbers are arbitrary). It is shown within the Hamiltonian formalism that the equations of motion derived in the small interface angle approximation admit exact solutions in the implicit form (see also Ref. [2]). The analysis of these solutions shows that, in the general case, weak root singularities are formed on the interface due to the Kelvin-Helmholz instability. The surface profile and its first derivative occur continuous functions close to the singularities, but the second derivative becomes infinite while approaching the collapse instant. For Atwood numbers close to unity in absolute values, the surface curvature has a definite sign correlated with the boundary deformation directed towards the light fluid. For the fluids with comparable densities, the curvature changes its sign in a singular point. In the particular case of the fluids with equal densities, the obtained results are consistent with those obtained by Moore.

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Posters

Tropical Limit in Statistical Physics

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This contribution proposes a new definition of tropical limit for macroscopic systems in equilibrium outlined in [1]. Tropical geometry is an emergent branch in mathematics and physics [2] with application in high-energy physics and complex systems. So far tropical limit in statistical models has been considered as a low-temperature limit [3].

We argue that a more adapted description of tropical limit is given by the double-scaling limit of Boltzmann constant $k \to 0$ and a characteristic cardinality $N \to \infty$ in such a way that $kN =$ constant.

It is shown that such a definition is well-adapted to thermodynamic and statistical analysis since it preserves thermodynamic relations leaving temperature as a free parameter; moreover, it allows us to deal with systems with highly degenerated energy levels, e.g. spin ice, spin glasses and more general frustrated systems. Tropical free energy $F_{tr}(T)$ is a piecewise linear function of temperature T, tropical entropy is a piecewise constant function and the system has energy for which tropical Gibbs' probability has maximum.

Such a formalism is a natural tool in the investigation of phenomena related to exponential degenerations: among these, we describe the tropical analogue of limiting temperatures [4] in systems with infinitely many energy levels.

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Totally asymmetric simple exclusion process on long chains with a shortcut in the bulk

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The one-dimensional totally asymmetric simple exclusion process (TASEP) is one of the rare examples of exactly solvable models with non-equilibrium steady states and boundary induced phase transitions. The process was first introduced in [1] as a model of kinetics of protein synthesis, describing the ribosome translocation along a messenger ribonucleic acid (mRNA). Another natural interpretations of TASEP is given in terms of a single-lane vehicular traffic, see the reviews [2, 3] and references therein. Various extensions of the basic model were devised to describe different driving conditions and drivers strategies. In this interpretation, the domain wall, or the shock in the density profile, which appears on the coexistence line between the low-density and high-density phases, models the front of a traffic jam. On simple chains the stationary properties of TASEP have been extensively studied and exactly solved in the thermodynamic limit for periodic, closed and open boundary conditions, first for stochastic continuous-time dynamics and then for a number of update rules in discrete time [2, 3].

Studies of TASEP on different networks with points of bifurcation and merging of chains have recently attracted much attention due to the variety of novel features that have been observed in such complex non-equilibrium systems. The special case of a network with a section of two parallel chains of equal length inserted in the bulk of a long chain was studied in our paper [4]. Since there are no exact results for TASEP on networks with junctions, effective injection and ejection rates we introduced for each chain segment and the possible phase structures of the system in terms of these rates were studied. A coexistence phase in the double-chain segment was found when the head and tail single chains are of equal length and in the maximum current phase. Recently, the dependence of the phase in the double-chain segment on its position in a long but finite network has been studied too [5]. It was found that a simple translation of this section forward or backward along the backbone leads to a sharp change in the shape of the density profiles in the parallel chains. The extreme case of TASEP on open chains with a zero-length shortcut in the bulk was reexamined in our recent paper [6]. It was shown that the shunted segment can exist in both low-density and high-density phases, as well as in the coexistence (shock) phase. The main parameters of that shock phase were found to be governed by a positive root of a cubic equation the coefficients of which linearly depend on the probability of choosing the zero-length shortcut.

This contribution presents a generalization of the above studies to the general case of a shortcut of arbitrary length $L^{\text{sc}} \geq 2$. We present both numerical and analytical results on the conditions for the appearance and the statistical properties of the coexistence state in the double-chain segment inserted in the bulk of open chains of length $L = 400$, carrying the maximum current $J_{\max} = 1/4$ (in the thermodynamic limit). The problem is interesting on its own because the conditions for coexistence of low- and high-density phases are essentially different from those for a simple chain between two reservoirs. Our main results are: (1) For any values of the external rates in the domain of the maximum current phase, there exists a position of the shortcut where the shunted segment is in a phase of coexistence with a completely delocalized domain wall; (2) The main features of the coexistence phase and the density profiles in the whole network are well described by the domain wall theory. Apart from the negligible inter-chain correlations, they depend only on the current through the shortcut; (3) The model displays an unexpected feature - the current through the longer shunted segment is larger than the one through the shortcut. In particular, we show that this effect is due to the fact that the nearest-neighbor correlation between the first segment and the shortcut, $G_{1,\mathrm{sc}}$, are greater than the correlations between the first segment and the second one, $G_{1,2}$, for all $L^{\mathrm{sc}} < L$:

$$J^{(2)} > \frac{1}{8} + \frac{1}{4}\left(G_{1,\mathrm{sc}} - G_{1,2}\right), \quad J^{\mathrm{sc}} < \frac{1}{8} - \frac{1}{4}\left(G_{1,\mathrm{sc}} - G_{1,2}\right), \quad L^{\mathrm{sc}} < L.$$

