

Parabolic regularization of the gradient catastrophes for the Burgers-Hopf equation and Jordan chain

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Non-standard parabolic regularization of gradient catastrophes for the Burgers-Hopf equation is proposed. It is based on the analysis of all (generic and higher order) gradient catastrophes and their step by step regularization by embedding the Burgers-Hopf equation into multi-component parabolic systems of quasilinear PDEs with the most degenerate Jordan block. Probabilistic realization of such procedure is presented. The complete regularization of the Burgers-Hopf equation is achieved by embedding it into the infinite parabolic Jordan chain. It is shown that the Burgers equation is a particular reduction of the Jordan chain. Gradient catastrophes for the parabolic Jordan systems are also studied.