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Book of abstracts

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ICCCM 2023

VII International Conference on Computational Contact Mechanics

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SCIENTIFIC PROGRAM

Wednesday, 5 July		
8:00	**Registration**	
9:00	**Opening**	
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	Peter Wriggers	Contact of flexible polyhedra modelled by virtual elements
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	Rolling contact	
	10:00	A novel Boundary Element formulation for steady-state viscoelastic circular contacts Santeramo M., Putignano C., Vorlaufer G., Krenn S., Carbone G.
	10:20	Wear and power dissipation modelling in wheel-rail contact Myśliński A., Chudzikiewicz A.
	10:40	Non-linear dependencies of filler-reinforced elastomers in rolling contact: An experimental and numerical investigation de Lorenzo Oliveira M., Le Tallec P., Bussetta P., Berger E., Nuytten S.
11:00	**Coffee break**	
11:20	**Session 2**	
	A - Adhesion and Debonding	
	11:20	A thermodynamic motivated RCCM damage interface model using explicit dynamics CD-Lagrange scheme Larousse P., Dureisseix D., Gravouil A., Georges G.
	11:40	Numerical simulation of adhesive contacts of an elastic quarter space with freely sliding side Li Q., Popov V.L.
	12:00	The effect of adhesion in viscoelastic contact mechanics Mandriota C., Carbone G., Menga N.
	B - Delamination, fracture and failure processes	
	12:20	Numerical simulation for contact problems in crack growth with phase field approach Takaishi T., Kimura M.

	12.40	Extending non-smooth contact mechanics to cohesive zone modelling Collins-Craft N.A., Bourrier F., Acary V.
	13:00	Double peeling of thin viscoelastic tapes from rigid substrate Ceglie M., Menga N., Carbone G.
13:20	**Lunch**	
14:20	**Keynote 2**	
	Mike Puso	Recent topics on immersed boundary methods, unbiased mortar contact and contact dynamics
15:00	**Session 3**	
	Solution algorithms and numerical efficiency – Section 1	
	15:00	Combination of adaptive mesh refinement and high performance computing for accurate solution of elastostatics contact mechanics problems Epalle A., Ramière I., Latu G., Lebon F.
	15:20	Efficient and accurate numerical time integration of arbitrary fine meshed small sliding contact via contact modes and hyper reduction Witteveen W., Koller L.
	15:40	Hybrid domain decomposition for huge contact problems Dostal Z., Brzobohatý T., Horák D., Vlach O.
	16:00	Stabilized FEM for contact problems Gustafsson T., Stenberg R., Videman J.
	16:20	An adaptive quasistatic contact model based on IGA applied to impact analysis in flexible multibody systems Rückwald T., Held A., Seifried R.
16:40	**Coffee break**	
17:00	**Departure for the visit of the Royal Palace**	

Thursday, 6 July		
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8:40	**Keynote 3**	
	Rolf Krause	Multilevel Strategies for Contact Problems - from inner to outer approximations of the feasible set
9:20	**Session 4**	
	Solution algorithms and numerical efficiency – Section 2	
	9:20	A Novel Method for Multibody Dynamics Contact Represented by Linear Complementarity Problems Lou Q., Kovecses J.
	9:40	Multiscale methods for substructuring multibody systems with contact Hutchison C., Hewlett J., Kövecses J.
	10:00	Targeting a faster time-to-solution of mortar-based contact problems Steimer C., Mayr M. Popp A.
	10:20	Model order reduction for the fatigue life prediction of wire ropes in tension and bending Guidault P.A., Zeka D., Néron D., Guiton M., Enchéry G.
	10:40	Numerical analysis of a non-clamped dynamic thermoviscoelastic contact problem Bartman P., Bartosz K., Jureczka M., Szafraniec P.
11:00	**Coffee break**	
11:20	**Session 5 (parallel session)**	
	Solution algorithms and numerical efficiency – section 3	
	11:20	Error analysis of the Hybrid Hyper-Reduction method for frictionless contact problems. Le Berre S., Ramière I., Ryckelynck D.
	11:40	Contact mechanics and inseparability: towards dictionary-based sparse approximations Kollepara K.S., Aguado J., Le Guennec Y., Silva L.
	12:00	Parallel, High Performance Contact Solvers Dokken J., Richardson C., Roggendorf S., Wells G.N.
	12:20	Energy conserving contact-impact algorithm using the method of Lagrange multipliers and the explicit central difference time integration scheme Markovic D., Casadei F., Larcher M.

	12:40	Interior point methods for computing frictional contact problems with hyperstaticity Nguyen M.H., Acary V., Armand P.
	13:00	A monolithic computational method for elasto-dynamics with plasticity and contact based on variational approach. Acary V., Bourrier F., Viano B.
11:20	**Session 5B (parallel session)**	
	A - Modelling of friction joints under dynamical loading	
	11:20	Variability of vibration response in friction-damped structures due to non-unique residual tractions: Computation of bounds Ferhatoglu E., Groß J., Krack M.
	11:40	Competing dry friction contact models for underplatform dampers Gastaldi C., Gola M.
	B - Multi-scale approaches	
	12:00	Homogenization based two-scale modelling of fluid-saturated porous media with self-contact in micropores Rohan E., Heczko J.
	12:20	Multi-scale analysis of contact of rough surfaces through FEM/BEM code coupling Shaw R., Mayr M., Popp A.
	C - Multifield problems with contact constraints	
	12:40	A thermodynamically consistent computational framework for brittle crack propagation along contact interfaces Athanasiadis I., Shvarts A., Lewandowski K., Pearce C., Kaczmarczyk L.
	D - Contact in biomechanics	
	13:00	An Embedded Approach for Fluid-Structure-Contact Interaction Problems and application to the aortic flow. Nestola M.G.C., Zulian P., Rossinelli D., Krause R.
13:20	**Lunch**	
14:20	**Session 6 (parallel session)**	
	A - Solution algorithms and numerical efficiency – Section 4	
	14:20	Rattle for mechanical systems with frictional unilateral constraints Harsch J., Capobianco G., Eberhardt L., Eugster S.R., Leine R.I.
	14:40	An algebraic domain decomposition strategy for the solving contact problems

		Kothari H., Zulian P., Krause R.
	15:00	Novel approach for accurate identification of the real contact area for numerical modelling of triboelectric nanogenerators Shvarts A.G., McGinn T., Athanasiadis I., Kaczmarczyk L., Pearce C.J., Kumar C., Mulvihill D.M.
	15:20	Improving performance of augmented Lagrangians Horak D., Dostal Z., Kruzik J., Vlach O.
	B - General papers – Section 1	
	15:40	Beam-inside-beam contact Magliulo M., Lengiewicz J., Zilian A., Beex L.
	16:00	Physics-informed neural networks for contact mechanics Sahin T., von Danwitz M., Popp A.
14:20	**Session 6B (parallel session)**	
	A - Discrete element methods for contact	
	14:20	Simulating tribocharging of flowing granular materials with patchy particles Preud'homme N., Opsomer E., Lumay G.
	14:40	An Improved Normal Compliance Method for Non-Smooth Contact Dynamics Abide S., Barboteu M., Dumont S., Nacry F., Nguyen V.A.T.
	15:00	Comparative analysis of experimental and numerical results for viscoelastic indentation of thin layers Mikayilov E., De Carolis S., Santeramo M., Carbone G., Putignano C.
	B - Constraints enforcement methods	
	15:20	Third-medium model for contact and pneumatic actuation Faltus O., Rokoš O., Horák M., Doškář M.
	15:40	Nested objective functions for frictional contact Hurtado D.R., Beex L.
	C - Contact at the nanoscales	
	16:00	A coarse-grained molecular dynamics method for simulating fatigue crack propagation Niknafs S., Silani M., Concli F., Aghababaei R.
16:20	**Coffee break**	
16:40	**Session 7**	
	Friction and wear	

	16:40	Breakdown of Reye's theory in single-asperity wear Garcia-Suarez J., Brink T., Molinari J.F.
	17:00	Railway wheel wear calculation: comparison of local and global applications of Archard's law Bosso N., Magelli M., Zampieri N.
	17:20	A Novel Single Pass Unbiased Frictional Contact Algorithm Sahu I., Petrinic N.
	17:40	Effect of frictional weakening in fretting wear Yastrebov V.A., Basseville S.
18:00	**Session end**	
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20:40	**Conference dinner at the restaurant of the "Società Canottieri Caprera"**	

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8:10	**Registration**		
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	Loïc Salles	Computational methods in Tribomechadynamics	
9:20	**Session 8**		
	Contact detection algorithms		
	9:20	Novel framework for modelling contact of curved surfaces Bignold S., de Frias G., Shelton T., Buche M., Wagman E., Miller S., Beckwith F., Manktelow K., Merewether M., Parmar K, Thomas J., Trageser J., Treweek B., Veilleux M.	
	9:40	Strategy to address two-dimensional pointwise concave contact problems da Silva L., Craveiro M.V., Gay Neto A.	
	10:00	High-Fidelity Stress Fields in Contact Problems using Beam, Plates, and Shells Layer-Wise Models Saputo S., Petrolo M., Pagani A., Carrera E.	
	10:20	Robust and generic contact detection strategy using tandem traversal of Bounding Volumes Hierarchies and spatio-temporal intersection Motte A., Bovet C., Chiaruttini V., Jamond O., Prabel B.	
	10:40	A refined algorithm for hierarchical face clustering and contact detection for segment-to-segment contact search between three-dimensional deformable bodies with irregular surface meshes Chuo M.C.K., Izzuddin B.A.	
11:00	**Coffee break**		
11:20	**Session 9**		
	Discretization techniques		
	11:20	Virtual element methods and higher order penalty-based Node-to-Segment contact Moherdaui T.F., Gay Neto A., Wriggers P.	
	11:40	Use of nonsymmetric unilateral cinematic constraints to solve Coulomb contact/friction problem Verpeaux P., Breuzé M.	
	12:00	Towards an Embedded Mesh Approach for Isogeometric Boundary Layers in Contact Mechanics Loera Villeda E.G., Steinbrecher I., Popp A.	