In conclusion, from the viewpoint of vehicular traffic, most comfortable conditions for the drivers are provided when the shortcut is shifted downstream from the position of coexistence, when both the shunted segment and the shortcut exhibit low-density lamellar flow. Most unfavorable is the opposite case of upstream shifted shortcut, when both the shunted segment and the shortcut are in a high-density phase describing congested traffic of slowly moving cars. The above results are relevant also to phenomena like crowding of molecular motors moving along twisted protofilaments.

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Chaos control and function projective synchronization of fractional order systems through back stepping method

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In this article the authors have studied the chaos control and the function projective synchronization between fractional order identical T-system, and non-identical T-system and Lorenz chaotic system using back stepping method. According to the stability theory, the conditions for local stability of nonlinear three-dimensional commensurate and incommensurate fractional order systems are discussed. Feedback control method is used to control the chaos in the considered fractional order T-system. Numerical simulations are carried out using MATLAB and the results are depicted through graphs.

Analytic solutions of the Somos 6 recurrence via hyperelliptic Prym varieties

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Somos sequences are integer sequences generated by bilinear recurrence relations. They have appeared in number theory, statistical mechanics, as well as arising from reductions of bilinear PDE in the theory of discrete integrable systems. General bilinear recurrences of order 3,4, and 5 generate sequences of Fibonacci-type numbers, which can be written in terms of the sigma-function of appropriate elliptic curves.

This presentation concerns with the general form of the order 6 recurrence

$$\tau_{n+6}\tau_n = \alpha\tau_{n+5}\tau_{n+1} + \beta\tau_{n+4}\tau_{n+2} + \gamma\tau_{n+3}^2, \quad n \in N$$

with arbitrary coefficients α, β, γ, which can be described as an integrable birational map φ on C^4 having 2 independent algebraic integrals (the Somos 6 map). As was shown in [1], the solutions of φ are the first ones which are beyond genus one: they are parameterized by sigma-function of genus 2 curves.

Our goal is to reconstruct the sigma-function solutions of the Somos 6 map from the initial data: Namely, given the first 6 terms of the sequence $\{\tau_n\}$ we determine the equation of the corresponding hyperelliptic curve X and the translation vector $\mathbf{v} \in \mathrm{Jac}(X)$ in the sigma-function solution.

One of our main tools is a 3×3 Lax representation for the map φ, which was recently derived from the similar Lax pair for the discrete BKP equation, as was announced in [1]. The corresponding spectral curve S is trigonal of genus 4 having an involution σ with 2 fixed points. Then the 2-dimensional Jacobian of X, the complex invariant manifold of φ, is identified with a principally polarized Prym subvariety $\mathrm{Prym}(S, \sigma)$ of $\mathrm{Jac}(S)$.

To obtain an explicit algebraic description of $\mathrm{Prym}(S, \sigma)$ and, therefore, of X, we use the recent result of [2], which studies the general case of 2-fold coverings of hyperelliptic curves with 2 branch points.

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Quantum Boundary Conditions and Geometric Phases

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In Physics dynamical equations often have a differential form and are solved under various boundary conditions. Indeed in Quantum Mechanics the dynamics is encoded by the Schrödinger equation:

$$i\partial_t \psi = H\psi, \tag{1}$$

where H, the Hamiltonian operator, is a self-adjoint operator. Thus, the dynamics described by this equation is consistently described by a unitary operator and the spectrum associated to H is purely real.

In this contribution we are going to concentrate on the mutual relation between self-adjointness and boundary conditions. In particular following [1, 2] we are going to concentrate on the one-dimensional case and using the technique of *boundary triples* we are going to classify all boundary conditions.

Moreover we are going to show the existence of a non-trivial geometric phase in a quantum system with moving boundaries. The problem of a non-relativistic quantum particle confined in a one dimensional box with moving walls provided with Dirichlet boundary conditions has been investigated in [3].

Here we are going to take into account more general boundary conditions and study the geometric phases that emerge. Non-Euclidean geometries naturally arise in this context unfolding new geometrical ideas in the field of quantum mechanics.