	12:20	HHT-α and TR-BDF2 schemes for dynamic contact problems Huang H., Pignet N., Drouet G., Chouly F.
	12:40	Application of a posteriori analysis to contact problems Fontana I., Di Pietro D.A., Kazymyrenko K.
	13:00	An arbitrary order contact formulation using Lagrange multipliers from Raviart-Thomas space Kaczmarczyk L., Athanasiadis I., Shvarts A.G., Lewandowski K., Pearce C.J.
13:20	**Lunch**	
14:20	**Session 10**	
	General papers – Section 2	
	14:20	Tube/projectile interaction modeling using finite element simulation Collas T., Lebon F., Rosu I., Ningre C.
	14:40	Fast simulations of parametric problem with contact non-linearity Pawar G.S., Kulkarni S.S.
	15:00	Identification of contact traction and material parameters for soft bodies Lavigne T., Bordas S.P.A., Lengiewicz J.
	15:20	Third Medium Contact Method for Topology Optimization Frederiksen A.H., Sigmund O., Poulios K.
	15:40	Beam lattice metamaterials with internal contact and instabilities Horak M., La Malfa Ribolla E., Jirásek M.
	16:00	Still on the shifted penalty method Zavarise G.
16:20	**Coffee break**	
16:40	**Conference closure**	

SESSION 1
Rolling contact
Wednesday, 10:00 – 11:00

A novel Boundary Element formulation for steady-state viscoelastic circular contacts

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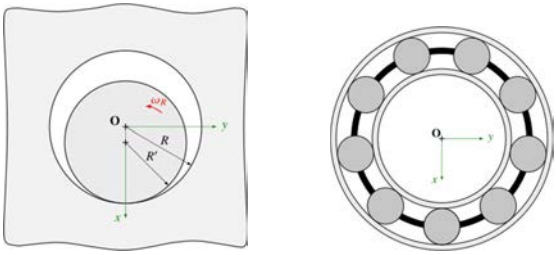
Keywords: Viscoelasticity, Friction, Soft matter.

In the last decades, soft mechanics has attracted more and more attention from a broad cross-disciplinary community, due to the really ubiquitous presence of soft materials in everyday life. As a matter of fact, the latter are widely spreading in industry, where an increasing shift from hard metals to polymeric materials is observed. Indeed, because of their advantageous characteristics in terms of lighter weights, environmental resilience, and lower manufacturing costs, polymers and polymer composites are now frequently deployed in power transmission components. Examples include gears, seat belts, tires, bearings, seals, etc. However, difficulties arise in predicting the mechanical response of such materials. Thus, the main angle of worldwide researchers is to provide a better understanding of viscoelastic materials, which exhibit a pronounced time- and temperature-dependent behavior.
Therefore, the intricate rheology that distinguishes these materials contributes to the complexity of contact problem formulation, which is exacerbated when soft bodies come into contact. In particular, it is well known that at very low speeds the deformable solid behaves as a soft elastic body, as it enters the so-called elastic rubbery region, and viscoelastic energy dissipation is negligible. On the other hand, at very high speeds, the material is in the glassy region and behaves elastically, but it is much stiffer than in the rubbery zone. Instead, the transition region is mainly the region in which energy dissipation occurs during rolling or sliding conditions, and thus viscoelasticity becomes crucial.
A variety of analytical, numerical, and experimental approaches have helped to enhance our comprehension in the field of viscoelastic mechanics; though, attention has been paid to the paradigmatic example of non-conformal contact, where a rigid indenter slides/rolls over a viscoelastic half-plane. Nonetheless, circular contact problems are of the utmost importance in mechanical applications: the pin-joint or the rolling element bearing are representatives of conformal and conformal-like problems. Furthermore, this type of problem is very often encountered in biomechanics, where hip joints and prostheses are representatives.
The focus of this work is then to provide a further contribution to the field by investigating the steady-state rolling/sliding circular contact. Specifically, we provide a Boundary Element methodology for the paradigmatic circular contact problem of a rigid cylindrical pin rolling into a holed infinite viscoelastic space, as sketched in Figure 1a. Validation of numerical predictions with the existing analytical solution provided by A. Persson [1] for the elastic case assessed the approach effectiveness, blazing the trail to the viscoelasticity regime investigation. It is crucial to emphasize that the proposed methodology is not limited to conformal contacts because the formulation is generally developed for both conformal and non-conformal surfaces. In particular, it is based on the

definition of ad-hoc Green's functions [2], via a complex-variable method [3,4], which take into account the circular hallmark of the contact domain, and it is able to deal with real viscoelastic materials, characterized by a continuous spectrum of relaxation times.
The present methodology paves the way for further research in lubricated contacts, such as the case of a journal bearing where the shaft and the bushing are both linearly viscoelastic, and in multiple contact problems, such as the case of a rolling element bearing with linearly viscoelastic races and metallic rolling elements (see Figure 1b), where viscoelasticity plays a fundamental role in the bearing load distribution. Furthermore, early experimental outcomes seem to corroborate, in terms of friction, the numerical results.



(a) (b)

Figure 1: Schematic of a rigid cylindrical pin rolling into a holed infinite viscoelastic space (a); schematic of a rolling element bearing (b).

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Wear and power dissipation modelling in wheel-rail contact

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Keywords: elasto-plastic contact, friction, wear, energy dissipation, FE model, generalized Newton method, wear distribution.

The paper is concerned with the numerical evaluation of energy dissipation flow and wear [1,3] in wheel-rail contact problems. Wear is a complex physical process characterized by the deformation and removal of material from a solid surface due to the mechanical action exerted by the another solid [7,8]. Many different physical and/or chemical factors may generate the occurrence of the wear phenomenon on contacting surfaces [2,4,6,10].

The two-dimensional wheel-rail contact problem between a rigid wheel and an elasto-plastic rail lying on a rigid foundation is considered. The contact phenomenon includes Coulomb friction, frictional heat generation as well as the wear of the contacting surfaces. The displacement and stress of the rail in contact are governed by the coupled elasto-plastic and heat conductive equations. The elastic and plastic responses are approximated, respectively, by Hooke's law and by von Mises yield criterion with isotropic power law hardening.The wear depth function appears as an internal variable in the non-penetration condition updating the gap between the worn surfaces of the bodies. Moreover the dissipated energy due to friction is calculated to evaluate the loss of the rail material and to determine the shape of the contacting surfaces during the wear evolution process. Therefore the wear phenomenon is modeled by the combined Archard and power dissipation models. Focusing on energy dissipation evolution in wheel-rail elasto-plastic contact problems extends the authors results from [9].

The contact problem is solved numerically. The finite element method is used to discretize it. The original coupled problem is solved numerically using the splitting method [5,9]. In this approach first for a given temperature the displacements, stresses and wear depth are calculated using the generalized Newton method. The plastic flow and friction inequality conditions are reformulated as equality conditions using the nonlinear complementarity functions [5]. In the next step, for a given displacement and stress, the temperature is updated using Cholesky method. The distribution of the surface flash temperatures and stresses as well as the evolution of the dissipated energy as well as the wear depth and the shape of the contact surfaces due to wear are reported and discussed.

The presented numerical results indicate that the obtained contact patches are characterized by longer zones and lower stress intensity than in the elastic case. The dissipated energy method and contact interface shape update strategy are efficient and precise tools to evaluate the wear distribution. The impact of hardening parameters as well as temperature dependent material parameters and creepage on the wear evolution process requires further research.

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Non-linear dependencies of filler-reinforced elastomers in rolling contact: An experimental and numerical investigation

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Keywords: Contact, Friction, Elastomer, Viscoelasticity, Constitutive Law, Non-linear, Tribometer, Finite Element Method.

In the automotive industry, the research and development of tires often requires careful consideration of many tribological contributions to sliding friction between industrial filled-rubber elastomers and the road. Among many potential mechanisms in place, we highlight: hysteresis, adhesion, lubrication, flash-heating, dewetting and wear, which can be coupled across different asperities sizes/scales at the rough interface [1]. Within a multi-scale friction framework, two frequent numerical strategies are mentioned: half-space approximations in a Boundary Element formulation [2]; and contact homogenization techniques for Finite Elements [3]. The first approach provides a fast and practical methodology to compute complex contact interactions, however in many cases this choice is, by construction, restrained to small transformations and linear constitutive models. The second approach can be extended to finite strains and more complex constitutive models at the cost of a higher computational complexity.

Due to the complex numerical trade-off between fast and straightforward implementation versus robust and accurate multi-scale approaches added with the convoluted interaction of experimental processes, the impact of a non-linear constitutive law in the resulting friction is often overlooked. What makes the subject elusive in tribology, from an experimental standpoint, is the difficulty to devise a procedure that isolates the hysteresis contribution and diminishes the other mechanisms occurring at the interface. From a numerical perspective, even if a simpler contact indentation is considered on a Finite Elements environment, many softwares utilize Hyper-viscoelastic models, based on a division of the stress tensor between an equilibrium hyperelastic contribution and a non-equilibrium viscoelastic contribution (N parallel Maxwell rheological models). However, there is no guarantee, *a priori*, that this class of constitutive models satisfies the second principle of thermodynamics [4].

In this work, to address the considerations made above, a simple experimental procedure was devised. A landing contact procedure (compression with sliding) for a range of compressive loads and velocities was imposed between a filler-reinforced rubber provided by Michelin and a simple substrate by means of a hydraulic angular tribometer to recover its friction response, as shown in Figure 1.

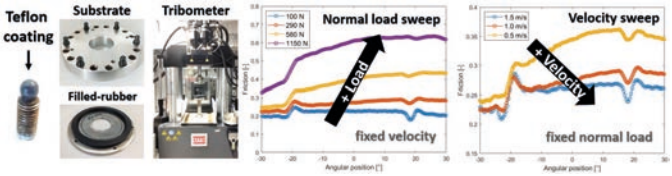

Figure 1: Experimental main components; friction results for varying velocity and/or compressive load.

The substrate component is a cylindrical base containing a ring of detachable spherical indenters which will enter in contact with the elastomer. To reduce the multi-physical coupling, the spheres were coated in Teflon to minimize the contribution of smaller scales, the contact test always occurs submerged in water with controlled temperature, the water was mixed with small doses of an emulsion to reduce its surface tension, thereof its risk of dewetting.

The experimental conditions are to be compared with its numerical reconstruction in an Finite Element framework with a second order explicit solver (commercial software *Impetus Afea*) with a penalty contact method, as shown schematically in Figure 2. To study the consequence of non-linear constitutive effects, two material models calibrated with Dynamic Mechanical Analysis procedures are considered: an exemplary hyper-viscoelastic formulation with a Mooney-Rivlin free-energy and 4 maxwell branches and another, thermodynamicaly stable, constitutive law [5] based on the work of Lopez-Pamies [6].

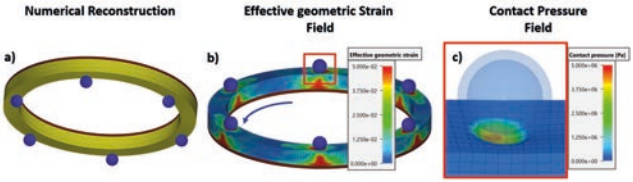

Figure 2: (a) Numerical reconstruction into. (b) Effective Geometrical Strain Field suring sliding. (c) Contact pressure field on the highlighted (in red) region from (b).

This proposition evaluates the pertinence of the experimental test to represent the hysteresis contribution to friction and the impact of non-linear coupled deformation and deformation rate sensitivities to represent the experimental tribological response.

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SESSION 2
Adhesion and debonding, Delamination, fracture and failure processes
Wednesday, 11:20 – 13:20

A thermodynamic motivated RCCM damage interface model using explicit dynamics CD-Lagrange scheme

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Keywords: Contact, Cohesive Zone Model, Symplectic Scheme.

The unmoulding process of a tyre is a fast dynamic problem involving a non smooth interface behaviour with contact and decohesion. In previous works, unilateral contact and impact have been studied in explicit dynamics [1, 2] but no damage nor cohesion were involved. Combining a contact problem and a thermodynamically motivated damage model within the so-called CD-Lagrange explicit dynamics scheme is the aim of this work. To do so, the RCCM macroscopic model [3] of adhesion with damage of the interface is studied. The thermodynamic motivation of the model and the use of a symplectic explicit scheme creates a framework based on good energy balance property.

Predicting unmoulding tyre process is a motivation for developing numerical simulation tools, and a robust dynamic scheme is mandatory. Indeed, this process leads to fast dynamic events, such as impacts and interface fracture, and implicit schemes exhibit convergence issues and/or possibly high numerical cost, so explicit schemes are of interest. The explicit CD-Lagrange scheme provides interesting properties due to its symplectic nature, allowing good conservation properties, among which the discrete energy conservation. The aim is therefore herein to use this scheme as a framework for modeling more complex interfaces properties in a modular implementation.

To describe a rigid-deformable contact (as between a mould and a polymeric part), the Hertz-Signorini and Moreau [2] condition of unilateral contact with a frictional Coulomb's law written in velocity and impulse are used as in (1), (2) for the normal contact and (3), (4) for tangential contact:

$$\textbf{if } g > 0\textbf{, then } r_{c,N} = 0, \tag{1}$$

$$\textbf{else } g = 0\textbf{, and } 0 \leq r_{c,N} \perp v_{c,N} \geq 0, \tag{2}$$

$$\textbf{and if } g > 0\textbf{, then } r_{c,T} = 0, \tag{3}$$

$$\textbf{else } g = 0\textbf{, and } 0 \leq (\mu r_{c,N} - r_{c,T}) \perp ||v_{c,T}|| \geq 0, \tag{4}$$

g is the gap between both solids involved in the contact, $v_{c,N}$ and $v_{c,T}$ are respectively the normal and tangential velocity, $r_{c,N}$ and $r_{c,T}$ are respectively the normal and tangential impulses and μ is the friction coefficient.

Our work aim is to add an interface behaviour to the contact. To do so, the phenomena of adhesion and damage are highlighted. The physical interpretation of damage, especially on interface, is usually settled at microscopic scale. A macroscopic model is a phenomenological one, aimed to traduce local behaviours with macroscopic quantities such as a damage parameter [3]. For this study, a macroscopic model has been chosen to deal with the impact and the damage evolution, as a generalization of the RCCM model [3]. According to the value of this parameter, the stiffness of the interface is modified,

the higher is the damage, the lower is the stiffness. Developing a modular framework is therefore useful for implementation of such models, which is a perspective to the present study. The definition of this model is given in (5),(6):

$$\textbf{if } [u] = \alpha \textbf{ and } \alpha < u_r, \dot{\alpha} =< v_d >_+, \tag{5}$$

$$\textbf{else } \dot{\alpha} = 0, \tag{6}$$

$[u]$ is the relative displacement between both solids, v_d for the relative velocity, α the damage parameter, u_r a constant parameter which is the damage rupture and $\dot{\alpha}$ the damage rate. Only small displacements are studied herein, but the nature of the explicit scheme, such as the explicit computation of the structure configuration will bear interesting features for future developments for large displacements and rotations. To solve our problem, a Finite Element (FE) resolution code is used as in Figure (1) with a test case solution after application of a vertical traction force on the left hand side of the structure.

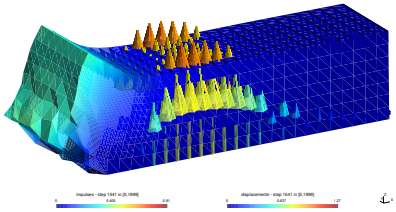

Figure 1: Mesh displacement (colors) and contact impulses (arrows) during Peeling Test

In conclusion, a modular framework composed of a thermodynamic motivated interface behaviour implemented in the explicit CD-Lagrange scheme formalism is created to solve contact problems with cohesive zone models in dynamics.

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Numerical simulation of adhesive contacts of an elastic quarter space with freely sliding side

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Keywords: Contact mechanics, quarter space, adhesion, edge effect, Boundary Element Method

In contact mechanics it is very often assumed that the loading is applied to a half space. However, this assumption is no longer valid if the contact region is on or very close to the sharp corners. This edge effect can be very often observed, for example in rail-wheel contacts, cylindrical bearings, gears or connections between turbine blades and discs. In this work, non-adhesive and adhesive contacts between a rigid body and an elastic quarter space is studied, where the movement of the side surface of the quarter-space is constrained by a rigid wall: it can slide freely along the wall. This is the simple case of Hetényi's contact problem [1][2]: one can obtain its solution easily by applying an additional mirror load to an elastic half-space. Numerical simulation is carried out using the Fast-Fourier-Transform (FFT)-assisted Boundary element method. In the case of adhesive contacts, the stress criterion proposed in [3] is used to determine the detachment of elements at the contact boundary, which is derived from the comparison of the elastic energy stored in the element with its surface energy. This approach has been extensively validated by comparison with analytical solutions. It can accurately reproduce the classic Johnson-Kendall-Roberts (JKR) results for Hertzian contact.

Simulation results show that depending on the position of the indenter relative to the side edge, different contact behavior is observed. In the case of adhesive contact, the force of adhesion first increases with increasing the distance from the edge of the quarter-space, achieves a maximum and decreases further to the JKR-value in large distance from the edge. The enhancement of the force of adhesion compared to the half-space contact is associated with the pinning of the contact area at the edge. The maps of the force of adhesion and their analytical approximations are provided [4].

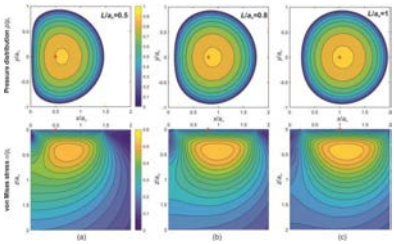

Figure 1: Contact area, pressure distribution on the surface and the internal von Mises stress for different indenter locations [4].

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The effect of adhesion in viscoelastic contact mechanics

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Keywords: Adhesion, viscoelasticity, friction

Predicting and understanding the tribological features resulting from the interplay between adhesion and viscoelasticity in sliding and rolling contacts between polymeric materials is a crucial issue in many engineering and industrial applications. To mention some examples: bio-inspired and biological systems, micro-electro-mechanical systems (MEMS), protective coatings, tire-road contact, micro-grippers, structural adhesives, pressure-sensitive adhesives, wear, sealings, touchscreens, windscreen wipers. Nowadays, a comprehensive theory of viscoelastic-adhesive sliding contacts is still lacking. Indeed, for sliding contacts, existing numerical and analytical approaches can predict the unknown contact domain only in adhesiveless conditions or by neglecting any viscoelastic effects [1]. Nevertheless, several existing experimental data highlight how adhesion might strongly affect the viscoelastic friction developed at the contact interface, both at low [2], intermediate, and high relative sliding/rolling velocity [3,4]. At low velocity the bulk of the viscoelastic material is excited at very low frequencies, and thus behaves as a soft elastic material, but local small-scale viscoelastic losses might still significantly affect the contact behavior, inducing an important adhesive frictional contribute (i.e., adhesion hysteresis). This has clearly been pointed out in [2], in which the authors report experimental data exploiting the tribological features of rolling contacts between rigid cylinders and rubbery-like substrates. One of the most relevant observed features is that the effect of local viscoelastic losses is to strongly increase the adhesive properties of the system, as the contact interface is able to withstand higher tensile forces with respect to static conditions as long as rolling occurs. The interplay between adhesion and viscoelasticity is still significant at intermediate and high velocity [3,4]. Specifically, a strongly increased friction coefficient with respect to adhesiveless conditions is experimentally measured, which is not fully understood yet. A possible explanation is that adhesion increases the contact area and, in turn, the amount of material experiencing viscoelastic hysteretic losses within the bulk (i.e., large scales viscoelasticity). Nevertheless, the experimental data from the Grosch's seminal work clearly indicate that the overall viscoelastic friction cannot be estimated by linearly superimposing the small- and large- scale viscoelastic losses. For this reason, predicting the size of the unknown contact area is of primary practical interest.