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Topological atlas of an integrable system with three degrees of freedom

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In this talk, we discuss the notion of a topological atlas of an integrable Hamiltonian system with three degrees of freedom having a set of physical parameters. Such an atlas gives a complete description of the so-called rough topology of the system. Since for any set of parameters we cannot give a clear three-dimensional picture of the bifurcation diagram, we need to investigate some cross-sections of this picture and their evolution with respect to the section value and the parameters. The natural value to build cross-sections is the energy value. In this approach, we in fact deal with iso-energy manifolds of the system and the bifurcation diagrams of the restriction of the momentum map to iso-energy manifolds. We present the method to obtain an analytical description of the separating set in the space "energy – physical parameters". This set classify so-called equipped iso-energy diagrams. The equipped diagram is the bifurcation diagram stratified by the rank of the momentum map and the types of critical points in the pre-image. Moreover, the notion of an equipped diagram includes its span, i.e., it is supplemented with two-dimensional chambers which are the components of the plane of the constants of the integrals additional to the energy cut out by the bifurcation diagram together with the number of the regular tori in the chambers and the way in which these tori are united in the so-called families. To each edge of the diagram we attach the notation of a 3-atom of the bifurcation occurring inside the iso-energy manifold when crossing this edge.

The result is presented for the Kowalevski top in a double field. It was proved integrable in [1, 2]. The critical subsystems forming the preimage of the bifurcation diagram were found and integrated in [3, 4, 5]. The complete topological classification of singularities was obtained in [6]. In [7], a new approach of visualizing two-dimensional topological invariants was given. These results are taken as an analytical basis for constructing the complete topological atlas of this irreducible problem with three degrees of freedom.

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On the reconstruction problem for nonlocal symmetries

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Let $\tau : \tilde{\mathcal{E}} \to \mathcal{E}^\infty$ be an arbitrary covering over an infinitely prolonged differential equation $\mathcal{E}^\infty$, $\mathcal{E} = \{F = 0\}$. In local theory, if $\varphi \in \mathcal{F}(\mathcal{E}^\infty) \bigcap \ker \ell_F$, where ℓ_F is the universal linearization operator, then evolutionary derivation $\ni_\varphi$ is a local symmetry of the equation $\mathcal{E}^\infty$. In nonlocal theory any symmetry in the covering τ is of the form $\tilde{\ni}_{\varphi,a} = \tilde{\ni}_\varphi + \sum_{i=1}^{N} a_j \frac{\partial}{\partial w_i}$, where function $\varphi \in \mathcal{F}(\tilde{\mathcal{E}}) \bigcap \ker \tilde{\ell}_F$, while functions $a_j \in \mathcal{F}(\tilde{\mathcal{E}})$ satisfy an additional system of equations. This system for a given φ (nonlocal shadow) may have no solution (in particular, not every local symmetry can be extended to a nonlocal symmetry in the covering $\tau : \tilde{\mathcal{E}} \to \mathcal{E}^\infty$). Nevertheless, one can try to find nonlocal symmetry $\tilde{\ni}_{\varphi,b}$ for a given φ in another covering. This problem is called the reconstruction problem for nonlocal symmetries or nonlocal shadows. This problem is of great importance both from theoretical and practical points of view. Solution this problem for one nonlocal shadow first was given in [2] (see also [4]), in the case of finite number nonlocal shadow $\varphi_1, \ldots, \varphi_m$ is given in [3], [5].

In this talk we review some old results and then revise and analyze constructions of coverings from [2] from geometrical point of view.

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On invariant solutions of the $k - \varepsilon$ turbulence model

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The classical symmetries of the $k - \varepsilon$ turbulence model have been calculated in [1]. In this paper we consider symmetry reductions of the $k - \varepsilon$ turbulence model and obtain families of exact solutions.

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On generalization of vertical bundles

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In the previous paper [5], the authors constructed generalized vertical Weil functors on the category of fibred manifolds with m-dimensional bases and fibred maps with embeddings as base maps.

In the present paper we observe that almost the same construction works on the category of all fibred manifolds and fibred maps. Next, we deduce that fibred product preserving bundle functors on all fibred manifold maps are the generalized vertical Weil functors on all fibred manifold maps.

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On the orientability of higher order contact elements

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In our contribution, we study higher order contact elements taking into account their orientability. We use methods of commutative algebra, in particular Weil algebras. Our examples of Weil algebras having the group of automorphisms connected, which we present, may serve as a new approach to study of certain geometrical objects not possessing orientation reversing maps. As to applications, there are scientific papers with interesting occurrence of nonorientable manifolds. For instance, in the material science (see [1]) knots and nonorientable surfaces in chiral nematics are studied, as an example of a phenomenon that topological concepts have come to play an increasingly significant role in characterizing materials across of diverse range of topics, e.g. in the study of defects, [2].

In the talk, we build on our earlier papers [3] and [4].