In this abstract, we report the results of a novel theory that we recently developed to study the adhesive sliding contact between viscoelastic bodies and rigid indenters [5,6]. The unknown contact domain is calculated by enforcing a proper energy balance for an infinitesimal contact area variation δa at each edge of the contact, i.e. Eq. (1), which generalizes the Griffith criterium to the viscoelastic case:

$$dU + \delta L_p = \Delta\gamma \delta a \tag{1}$$

where dU is the elastic energy variation, $\Delta\gamma$ is the so called Duprè work of adhesion, and δL_p is the non-conservative infinitesimal work done by the internal stress, which is directly related to the viscoelastic behavior of the material (vanishing in the case of purely elastic materials). The proposed theory provides possible insights into most of the existing experimental evidences. E.g., Fig.1 shows the predicted trend of the viscoelastic friction coefficient against the dimensionless sliding velocity

(a) and the corresponding experimental curve (b) from Ref. [4].

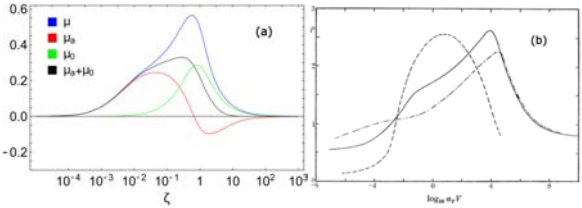

Figure 1: The trend of the friction coefficient against the sliding velocity: comparison between the theoretical predictions (a), and experimental data (b).

The blue line in Fig.1 (a) is the overall viscoelastic friction coefficient. The red line (μ_a) is the adhesive friction coefficient, which is proportional to the quantity $G_1 - G_2$, being G_1 and G_2 the energy release rates at the contact trailing and leading edge, respectively. The quantity μ_0 (green line) is the viscoelastic friction coefficient calculated in adhesiveless conditions. The predicted trends suggest that at low velocity friction mostly originates from the small-scale viscoelastic losses (i.e. adhesion hysteresis) as $\mu \approx \mu_a$. In such a range of speeds, a significative viscoelastic-induced enhanced adhesion is predicted, in agreement with experimental data reported in Ref. [2]. At intermediate and high velocity, the overall friction μ is highly increased with respect to the adhesiveless condition. The figure clearly indicates that $\mu \gg \mu_a + \mu_0$ (see black line), thus confirming that the small- and large- scale losses cannot be linearly separated. This is a relevant result, which indicates that models based on such assumption may fall short in describing the real frictional response. Finally, at very-low and very-high velocity, the material behaves elastically, thus friction vanishes. The experimental trends from Fig.1 (b) strongly support the theoretical predictions: adhesive friction (dashed line) is measured on a smooth sliding glass surface, whereas the adhesiveless condition (dot-dashed line) is ensured with a dusted silicon carbide surface; the solid line instead, whose trend is very similar to the one predicted by our theory, is the friction measured on a clean silicon carbide paper (i.e., a rough surface), and thus takes into account both the adhesive and viscoelastic effects.

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Numerical simulation for contact problems in crack growth with phase field approach

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Keywords: crack growth, phase field, contact surface.

The authors introduced the gradient flow model of crack growth with phase field approach[1] that is based on the approximated energy of cracked plate with variational approach by Bourdin-Francfort-Marigo[2] using Ambrosio and Tortorelli's approximation[3]. Here, we consider the crack growth in n-dimensional elastic material. Let $\Omega \subset \mathbf{R^n}$ be a bounded elastic body with crack. Let $u \in \mathbf{R^n}$ be a displacement, $z \in [0,1]$ be an phase field where $z = 1$ at cracked region and $z = 0$ at non-cracked region. The gradient flow model can be written as follows[1, 4]:

$$\begin{cases} -\mathrm{div}\left((1-z)^2\sigma[u]\right) = f(x,t) & \text{in } \Omega \times [0,T], \\ \alpha_z \dfrac{\partial z}{\partial t} = \left(\epsilon\ \mathrm{div}\left(\gamma(x)\nabla z\right) - \dfrac{\gamma(x)}{\epsilon} z + \sigma[u] : e[u](1-z)\right)_+ & \text{in } \Omega \times (0,T]. \end{cases} \tag{1}$$

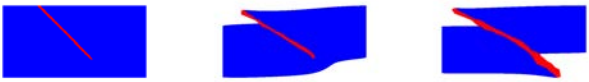
where $(a)_+ = \max(a, 0)$. $e[u] = (\nabla u + \nabla u^T)/2$ is strain tensor, $\sigma[u]$ is stress tensor. $g(x,t)$ is a given function of displacement at Dirichlet boundary Γ_D. Neumann boundary is set at $\Gamma_N = \Gamma \backslash \Gamma_D$. This model has many advantages for numerical simulation of crack growth: 1) Since the crack path is automatically selected, branching of the crack path can be expressed, 2) numerical simulation is performed in a fixed domain, so, there is no need to change the simulation boundaries during crack propagation, and complex shapes can be handled, 3) suppression of stress divergence at the crack tip by regularization with small parameter $\varepsilon (> 0)$ that represents the characteristic crack witdth, 4) extension to complex models while maintaing simplicity, e.g., crack growth in viscoealstic material[4].

Phase field can describe the complex crack surface, however, it cannot detect the contact of cracked surface because it plays the role that excludes elasticity of material. We propose new model that can treat crack surface with non-penetration condition using phase field . Our new model can treat the contact condition of crack surface also crack growth in material. The unilateral contact condition proposed by [5] can be easily included in our gradient flow type phase field model. Here we set $\bar{e}[u] := e[u] - \frac{1}{d}(\mathrm{div}u)I$, then we have $\sigma[u] = \beta(\mathrm{div}u)_+ I - \beta(\mathrm{div}u)_- I + 2\mu\bar{e}[u]$, where $(\mathrm{div}u)_+$ and $(\mathrm{div}u)_- = (-\mathrm{div}u)_+$ are the positive and negative parts of $\mathrm{div}u$, respectively, and we set $\beta := \lambda + 2\mu/d$. The gradient flow model with lateral contact condition can be written as follows[4]:

$$\begin{cases} -\mathrm{div}\left((1-z)^2\bar{\sigma}^+[u]\right) = f(x,t) & \text{in } \Omega \times [0,T], \\ \alpha \dfrac{\partial z}{\partial t} = \Big(\epsilon\ \mathrm{div}(\gamma(x)\nabla z) - \dfrac{\gamma(x)}{\epsilon} z \\ \qquad\qquad + \left(\beta(\mathrm{div}u)_+^2 + 2\mu|\bar{e}[u]|^2\right)(1-z)\Big)_+ & \text{in } \Omega \times (0,T]. \end{cases} \tag{2}$$

$t = 0$ $t = 0.05$

Figure 1: Defoamation of contacted plate. Displacements on right and left side are $u = (-1, 0)t$ and $u = (1, 0)t$, respectively.



$t = 0$ $t = 0.08$ $t = 0.09$

Figure 2: Defoamation of contacted plate with cack growth. Displacements on the right and left side are $u = (-1, 1)t$ and $u = (1, -1)t$, respectively.

We calculate deformation of 2 dimensional plate with Dirichlet bounday at the right and left side boundaries by FreeFem++[6]. Numerical results of compressive displacement for slanted crack shows the deformation with slip of the contact surface (Fig.1). Crack growth occurs of the half crack with slip displacement (Fig.2). Addressing the contact problem using phase field enables us to study deformation including crack growth in materials with complex crack surfaces.

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Extending non-smooth contact mechanics to cohesive zone modelling

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Keywords: Non-smooth, linear complementarity problem, cohesive zones.

The non-smooth approach to contact mechanics has been developed since the 1970s, following the pioneering works of Jean-Jacques Moreau [1], and has lead to many theoretically and computationally favourable formulations of contact problems, particularly for granular media, such as the contact dynamics method [2], in which the bodies are treated as rigid and the fundamental variables of the contact problem are the velocities before and after contact and the contact percussion. This dynamic formulation (as opposed to a static formulation relying on contact forces and displacements as the fundamental variables) has many advantageous aspects. A particular benefit of this formulation is that in the case of a pure normal contact problem, a two-dimensional contact and Coulomb friction problem, or a three-dimensional contact and friction problem modelled by an interior faceting of the Coulomb friction cone (following the remarkable contribution of Stewart and Trinkle [3]), the system may be written as a linear complementarity problem (LCP). This LCP formulation is particularly advantageous, as by exploiting results on this class of problems obtained by mathematicians, it can be shown that the evolution problem is well-posed (pure normal contact), or that the solution exists (frictional contact). In addition, the formulation as an LCP allows the problem to be addressed with implicit integration, with its concomitant benefits in terms of time-step size and thus numerical efficiency.
However, not all contacts can reasonably be described by laws as (conceptually) simple as unilateral contact with Coulomb friction. In particular, many systems have contacts that exhibit a degree of cohesion, and the formation of cracks in a solid body can also be modelled as the separation of two surfaces exerting a cohesive force on each other [4]. Indeed, this "cohesive zone" approach is regarded as the most physically accurate way of modelling crack propagation, as it serves to regularise the classical Griffith model of fracture so that it has finite stresses everywhere (as opposed to the diverging stress fields at the crack tip of the classical model). Many cohesive zone models (labelled "intrinsic") have an initial elasticity as the surfaces are drawn apart. This elasticity is physically spurious, and induces a phenomenon known as artificial compliance, that makes these intrinsic models essentially unusable for modelling dynamic phenomena. By contrast, "extrinsic" cohesive zone models do not possess this spurious elasticity as the surfaces are separated, and instead the cohesion immediately reduces as two surfaces are drawn apart. Another way of expressing this same idea is that extrinsic cohesive zone models are initially rigid, which immediately makes obvious the connection with the contact dynamics method. However, current extrinsic cohesive zone models, while initially rigid, do feature an unload–reload elasticity that remains physically spurious, and can serve to induce the artificial compliance phenomenon in the case of complex non-monotonic loading.
As such, we propose a non-smooth unification of contact mechanics with an extrinsic cohesive zone model. In the first instance, we focus on a purely normal problem (*i.e.* with only unilateral normal contact, and only mode I cohesion), where we specify appropriate non-smooth energy and dissipation

(pseudo-)potentials, from which we obtain our constitutive laws by exploiting the tools of convex analysis. Then, casting these laws in a dynamic form, we recover the principles of non-smooth contact dynamics [5], and are able to write our discrete-in-space-and-time problem as an LCP. We prove that this LCP is well-posed (and thus serves to regularise problems where quasi-static systems would demonstrate unphysical solution jumps), and further that our numerical integration scheme is dissipative (and symplectic in the absence of impacts). Comparison with results in the literature demonstrates that we are able to use orders-of-magnitude larger time-steps, and thus benefit from substantial gains in numerical efficiency [6].

Generalising further, we then write a model of unilateral contact with Coulomb friction (in two dimensions) that also encompasses mode I and mode II cohesion. Once again, by recasting our problem in terms of dynamics, we are able to write the discrete problem as an LCP, and prove the existence of a solution and the dissipativity of our numerical integration scheme.

By writing our problems in such a way, we connect the (otherwise disparate) traditions of non-smooth contact mechanics and cohesive zone modelling, and extend the domain of contact mechanics by carrying its mathematical tools into the domain of fracture mechanics.

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Double peeling of thin viscoelastic tapes from rigid substrate

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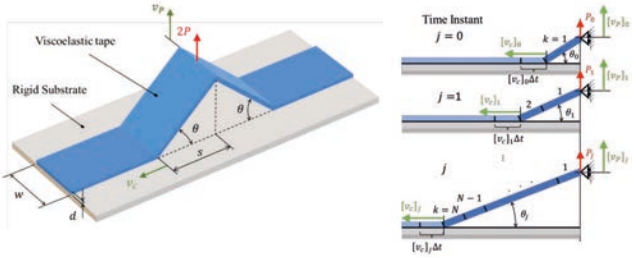
Keywords: viscoelasticity, adhesion, peeling, bio-inspired, tape.

Due to its adaptability, controllability, and reliability, peeling represents one of the common mechanisms of detachment exploited in nature and adopted in industrial applications. For this reason, from Kendall's pioneering study on the detachment of elastic tapes from rigid substrates [1], many studies have been led over decades focusing on peeling understanding and characterization, by accounting for the effects of large deformation, pre-strain, peel-rate, and different mechanical behaviors of films, adhesives, and foundations. Moreover, it has been suggested that the particular topology and geometrical arrangement of biological peeling systems may also be key factors for their superior adhesion strength [2], and that the interplay with the intrinsic non-elastic response of their tissues may contribute to enhance the resistance at heavy impact forces [3].



(a) (b)

Figure 1: V-shaped peeling scheme of a viscoelastic tape adhering to a rigid substrate (a). The non-uniform incremental tape discretization (b).

In this abstract, we report the results of our investigation on the peeling behavior of a viscoelastic thin tape arranged in double V-shaped configuration (see Fig. 1a). In the framework of the fracture mechanics, the peeling propagation is modeled by energy balance between the work per unite time W_P done by the peeling force, the work per unit time W_{in} done by internal stress in the detached viscoelastic tape during relaxation, and the rate of the surface adhesion energy W_{ad}.

$$W_P(t) + W_{in}(t) + W_{ad}(t) = 0 \tag{1}$$

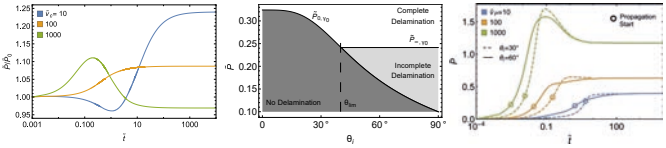
At the same time, the peeling angle θ must comply with the tape continuity equation

$$\frac{s}{cos\theta} = L + \int_L \varepsilon(x)dx \tag{2}$$

where $L + \int_L \varepsilon(x)dx$ is the total length of the non-adhering tape portion, and s is its projection on the substrate (see Fig. 1a).

To account for the peculiar non-stationarity of the V-peeling propagation, and the time dependent behavior of the viscoelastic tape, the process of peeling propagation is simulated by means of a numerical procedure based on a non-uniform spatial discretization of the tape, which is updated at each time step, as the peeling front moves (see Fig. 1b). Indeed, at each time instant, new equilibrium solution is calculated by iteratively solving the governing equations (1,2).

Our results show that, in contrast with the case of elastic tapes [4], the the viscoelastic peeling exhibits non-monotonic trends over time for both the peeling force P and angle θ, so that the delamination resistance of the system can both increase and decrease as the peeling front advances (see Fig. 2a). However, a long-term steady-state propagation regime is asymptotically approached, with the viscoelastic tape creeps completed, showing a constant peeling force. Furthermore, in the case of applied constant peeling force (see Fig. 2b), three different physical scenarios occur, depending on the initial peeling angle θ_i, in which the peeling process can either start and continue indefinitely, start and stop later, or even not start at all. Similarly, under controlled pulling-velocity v_P of the tape edge (see Fig. 1a), a time-varying peeling toughness behavior occurs with the peeling force that may exhibit a maximum for high values of v_P, leading to an enhancement in system resistance during the early stages of delamination, as shown in Fig. 2c.



(a) (b) (c)

Figure 2: The time-history of the normalized dimensionless peeling force $\tilde{P}/\tilde{P}_0$ for a constant peeling velocity $\tilde{v}_c$ propagation (a). The state map of the peeling process evolution for a constant dimensionless peeling force and initial angle (b). The time-history of the dimensionless peeling force $\tilde{P}$ for a constant pulling velocity $\tilde{v}_P$ (c).

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SESSION 3
Solution algorithms and numerical efficiency
Wednesday, 15:00 – 16:40

Combination of adaptive mesh refinement and high performance computing for accurate solution of elastostatics contact mechanics problems

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Keywords: High Performance Computing, Adaptive Mesh Refinement, Contact Mechanics Problems, Elastostaticity

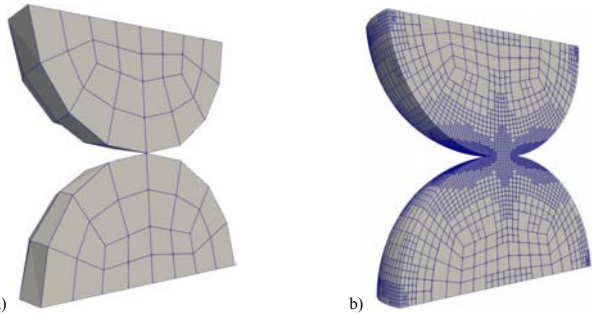
As contact mechanics problems are locally highly non-linear and non-regular, their numerical simulation is computationally challenging. Efficient numerical solutions of such problems often rely on adaptive mesh refinement (AMR). Even if efficient parallelizations of standard AMR techniques as h-adaptive methods begin to appear [1], their combination with contact mechanics problems remains a challenging task. Indeed, current developments on algorithms for contact mechanics problems are focusing either on non-parallelized new adaptive mesh refinement methods [2] or on parallelization methods for uniformly refined meshes [3].

The purpose of this work is to introduce a High Performance Computing (HPC) strategy for solving 3D contact elastostatics problems with AMR on hexahedral elements. The contact is treated by a node-to-node algorithm with a penalization technique in order to deal with primal variables only. Therefore, this algorithm presents the advantages of well modelling the phenomenon under study while not increasing the number of unknowns and not modifying the formulation in an intrusive manner. Concerning the AMR strategy, we rely on a non-conforming h-adaptive refinement solution. This method has already shown to be well scalable [1,4]. Superparametric elements are used in order to preserve the shape of hierarchical refined geometries, even for first-order finite elements solution (see Figure 1). An estimate-mark-refine approach with a local detection criterion based on a Zienkiewicz-Zhu (ZZ) type error estimator and a geometric-based stopping criterion is applied in order to perform the AMR process [5]. This combined strategy has recently proven its efficiency [6].

In this contribution, we propose to extend the combination of these contact mechanics and AMR strategies to a parallel framework. In order to carry out simulations, we place ourselves in the MFEM software [4] environment, an open-source finite elements method library. The proposed scalable contact algorithm is first based on a mesh partitioning that guarantees the contact paired nodes to be on the same processes. Furthermore, the contact stiffness matrix is locally built. The combined AMR-contact algorithm is ruled by two nested iterative loops. The external loop concerns the AMR process while the internal one deals with the contact solution. The penalized contact problem is solved thanks to a dedicated iterative solver. The iterative contact solution process is performed until the set of active contact nodes (detected by interpenetration) does not vary. Once the contact loop converged, the AMR strategy is locally applied and the mesh decomposition is rebalanced with the previously discussed partitioning contact constraints. Finally, the external iterative process ends once the AMR stopping criterion is satisfied. To perform the AMR on the MFEM software, the currently implemented h-adaptive method is enriched with our own estimate-mark-refine approach.