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Thermostatistics of basic-deformed bosons and fermions

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Based on the q-deformed oscillator algebra, we study the thermostatistics of q-deformed bosons and fermions and show that thermodynamics can be built on the formalism of q-calculus. The entire structure of thermodynamics is preserved if ordinary derivatives are replaced by the use of an appropriate basic-deformed Jackson derivative and q-integral [1, 2, 3]. In this context, we derive the most important thermodynamic functions and we study the q-boson and q-fermion ideal gas in the thermodynamic limit. Finally, we discuss the possible formulation of a basic-deformed quantum mechanics defined in the framework of the basic square-integrable wave functions space.

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Monodromy of the axially symmetric 1:1:-2 resonance

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We consider integrable Hamiltonian systems in three degrees of freedom (3–DOF), that are in m:m:$-n$ resonance, where m and n are coprime integers with $m, n \geq 1$. The integrability comes from the periodic motion of the quadratic part and an imposed rotational symmetry about the vertical axis. Namely, the Hamiltonian function H Poisson commutes with the oscillator

$$L \;=\; \frac{m}{2}(y_1^2 + x_1^2) \;+\; \frac{m}{2}(y_2^2 + x_2^2) \;-\; \frac{n}{2}(y_3^2 + x_3^2)$$

where x_i, y_i, $i = 1, 2, 3$ are canonical co-ordinates on R^6, $i = 1, 2, 3$. Further we assume that H has an axial symmetry in the physical space R^3 with respect to rotations about the x_3–axis, that is H Poisson commutes with the third component of the angular momentum N. Since all three functions H, L and N Poisson commute with each other they describe a 3–DOF integrable Hamiltonian system and the energy-momentum mapping

$$F = (L, N, H): \quad R^6 \quad \longrightarrow \quad R^3$$

defines an integrable Hamiltonian fibration in R^6 (IHF). Such class of systems can appear in practice after normalization and truncation of axially symmetric systems near an m:m:$-n$ resonant equilibrium.

The understanding of the qualitative properties of IHFs is one of the central problems of modern classical mechanics. For reviews of results and applications see [1, 2]. In particular, Duistermaat [3] showed that the monodromy of the IHF is the coarsest obstruction to the existence of global action angle variables.

Here we focus on systems near a 1:1:-2 resonance. The set of critical values of the energy momentum mapping is determined by the bifurcation diagram of the reduced 1–DOF system. We find a rich bifurcation diagram containing three parabolas of Hamiltonian Hopf bifurcations that join at one point. Further, we describe in detail the monodromy of the resulting ramified 3–torus bundle.

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Scattering analysis of a non-linear locally driven potential.

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In the context of both quantum and classical theory, we present an analysis of transmission through different non-linear potentials that are driven locally i.e that have a explicit time dependence only in a specific region of space. These potentials carry diferent information that the scenario when the driving is executed in all space and shows an interesting frame to make a comparison between de classical and quantum scattering behaviour.

Its known that a quantum periodically driven system could be solved by the use of the Floquet theory [1]. Integrating this theory with the Bloch theory for spatially periodic structures we develop a base of eigenfunctions able to describe the system in all space. After that its possible to calculate transmition an reflections coefficients [2]. Also its known in the literature that classic analogue of these type of potentials could induced chaos [3], a specific phenomenon that let us identify some kind of interesting phenomena in the quantum regime that occurs when the parameters of the potential are the same that in the chaotic classical regime.

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Classification of nonlinear equations with two-soliton solution

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There are nonlinear equations that admit N-soliton solutions. Such equations are sometimes called the completely integrable ones. For example, hierarchies of Lax, Sawada-Kotera (SK), Kaup-Kupershmidt (KK) equations are well known [1, 2, 3, 4, 5, 6, 7, 8]. At present paper, we discuss nonlinear equations with two-soliton solutions. We consider general form n-th order partial differential equation with weight-homogeneous polynomial nonlinearities in (1+1) variables $E^n(u, u_t, u_x u, u_{xx}u_x, {u_x}^2, \dots)$. We assume an equation has a meromorphic solution (only finite number of negative powers is contained in its Laurent expansion).

For Lax and Sawada-Kotera hierarchies the one- and two-soliton solution is represented by

$$u(x,t) = K\frac{\partial^2}{\partial x^2}\log\left(1+\mathsf{e}^{p_1 x - q_1 t}\right), \tag{1}$$

$$u(x,t) = K\frac{\partial^2}{\partial x^2}\ln\left(1+\mathsf{e}^{p_1 x - q_1 t}+\mathsf{e}^{p_2 x - q_2 t}+\alpha_{12}\mathsf{e}^{(p_1+p_2)x-(q_1+q_2)t}\right) \tag{2}$$

where $q_i = p_i^n$, $i = 1, 2$, n – order of a nonlinear equation.

The condition for two-soliton solution is considered in the paper [9]. Though, the structure of these equations and their hierarchy are not specified.

At this work we represent equations of seventh and higher order which have two-soliton solutions (2SS). We got one more equation of seventh order with 2SS which differs from Lax and Sawada-Kotera ones. For higher order this equation has a hierarchy. Also there is one more equation with 2SS solution but without hierarchy (table 1).