The proposed AMR-contact strategy is first evaluated on the 3D Hertzian contact problem with tens of millions of unknowns (see Figure 1). Our HPC approach turns out to be well scalable for

hundreds of cores. In addition, with the employed AMR strategy, an optimal mesh, reducing the number of unknowns by at least ten compared to an equivalent uniformly refined mesh, is found in few AMR iterations. This mesh is automatically refined around the contact area, especially in the zones where the contact status changes. Furthermore, our HPC strategy has been successfully confronted with industrial test cases derived from the nuclear industry, which confirms the potentiality of such approach.



a) b)

Figure 1: Parallel AMR-contact strategy for a 3D Hertzian contact (cross-sectional view) : (a) initial coarse mesh; (b) Hierarchical AMR with an error threshold of 8%.

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Efficient and accurate numerical time integration of arbitrary fine meshed small sliding contact via contact modes and hyper reduction

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Keywords: Small sliding contact, model reduction, hyper reduction

If the contact between two surfaces is described as "small sliding", only small relative motion occur between the two contact partners. This type of contact is of significant technical importance, as it occurs in all fixed connections. Examples are: Screwed connections, riveted connections, welded connections, press connections, etc. Inside the contact area, state-dependent contact and friction forces occur. Depending on the type of structure and type of load, these contact and friction forces can have both local and global effects. An example of a local impact would be "fretting" due to relative tangential displacement under high normal loads. An example of global impact would be an increased damping and load dependent eigenfrequencies of global vibration modes. The first one is utilized, for example, in turbine friction belts.
As shown in [1] the contact and friction forces lead to complex nonlinear phenomena such as load-dependent natural frequencies and damping and mode coupling. In addition, it was pointed out in [2] that the contact and friction forces due to vibrations have a very high variability and "move around" in the contact region in a very complex way. Within the contact area, there are simultaneously zones in which the contact is open and closed. The zone with closed contact can be subdivided again into zones in which there is sticking and sliding. These three zones (open contact, sticking, sliding) move through the contact area in a state-dependent manner. Note, that the insights of the former mentioned publications [1] and [2] are based on measurements.
An accurate numerical reproduction of these effects would require a very fine Finite Element (FE) mesh. The nonlinear contact and friction forces must be computed considering the current joint state and the dynamics needs to be regarded as well. These requirements make the direct application of the FE method impossible since it would lead to exorbitantly high computation times. In [1], weeks or even years were estimated for the simple structure used there.
In recent years, the authors and Pichler have developed a method that meets the requirements just mentioned with low computation times. The core of the method is (1) the reduced computation of the deformations within the contact area via contact modes and (2) the reduced computation of the contact and friction forces with contact and friction force modes, respectively.
For the reduced computation of the contact area a flat projection is used. This means, that the deformation of the structure (and the contact area) is computed based on a linear superposition of weighted time invariant trial vectors, often called “modes”. The weights are state variables and often called “modal coordinates”. There are many proposals for such trial vectors in the literature, such as the famous one of Craig and Bampton. All of them have in common that they are able to represent global deformations quiet well. However, they are not able to represent local deformations as they occur in contact regions. Therefore, such a mode base is extended by so-called contact modes. These are computed simulation-free (a-priori) based on modal derivatives. Detailed information can be found in [3] and [4]. Recently it was shown in [5] that with this reduction method the measured results of [1] and [2] can be reproduced in numerical simulations qualitatively excellent. The computation times predicted in [1] with years have been reduced to approx. 25 to 45 min (average desktop PC with the 64bit operating system Windows 10 (32GB RAM, Intel(R) Core(TM) i7-3820 CPU @3.6GHz)).

As mentioned, model reduction with contact modes already brings a significant performance gain without sacrificing accuracy. However, the contact and friction forces are still computed for each FE node. For this, all involved FE node displacements have to be computed from the modal coordinates. The finally obtained forces need to be projected back into the modal space. This process becomes the bottleneck of the numerical simulation if many FE nodes are involved in the contact area. To increase the efficiency of numerical time integration again, the contact and friction forces are also computed by a superposition of time invariant trial vectors. In [6] and [7] mathematical details about different methods applicable to different contact situations can be found.
As mentioned before, it was shown in [5] that model reduction with contact modes can reproduce all measured nonlinear effects. In this work it is reported that the application of hyper reduction to the simulation tasks documented in [5] leads to similar result quality with again reduced computation times. From the initially estimated huge amount of computation time just a view minutes remain, and all measured effects can be observed in FE result quality. The combination of model reduction and hyper reduction makes the approach quasi "mesh - free" since the fineness of the FE meshing does not play a role in the dynamic simulation anymore.
Finally, the potential of the approach is demonstrated by a particle impact simulation of a turbine blade rotation with 10000rpm. All contact and friction forces in the friction belt have been considered as well as the nonlinear rotation and the dynamics of the structure. The results are in good agreement with the expectations and the simulation time is low.

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Hybrid domain decomposition for huge contact problems

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Keywords: Hybrid domain decomposition, parallel algorithms, scalability.

The current development of supercomputers motivated the research of massively parallel algorithms for solving many challenging problems, including huge contact problems of elasticity. Here describe how to adapt the three-level hybrid domain decomposition method proposed by Klawonn and Rheinbach to solving multibody contact problems with billions of nodal variables. The basic idea is to decompose the domains occupied by the bodies into subdomains, use the transformation of variables to join some subdomains by the rigid body motion of adjacent subdomains into clusters so that each cluster has only six rigid body modes, and then use the standard FETI (finite elements tearing and interconnecting) methodology to get well conditioned dual quadratic programming problems with bound and equality problem which can be solved with an asymptotically optimal (linear complexity). The results of numerical experiments [1]–[4] show the considerable scope of scalability of both H-TFETI (hybrid total FETI) and H-TBETI total boundary. The observation is supported by fundamental estimates [4]

$$\|\mathbf{S}_{\rm subdom}\| \geq \|\mathbf{S}_{\rm cluster}\| \geq \frac{1}{4m}\overline{\lambda}_{\rm min}(\mathbf{S}_{\rm cluster}) \quad \text{and} \quad \overline{\kappa}(\mathbf{S}_{\rm cluster}) \approx m\overline{\kappa}(\mathbf{S}_{\rm cluster})$$

giving the bounds on the spectrum of $m \times m \times m$ cube clusters in terms of the bounds on the spectrum of the subdomains' Schur complements. Table 1 shows that the H-TFETI-DP iterations are very cheap so that for well-structured discretizations, the unpreconditioned hybrid TFETI method can outperform the very powerful TFETI-DP with Dirichlet preconditioner

clusters	subdomains	unknowns	iter/time[sec] H-TFETI	iter/time[sec] TFETI	iter/time[sec] DirTFETI
27	729	15,000,633	89/27.7	60/17.3	20/40.0
343	9261	190,563,597	105/35.1	59/20.3	19/46.2
4096	110,592	2,275,651,584	104/58.7	na	na

Table 1: Billion clumped cube - unpreconditioned H-TFETI and TFETI, and TFETI with Dirichlet preconditioner (DirTFETI), times include initiation, $m = 3$, linear problem [4]

The contact benchmark is the same clamped cube over a sinus-shape obstacle as in Fig. 1, loaded by own weight, decomposed into $4 \times 4 \times 4$ clusters, $H/h = 14$, using the ESPRESO [5] implementation of H-TFETI for contact problems. We can see that TFETI needs a much smaller number of iterations. However, H-TFETI is still faster due to 64 times smaller coarse space and better exploitation of the node-core memory organization. In general, if we use $m \times m \times m$ clusters, the hybrid strategy reduces the coarse problem's dimension and cost of the coarse problem by m^3 and m^6, respectively.

Clusters	Subdomains	Cores	Unknowns $\times 10^6$	H-TFETI iter/sec	TFETI iter/sec
64	4096	192	13	169/23.9	117/24.9
512	72,900	1536	99	208/30.2	152/115.1
1000	656,100	3000	193	206/42.6	173/279.9

Table 2: Clamped elastic cube over the sinus-shaped obstacle, total times, $m = 4$, $H_s/h = 14$

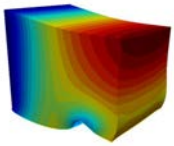

Figure 1: Displacements of a clamped elastic cube over the sinus-shaped obstacle

Moreover, the two-level structure of the coarse grids (split between the primal and dual variables) can be effectively exploited by the node-core design of the modern supercomputers' hardware. We conclude that H-TFETI-DP is a competitive algorithm for solving some huge contact problems.

Remark 1. Similar results can be obtained for the clusters of subdomains discretized by the boundary element and hybrid TBETI method.

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Stabilized FEM for Contact Problems

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Keywords: Stabilized FEM, Nitsche's method, contact problems.

Stabilization of mixed finite element methods for saddle point problems is a well-established technique that allows one to use finite element spaces that do not satisfy the Babuška–Brezzi condition. They were introduced and analysed in 80's by Hughes, Franca, Brezzi, Pitkäranta and others. The analysis has, however, suffered from the fact that full regularity of the exact solution needs to be assumed.

In this talk, we give an overview of our recent and ongoing work on stabilized FEM for contact problems. The problems are written in a mixed form, with the contact pressure acting as a Lagrange multiplier, and the stabilised formulation is derived by adding appropriately weighted residual terms to the discrete variational forms.

We show that the discrete formulation is uniformly stable and that it leads to a quasi-optimal a priori error estimate without further regularity assumptions. Moreover, we establish a posteriori estimates whose optimality is ensured by local lower bounds.

To implement the method, the discrete Lagrange multiplier can be locally eliminated, thus giving rise to a robust Nitsche-type method. We present a series of numerical results which confirm the optimality of our a posteriori estimates.

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An adaptive quasistatic contact model based on IGA applied to impact analysis in flexible multibody systems

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Keywords: IGA, adaptive, impact, quasistatic contact model, flexible multibody system.

The focus of this work is an adaptive algorithm for efficient and detailed impact simulations in flexible multibody systems. Multibody systems allow the efficient analysis of the overall motion of systems, e.g. in machine dynamics or robotics. Impacts within flexible multibody systems often consist of large rigid body motions before and after impact. Elastic deformations normally remain small if stiff materials, e.g. steel or aluminum, are selected. These conditions allow the use of the floating frame of reference formulation [3], which requires global shape functions of the flexible bodies. Usually, a finite element model consisting of isoparametric elements in combination with a model reduction is used to approximate the global shape functions. A disadvantage of isoparametric elements is that the geometry is discretized. However, impact simulations depend on an accurate representation of the geometry in the contact area. As an alternative approach, isogeometric elements can be used where there is no error in the representation of the geometry. For this reason, the isogeometric analysis (IGA) will be employed in this work to determine the global shape functions. For a more detailed introduction to the IGA, see, for instance, [1]. In order to preserve the local deformation of the contact region in the model reduction, a Craig-Bampton method is used. In a previous work [2], the IGA bodies are reduced with a Craig-Bampton method resulting in numerically stiff equations of motion despite additionally added numerical damping. For isoparametric elements, the literature [4] proposes a quasistatic contact model to solve this issue. In this work, the quasistatic contact model is applied to an IGA model. The contact model is then paired with a penalty method for contact treatment.
Setting up impact simulations usually requires heuristic testing. A selection of three challenges is listed below.
Firstly, the required penalty factor is determined heuristically. Thereby, the penalty factor should be chosen large enough such that the results become independent of the chosen parameter.
Secondly, the bodies in contact need to be refined in the contact area. In practice, this area is very small and requires a high element resolution. If the refined area is too large, the computation time increases. If the refined area is too small, the impact cannot be fully resolved.
Thirdly, the location of the contact area must be known prior to the simulation. In the simplest case, the impact starts in the first time step. However, if large rigid body motions, e.g. rotations, occur prior to the impact, the setup becomes more challenging.
Therefore, the aim of this work is to develop an algorithm to incorporate adaptivity into the setup of impact simulations. Thereby, the penalty factor, the location and width of the contact area are determined automatically.

Usually, an impact can be divided into three phases: the pre-impact phase, the impact phase and the post-impact phase. In the pre-impact phase, the elastic eigenmodes are not yet excited allowing larger step sizes. In contrast, the impact phase requires the smallest step sizes due to high impact dynamics. In the post-impact phase, larger step sizes are feasible again. Switching between the phases can be

achieved by minimizing the search function

$$f_{\mathrm{s}}(\xi_{\mathrm{C}}, \eta_{\mathrm{C}}, \xi_{\mathrm{T}}, \eta_{\mathrm{T}}) = \|\mathbf{x}_{\mathrm{C}}(\xi_{\mathrm{C}}, \eta_{\mathrm{C}}) - \mathbf{x}_{\mathrm{T}}(\xi_{\mathrm{T}}, \eta_{\mathrm{T}})\|, \tag{1}$$

where $\mathbf{x}_{\mathrm{C}}$ and $\mathbf{x}_{\mathrm{T}}$ are points on the contact and target body at which the two bodies are the closest. The local coordinates of the IGA are denoted by ξ_{C}, η_{C}, ξ_{T} and η_{T}. Minimizing Eq. (1) gives the time and location of the impact. This contact search can be found in Fig. 1, where the concept of an adaptive impact simulation is visualized.
The impact phase uses a quasistatic contact model [4] to evaluate the penalty based contact. In practice, the solution of the quasistatic contact equations fails for a penalty factor beyond its converging value. This property is used to adaptively determine the penalty factor iteratively. After the impact simulation, the elements in contact are identified and the contact area is adaptively adjusted. If required, the simulation is repeated to validate the newly refined mesh. Finally, in the post-impact phase, large rigid body motions can be simulated, and further impacts may occur.

As an application example, a planar setup consisting of two cylinders is simulated. The initial distance, as well as the translation and rotational velocity can be chosen arbitrarily. This setup includes large rigid body motions in the pre-impact phase, where the location of the impact is unknown before the simulation begins. The application example is therefore well suited to test the adaptive contact algorithm.

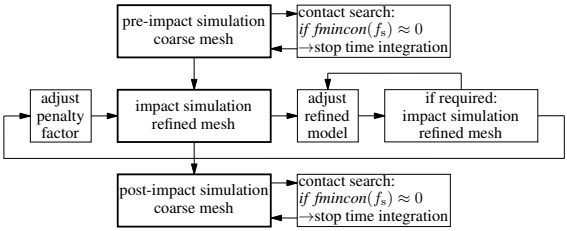

Figure 1: Concept of the time simulation

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SESSION 4
Solution algorithms and numerical efficiency
Thursday, 9:20 – 11:00

A Novel Method for Multibody Dynamics Contact Represented by Linear Complementarity Problems

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Keywords: LCP, complementarity, dynamics

Unilateral contact and friction in multibody dynamics systems present a complex challenge. The representation using unilateral constraints and Coulomb model forms the classic friction cone and generally results in a nonlinear complementarity problem (NCP). One common approach to address the nonlinearity is to use a faceted approximation of the friction cone. However, due to the fact that in the tangential plane there are only two independent directions, any representation that uses more than that for the base vectors will introduce an artificial redundancy problem. To overcome this issue, the so-called box friction model can be used [1]. This model associates each contact point with one normal force and decomposes the friction force along two orthogonal directions in the tangent plane. This results in a linear complementarity problem (LCP).

Two categories of methods for solving LCPs can be distinguished: iterative and pivoting. Iterative methods, such as the Projected Gauss-Seidel method (PGS), the Projected Jacobi method, and the PGS subspace minimization method, solve LCPs through a series of Gauss-Seidel iterations and converge to an approximate solution. These methods usually do not find an exact solution for the problem as they do not check the complementarity condition. On the other hand, pivoting methods, such as Lemke's algorithm, Murty's algorithm, and Judice's algorithm, pivot complementarity variables among separated ranges defined by complementarity conditions and search for a feasible solution [2]. These methods were generally developed for the mathematical problem of LCPs without considering the physical nature of contact problems.

Despite the existence of numerous methods, the force coupling issue remains a common challenge in solving contact problems. Many of these solution algorithms consider the contact formulations as pure mathematical problems, and treat normal and friction forces as separate variables and require given bounds to define the complementarity conditions. However, the bounds of the friction force depend on the normal force, which is unknown and must be solved concurrently with the friction forces. The conventional approach for addressing this issue is to estimate a value for the normal force in the first step before applying the solution algorithm for the LCP. However, this process typically involves repeated iterations, which can introduce additional errors and computational costs.

This paper presents a novel algorithm for complementarity problems, which can overcome the challenge of force coupling in more accurate and efficient way compared to other algorithms. It is particularly devised for the contact problems in multibody dynamics systems. The method falls under the pivoting category. The main concept behind the method is to pivot complete contact points instead of individual force components. This approach preserves the force coupling relationships and enables the simultaneous resolution of both normal and friction forces without the need of per-imposed bounds. The high-level architecture of the method is presented in Fig. 1 and consists of three stages: grouping, solving, and pivoting.

At a given time, a contact point can be classified into ten different groups based on its physical motion. These groups comprise movement in the normal direction: either toward or away from the

constraint plane, as well as movement in two tangential directions: positive sliding, locked, or negative sliding. The objective of the grouping stage is to initialize a group configuration variable g, which reflects the distribution of all contact points among these groups. Once the groups have been identified, the *free* (basic) and *tight* (non-basic) variables associated with each contact point can be determined. The free variables are unknowns that lie within the defined bounds, while the tight variables are known values at either zero or the bound. For example, in a group associated with negative sliding along the x-axis, the friction force is a tight variable that is at its upper bound, whereas the velocity would be a free variable with an unknown negative value.

Figure 1: Algorithm flowchart for the proposed method

The solving stage is to substitute the tight varaibles into the LCP formulation and solve for the free variables. However, the formulations cannot be solved directly due to the issue of force coupling. To address this, a four-step process has been established, which includes tight force elimination, tight velocity separation, free variable solving, and index restoration. These steps iterate over each contact point, resolve the force coupling relations, and obtain a solution for all complementarity variables.

The pivoting stage involves pivoting the contact points by comparing the solution obtained in the solving stage with the specified bounds of the free variables. Special rules have been developed to ensure that the points are effectively pivoted among the ten groups. The process outputs an updated group configuration variable, g^+, along with the total count of infeasible points, n_f. The proposed algorithm iterates between the solving and pivoting stages until a solution that satisfies the complementarity conditions is achieved.

To illustrate the proposed method, examples of rigid body contact systems were considered. The simulation results using the proposed algorithm were compared to those obtained using the Judice's algorithm. The results demonstrate that the proposed method can give significant improvements in both efficiency and accuracy, with a reduction of over 30% in computational time and a substantial increase in the accuracy of the simulation results.