The equation of type *new* is $E^7(u)$

$$u_t+u_{x,7}+\frac{56}{K}uu_{x,5}+\frac{56}{K}u_{x,4}u_x+\frac{140}{K}u_{x,3}u_{x,2}+\frac{840}{K^2}u^2u_{x,3}+\frac{1680}{K^2}uu_{x,2}u_x+\frac{3360}{K^3}u^3u_x=0 \tag{3}$$

The equation (3) differs from Lax and Sawada-Kotera equations. There is no term u_x^3 of same weight as another ones.

We classified high order PDEs with polynomial nonlinearities that accept 2SS solution in Hirota's form using the substitution (2). All $E^n(u)$ equations admit dispersion law in form $q_i = p_i^n$. At fig. 1 complete structure is represented.

In integrable case of 2SS coefficient α_{12} uniquely determines single equation or hierarchy of equations. We discovered one hierarchy of 2SS equations of *new*

Type of equation	Order of equation										Property		
	3	5	7	9	11	13	15	17	19	21	2SS	3SS	hierarchy
Lax	y	y	y	y	y	y	y	y	y	y	z	z	z
SK		y	y		y	y		y	y		z	z	z
new			y			y			y		z		z
tail				y	y	y	y	y	y	y	z		

Table 1: E^n classification; y denotes existence of an equation at given n, z – existence of property

type and countable family of isolated 2SS equations of *tail* type. By a direct calculation *new* and *tail* equations have no 3SS.

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Thermal Expansion of Crystalline Silicon
Nonlinear Models and Bayesian Model Selection

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In this paper we compare two different models, Legendre polynomials and a semi-empirical model that combines Grüneisen equation and Einstein model of a solid [1] to explain a set of thermal expansion measurements of monocrystalline silicon in the temperature range from 7 K to 293 K, that were performed at PTB [2].

Given the data $\mathbf{x}$, a list of models M_i, a set of nuisance parameters Θ_{M_i} and the likelihood $L(\theta_{M_i}, M_i|\mathbf{x})$, the Bayesian approach to the data analysis consists of assigning a prior probability distribution $\pi(\theta_{M_i}|M_i)$ to the model-dependent parameters and a prior probability $\Pi(M_i)$ to each model. By a little bit of algebra and introducing the *evidence* $Z(\mathbf{x}|M_i) = P(\mathbf{x}|M_i)$, the updated probability for the model M_i provided by the data is

$$\mathrm{Prob}(M_i|\mathbf{x}) = \frac{Z(\mathbf{x}|M_i)\Pi(M_i)}{\sum_j Z(\mathbf{x}|M_j)\Pi(M_j)} .$$

The evidence can be calculated by integrating over the parameter space Θ_{M_i}

$$Z(\mathbf{x}|M_i) = \int_{\Theta_{M_i}} L(\theta_{M_i}, M_i|\mathbf{x})\pi(\theta, \theta_{M_i}|M_i)\,\mathrm{d}\theta_{M_i} .$$

The evaluation of the above integral becomes a formidable task when the parameter space has more than very few dimensions. Among the algorithms for carrying out these integrations numerically, we exploited a nested sampling technique relating the likelihood values to the prior volume [3, 4]. By assuming that data are normally distributed with given mean and variance, we calculate the integral for evidence over multidimensional spaces and show that under these hypothesis the model for thermal expansion coefficient consisting of the Grüneisen equation combined with Einstein model of a solid is more probable than the Legendre polynomial model.

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New brackets in Hamiltonian systems with non-zero Berry curvature

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We have studied the Hamiltonian systems with non-zero Berry curvature. We focused on the equations of motion and found new brackets. They generalize the Poisson brackets and provide new bracket relations between coordinate, momentum and angular momentum.

Approximate symmetries of partial differential equations in viscoelasticity

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In many problems of physical interest differential equations contain terms involving "small" parameters. The combined treatment of the theory of Lie groups and perturbation analysis leads to the development of the Theory of approximate symmetries. To investigate equations involving such small terms, the approximate symmetry perturbation approach can be used. In the framework of nonlinear viscoelasticity, we consider the 2×2 system of partial differential equations

$$u_t - v_x = 0, \tag{1}$$
$$v_t - f(u)\, u_x = [\lambda(u)\, v_x]_x\,. \tag{2}$$

By considering u as the specific volume, $p(u) = \int^u f(s)\, ds$ the pressure, $\lambda(u)$ the viscous variable coefficient and v the velocity, the system (1)-(2) physically describes the one-dimensional, compressible, viscous flow of a fluid, treated from the lagrangian point of view. Particular cases of equations belonging to the class of the system (1)-(2) can be found in [1]-[2]. Having in mind to perform an "approximate symmetry analysis", we introduce a small parameter ε

$$u_t - v_x = 0, \tag{3}$$
$$v_t - f(u)\, u_x = \varepsilon\, [\lambda(u)\, v_x]_x\,. \tag{4}$$

Following [3]-[5], we perform the approximate symmetry analysis of the model (3)-(4) and, in some physical cases, approximate solutions are computed by means of the approximate generator of the first order approximate group of transformations.

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Two-dimensional superintegrable quantum systems with potentials expressed in terms of Painlevé transcendents

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We consider quantum superintegrable Hamiltonians that admit separation of variables in Cartesian coordinates and allow the existence of a fourth-order integral of motion in the two-dimensional Euclidean space. The most interesting ones involve potentials expressed in terms of Painlevé transcendents. We show how the results are related to the third-order superintegrable systems.

On solutions to the Burgers equation with a periodic boundary condition on an interval

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Properties of the solutions to the Burgers equation $u_t = \varepsilon^2 u_{xx} - 2uu_x$ on a finite interval $x \in [0,\, L]$ are studied. The initial value/ boundary conditions model a periodic perturbation on the left boundary:

$$u(x,0) = a, \quad u(0,t) = a + b\sin(\omega t), \quad u_x(L,t) = 0$$

The asymptotics of the solution for this problem at $L \to \infty$ coincides with the well known Fay solution [1]. In particular, $\lim_{x\to+\infty} u(x,t) = a$, which is the solution's average value over $x > 0$.

Not so for another asymptotics, at $t \to +\infty$. The form of the solution retains the sawtooth profile [2],[3], yet its average over $[0, L]$ differs from a and depends also on the perturbation amplitude b. Interaction between two perturbations of different frequencies is discussed.

Solutions to the nonhomogeneous Burgers equation $u_t - \varepsilon^2 u_{xx} + 2uu_x = A\sin(\omega t)$, [3], are shown to have similar properties.

The action of symmetries on the sawtooth solutions is studied.

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Hamiltonian fluid reduction of drift-kinetic equations for non-dissipative plasmas

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Drift-kinetic models (see, e.g. Ref. [1]) are commonly adopted to describe the dynamics of plasmas in the presence of an intense magnetic field. Also, from drift-kinetic equations, fluid models can be derived, which describe the evolution of a finite number of moments of the drift-kinetic distribution function. For plasmas where dissipative effects are negligible, such as the essentially collisionless plasmas of the magnetosphere or of the core of tokamak fusion devices, drift-kinetic equations are supposed to possess a Hamiltonian structure. Their reduction to a finite set of fluid equations, however, might in general violate the Hamiltonian structure of the parent drift-kinetic model, depending on the adopted closure relation. An uncontrolled closure could in particular lead to the introduction of unphysical dissipation in the resulting fluid model.

In this contribution I will describe a closure relation that leads to a Hamiltonian fluid model, starting from a Hamiltonian drift-kinetic model. In the two-dimensional (2D) limit, where translational invariance is imposed along the direction of the dominant component of the magnetic field, the Poisson bracket of the fluid model is obtained from a non-trivial extension of the Lie algebra associated with the Lie-Poisson bracket of the 2D Euler equation for an incompressible fluid. The extension to the 3D case can be carried out by means of a procedure described in Ref. [2], which allows to extend to 3D a 2D Poisson bracket from plasma fluid models assuming a strong magnetic field along a spatial direction. Alternatively, the Poisson structure of the fluid model can be obtained by carrying out a change of variables which unveils the direct sum structure of the Poisson bracket. In terms of the new set of variables, Casimirs of the Poisson bracket for the fluid model, which turn out to be associated with Lagrangian invariants, can also be easily derived.

The results presented in this contribution extend those described in Ref. [3].

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On a quaternionic Riccati differential equation and generalized analytic functions

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We consider a nonlinear partial differential equation for complex quaternion-valued functions, related to the three-dimensional stationary Schrodinger equation and supersymmetric quantum mechanics [1, 2, 3], and enjoys many properties similar to those of the ordinary differential Riccati equation such as Euler and Picard theorems. Complex quaternionic Vekua type equations arising from the factorization of the three-dimensional stationary Schrodinger equation are studied. Some concepts from classical pseudoanalytic function theory [4] are generalized onto the considered spatial case [5]. The derivative and antiderivative of spatial pseudoanalytic function are introduced and their applications to the quaternionic Riccati are considered.

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Integrable Burgers-type two-component systems with non-diagonalizable linearity

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We present the results of our higher symmetry classification of two-component, second and third-order ($N = 2, 3$) evolutionary systems homogeneous in Burgers-mKdV-pKdV weighting that have non-diagonalizable ($\epsilon \neq 0$) constant matrices ($a = 0, 1$) as the coefficient of highest order x-derivative terms:

$$\begin{pmatrix} u \\ v \end{pmatrix}_t = \begin{pmatrix} a & \epsilon \\ 0 & a \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}_{Nx} + \begin{array}{c} \text{lower order polynomial terms} \\ \text{homogeneous in Burgers weighting.} \end{array}$$

Some new second and third order (symmetry) integrable systems with their master symmetries are presented. Five out of the eight third-order systems are observed to possess conservation laws also. One of the third-order systems is found to be related to the Sasa-Satsuma system. We give a bi-Hamiltonian structures of one of the new systems and comment on the structures of the remaining three.

In the classifications of Burgers-type systems rewieved in [1], two-component cases of [2] and those in [3] the coefficient matrix of the leading order (linear) terms is taken to be the identity matrix. In [4] and two-component cases of [5] the matrix is taken to be arbitrary but diagonal. Our classification covers the non-diagonal(izable) cases of the coefficient matrix of the leading order terms.

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The inverse scattering transform for the focusing nonlinear Schrödinger equation with a one-sided non-zero boundary condition

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We present the inverse scattering transform as a tool to solve the initial-value problem for the focusing nonlinear Schrödinger equation with one-sided non-zero boundary value $q_r(t) \equiv A_r\, e^{-2iA_r^2 t + i\theta_r}$, $A_r \geq 0$, $0 \leq \theta_r < 2\pi$, as $x \to +\infty$. The direct problem is shown to be well-defined for solutions $q(x,t)$ to the focusing nonlinear Schrödinger equation such that $[q(x,t) - q_r(t)\vartheta(x)] \in L^{1,1}(R)$ [$\vartheta(x)$ denotes the Heaviside function] with respect to $x \in R$ for all $t \geq 0$, for which analyticity properties of eigenfunctions and scattering data are established. The inverse scattering problem is formulated both via (left and right) Marchenko integral equations and as a Riemann-Hilbert problem on a single sheet of the scattering variables $\lambda_r = \sqrt{k^2 + A_r^2}$, where k is the usual complex scattering parameter in the inverse scattering transform. Unlike the case of fully asymmetric boundary conditions [2] and similarly to the same-amplitude case dealt with in [1], the direct and inverse problems are also formulated in terms of a suitable uniformization variable that maps the two-sheeted Riemann surface for k into a single copy of the complex plane. The time evolution of the scattering coefficients is then derived, showing that, unlike the case of solutions with the same amplitude as $x \to \pm\infty$, here both reflection and transmission coefficients have a nontrivial (although explicit) time dependence. These results will be instrumental for the investigation of the long-time asymptotic behavior of physically relevant solutions to the focusing nonlinear Schrödinger equation with nontrivial boundary conditions, either via the nonlinear steepest descent method on the Riemann-Hilbert problem, or via matched asymptotic expansions on the Marchenko integral equations.

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On the Hamiltonian structure of hydrodynamic-type systems of conservation laws

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The theory of quasilinear equations of first order is one of the most developed parts in integrable systems. However their Hamiltonian formulation is an open question in general. B.A. Dubrovin and S.P. Novikov introduced the concept of homogeneous differential-geometric Poisson brackets in 1983-1984. A subclass of the hydrodynamic type systems can be equipped by first order homogeneous Hamiltonian operators. However, some hydrodynamic type systems can be equipped by third order homogeneous Hamiltonian operators.

The first examples were found by O.I. Mokhov (2-component case, Chaplygin gas equation) [1] and by E.V. Ferapontov, O.I. Mokhov, C.A.P. Galvao and Ya. Nutku (3-component reformulation of WDVV equations) [2]. Recently, R.F. Vitolo and M.V. Pavlov found a 6-component example (from the WDVV hierarchy) [4].

In this poster we find a new criterion which allows us to effectively reconstruct such a Hamiltonian operator if the corresponding hydrodynamic type system is written in Casimirs, hence if the system is written as a system of conservation laws. Conversely, the criterion can be used to describe all possible hydrodynamic type systems for each given third order homogeneous Hamiltonian operator. We solve this problem using our classification of 3-component homogeneous Hamiltonian operators [3].

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On local and nonlocal variational constants of motion

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Let $q(t)$ be a solution to Euler-Lagrange equation for a smooth Lagrangian $L(t, q, \dot{q})$, with q in an open set of R^n, and let $q_\lambda(t)$, $\lambda \in R$, be a smooth family of perturbed motions, such that $q_0(t) \equiv q(t)$. Then the following function is constant:

$$t \mapsto \partial_{\dot{q}} L\big(t, q(t), \dot{q}(t)\big) \cdot \partial_\lambda q_\lambda(t)\big|_{\lambda=0} - \int_{t_0}^{t} \frac{\partial}{\partial\lambda} L\big(s, q_\lambda(s), \dot{q}_\lambda(s)\big)\Big|_{\lambda=0} ds$$

($\partial_{\dot{q}}$ gradient with respect to the vector $\dot{q}$ and $\cdot$ scalar product in R^n). This constant of motion is generally *nonlocal*, and while it is often trivial or of no apparent practical value, there are cases when it is interesting and useful.

We can get genuine first integrals for $L = \frac{1}{2}\|\dot{q}\|^2 - U(q)$ with U homogeneous of degree -2, in particular Calogero's potential, and $q_\lambda(t) = e^\lambda\, q\big(e^{-2\lambda} t\big)$. This example is taken from [1], while the other applications come from [2].

We also find nonlocal constants of motion which give global existence and estimates for the solutions of the dissipative equation $\ddot{q} = -k\dot{q} - \partial_q U(q)$, when $k > 0$ and $U : R^n \to R$ is bounded from below. In this case the Lagrangian is $L = e^{kt}\big(\frac{1}{2}\,|\dot{q}|^2 - U(q)\big)$ and the family is $q_\lambda(t) = q\big(t + \lambda\, e^{kt}\big)$.

Finally, we show a nonlocal constant of motion for the Maxwell-Bloch system in Lagrangian formulation which leads to separation of one of the variables. This is done with the Lagrangian $L = \frac{1}{2}\big(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2 + \dot{q}_3\big(q_1^2 + q_2^2\big)\big)$ and the family $q_\lambda(t) = \big(e^\lambda q_1(t), e^\lambda q_2(t), e^{-2\lambda} q_3(t)\big)$.

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Unique characteristics of multi-front solutions of Sine-Gordon equation in higher dimensions

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The Sine-Gordon equation in (1+2) and (1+3) dimensions is not integrable. Still, the Hirota algorithm generates N-front solutions of that equation for all N. Non-integrability of the equation in these higher space dimensions affects the physical characteristics of the solutions. In (1+2) dimensions, each multi-front solution propagates rigidly at a constant velocity, v. The solutions are divided into two unconnected subspaces. In one sunspace the velocity of each solution obeys $v > c$ or $v = c$; in the other subspace, $v < c$ ($c = 1$). Each subspace is connected by an invertible transformation (rotation plus dilation) to the space of front solutions of an integrable Sine-Gordon equation in two dimensions. The faster-than-light solutions are connected to the solutions in (1+1)-dimensional Minkowski space by a linear (rotation + dilation) transformation. The slower-than-light solutions are connected to the solutions in 2-dimensional Euclidean space by a similar transformation and also by Lorentz transformations. The Sine-Gordon equation in (1+3)-dimensional Minkowski space has a richer variety of multi-front solutions. Its slower-than-light solutions are connected to the solutions of the integrable equation in 2-dimensional Euclidean space. However, only a subset of its faster-than-light solutions is connected to the solutions of the integrable equation in (1+1)-dimensional Minkowski space.

New mechanism for mass generation: Coupled linear wave equation and Sine-Gordon equation in (1+2) and (1+3) dimensions

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Coupling of the linear wave equation and the Sine-Gordon equation in (1+2) and (1+3) dimensions offers a new mechanism for mass generation. A driving term, which is generated from a slower-than-light, multi-front solution of the Sine-Gordon equation, enables the linear wave equation, which in itself would generate solutions that represent massless particles, to admit a solution that is localized in space and emulates a free, spatially extended, massive relativistic particle. The localized solution is an image of the junction, or junctions, at which the Sine-Gordon fronts intersect. It propagates together with the multi-front solution at the velocity of the latter. This result can be also formulated through the expansion in powers of a small coupling coefficient of the Euler-Lagrange equations of a Lagrangian system.

Equilibrium configurations of the surface of a conducting fluid in the nonuniform magnetic field

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A magnetic field leads to the deformation of the free surface of the fluid placed in it. A high-frequency magnetic field penetrates only into a thin surface layer of a conducting fluid. If the thickness of the layer is smaller than the characteristic size of the structures on the surface, it can be assumed that the field does not penetrate into the fluid. At times far exceeding the oscillation period, the problem can be considered quasi-stationary. Under certain conditions, the capillary and time-averaged magnetic pressures can equilibrate each other. The problem of finding the corresponding equilibrium configurations is formally equivalent to the problem of a perfectly conducting fluid in a constant magnetic field not penetrating into the medium. Neglecting capillary forces, Shercliff [1] applied this approach for analyzing the configurations of liquid metal columns. Note that numerical solutions for a similar problem were obtained in Ref. [2].

We consider a problem of determining the equilibrium configurations of the free surface of a perfectly conducting fluid that is deformed by a nonuniform magnetic field generated by the system of parallel current-carrying linear conductors. In the case of a plane geometry of the problem, where the method of conformal mapping can be used, we obtain exact solutions of two different problems concerning the deformation of (i) the initially flat fluid surface and of (ii) the surface of a cylindrical jet. In the first case, two-dimensional holes are formed under the current-carrying linear conductors located parallel to the fluid surface. With the current growth, they are transformed into two-dimensional bubbles covering the conductors (see also Ref. [3]). In the second case, the jet is deformed by the transverse magnetic field until its splitting into separate jets.

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