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Multiscale methods for substructuring multibody systems with contact

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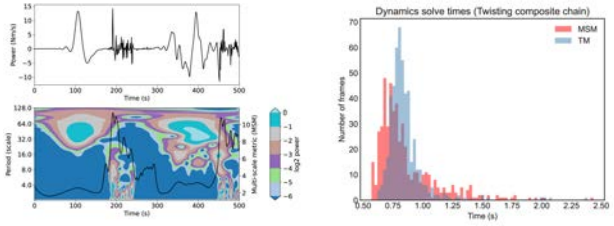
Keywords: Wavelets, substructuring, multibody simulation

Dynamic simulation of multibody systems, in particular when including contact, has many applications from robotics to vehicle design to haptic systems. However, simulation of very large systems can impact performance, and many methods have been employed to simplify or parallelize the contact problem over multi-core processors [1]. Substructuring involves splitting the multibody system into smaller subsystems of bodies which are solved in parallel. The Schur complement method uses the effective mass of the subsystems to incorporate the internal dynamics of the subsystems into the solution for the interface constraints [2]. In this way, the internal constraints of each subsystem can be solved in parallel to further reduce computational time. To assign bodies to a subsystem, local topological information about the corresponding body-constraint graph has been used, where bodies are nodes and constraints between bodies are represented by edges. A minimum degree algorithm grows subsystems from bodies with the lowest degree. Using only topological information, such as the degree of a body, will generically exhibit a lot of degeneracy in the subsystems generated. For example, a simple chain exhibits degeneracy because bodies each have degree two, as every body is connected to two other bodies.

We investigate whether incorporating dynamical information about the interactions beween bodies in a multibody system may help lift these degeneracies, while also giving a physically motivated basis for choosing which bodies belong to which subsystem. We consider the case where the body-constraint graph has edges with weights which contain information about the constrained interactions between bodies, not just the graph topology (i.e. a topological metric, abbreviated here as TM). Our criteria for determining whether two bodies are put into the same subsystem is that high-complexity interactions should generally be grouped together. To measure complexity, we use spectral information from a wavelet anlaysis using the time scales involved in interactions between two bodies (i.e. a multiscale metric, abbreviated here as MSM). An interaction here is chosen to be the physical power associated to generalized, relaxed constraint forces. Wavelet methods provide a way of quantifying multiscale behaviour in dynamical systems that exhibit non-stationarity, which is characteristic of multibody systems with contact. Figure 1(a) shows an example of the temporal profile for the physical power of a bilateral constraints in a multibody system. We introduce a novel metric which incorporates multiscale wavelet coefficients to develop a dynamics-based formulation of substructuring.

We apply our partitioning metrics to two use cases: a monolithic simulation of a twisting composite chain and of a parachute mechanism with an applied force. The composite chain is constructed to be highly symmetrical to demonstrate the usefulness of our method in resolving degeneracies. We also consider an asymmetrical assembly resembling a parachute, which also consists of bodies that have large mass-ratios. We choose which bodies are part of a subsystem based on a monolithic simulation, since this can be done offline. At each time step, a MSM is assigned to each edge (i.e. interaction) on a body-constraint graph of each case, then a minimum cut algorithm [3] is used to grow partitions, generally putting bodies with large MSM values in the same partition. To determine a final partitioning



(a) (b)

Figure 1: (a) Example wavelet spectrum based on the power of a generalized, relaxed constraint from a bilateral constraint. (b) Dynamics solve time for the twisting composite chain.

for the whole monolithic simulation, how often two bodies were put in the same partition at each time step is used. Our method shows strong potential to be able to partition multibody systems to improve dynamics solve times.

Figure 1(b) for the symmetrical, twisting composite chain shows improvements in the dynamics solve times for the MSM-based partitioning scheme compared to a TM scheme that uses the degree of a body to partition. We find a reduction in the mode of the dynamics solve time by 18% and in the mean by 1.24% when dynamical information is included. The complexity of the contact interactions in the twisting chain adds to the multiscale behaviour since there are sliding contacts with relative motion due to the applied torque which twists the system, breaking ties. For the asymmetrical parachute mechanism a reduction in the mode of the dynamics solve time by 8.77% and in the mean by 6.96% is observed. The MSM provides more of a semantic partitioning (i.e. bodies that make a chain are put together) compared to using a TM, which may be due to more complex bilateral interactions in a chain of the parachute.

Adding dynamical information can help remove topological redundancies in assigning bodies to groups, which we find can lead to improvements in the performance of multibody simulation. Our study demonstrates that using the dynamics, as encoded by the interactions between bodies, is an important avenue to investigate when grouping bodies for parallel computation.

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Targeting a faster time-to-solution of mortar-based contact problems

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Keywords: Contact Mechanics, Mortar Methods, Algebraic Multigrid, Parallel Algorithms

For some time, mortar methods have been the preferred modeling approach for surface-coupled problems with non-matching grids, as they provide high accuracy and variational consistency. The considerable additional numerical effort introduced by the evaluation of the mortar integrals (especially in three dimensions) requires an efficient and parallelizable framework, that scales well on parallel hardware architectures and is, thus, suitable for the solution of high-fidelity models with potentially several million degrees of freedom. Further challenges present themselves in the efficient solution of the arising system of linear equations on parallel computing clusters.

Although mortar methods have been particularly popular in contact mechanics, the efficiency of the computations has only recently been targeted and many opportunities for improvement remain.

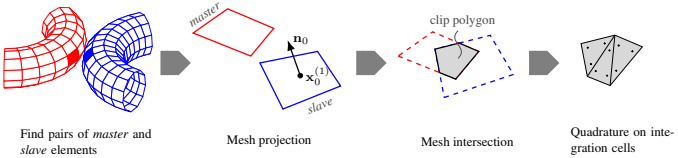

Figure 1: Evaluation steps of the mortar integrals to be performed in *every* nonlinear iteration [1]

When targeting a fast time-to-solution of mortar-based contact simulations, two components of the simulation play a decisive role:

1. Due to the large size of the linear system, which increases even further when using discrete Lagrange multipliers to enforce the contact constraints, direct solvers cannot be used anymore and iterative solvers with effective preconditioning are required. In many application cases the linear solver dominates the time-to-solution. It has recently been shown that aggregation based algebraic multigrid (AMG) methods can be used as highly efficient and scalable preconditioners for mortar-based contact problems in saddle-point formulation [2]. The proposed AMG method preserves the contact constraints and the saddle point structure on all multigrid levels, thus accelerating the convergence of the iterative linear solver.

2. Since the computational effort to evaluate the mortar integrals is solely related to the slave side of the contact interface, the workload may potentially be concentrated on very few processors, depending on the distribution of the interface discretization (see Figure 2). Dynamic load balancing of core kernels, as proposed in [1], allows for a parallel evaluation of mortar operators and their linearizations. The independent redistribution of the interface discretization becomes increasingly important for contact problems with evolving contact zones and can lead to a significant reduction of the time-to-solution of the contact evaluator.

Figure 2: Rolling contact with concentrated mortar workload

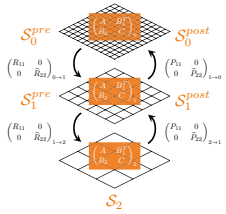

Figure 3: Multigrid hierarchy for a 2×2 block system in contact simulations

This contribution discusses recent findings on the combined application of AMG and dynamic load balancing techniques to mortar-based, large deformation contact formulations. Although weak [1][2] and strong scaling [1] has been proven for the methods individually, the scalability of the interplay of AMG and dynamic load balancing has yet to be studied and will be presented. We will assess the effect and potential speedup of the HPC framework through the proposed methods in a high fidelity contact simulation.

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Model order reduction for the fatigue life prediction of wire ropes in tension and bending

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Keywords: Model order reduction, PGD, frictional contact, LATIN method, DDM, wire rope

Helically wounded steel wires in tension and bending are widely encountered, especially in the energy and power transmission fields, e.g. mooring lines for floating offshore wind turbines (FOWT) or overhead electrical conductors. The life prediction of such structures involving fretting fatigue phenomena between their constituent wires is crucial. However, wire-scale simulations for complex and multiple loads requires effective computational and modeling strategies. Examples include the use of wire models and beam-to-beam contact algorithms [1] or the use of homogenization techniques [2]. Model reduction techniques and multi-scale domain decomposition methods (DDM) are investigated here in order to appreciate their ability to deal with this particular class of problem.

Model reduction for frictional contact problems is a challenging issue mainly due to the non-linear and non-regular character of frictional phenomena. It is indeed difficult to represent rapid changes in contact status and complex propagative phenomena of sliding/sticking zones as well as their strong multiscale content. Few works exist in the literature. Among them, one can cite a posteriori approaches with reduced basis enrichment such as the non-negative matrix factorization method [3] or the cone-projected greedy algorithm [4]. Among a priori methods that build the reduced basis on the fly throughout the computation by a progressive Proper Generalized Decomposition (PGD), one can cite the works developed in the framework of the non-incremental nonlinear solver called the LATIN method [5] and its application to contact problems [6, 7]. In this case, the resulting algorithm shares similar features with augmented Lagrangian methods known for their robustness in dealing with frictional contact problems [8]. The main difference with the LATIN method comes from the fact that, unlike classical incremental nonlinear solvers, an iterate of the solution is generated at each iteration over the entire space-time domain, which makes this method particularly suitable for a space-time separate representation of the solution for PGD model order reduction.

In this presentation, an SVD analysis of simulation results from [9] for a metric length of wire rope belonging to a FOWT mooring line is first proposed to illustrate its reduction potential. The analysis of contact quantities performed layer by layer for the spiral strand wire rope, shows that quantities for the outer layers where slip occurs mainly due to bending are more difficult to represent with a reduced order model. It is also more difficult to accurately represent frictional contact forces than normal contact forces and slip. The multiscale content of the problem (global modes corresponding to the wire rope tension and bending and local modes corresponding to contact and friction between wires) suggests that the use of multiscale methods may be beneficial.

In a second step, a model reduction method by PGD combined with the nonlinear solver LATIN is used on a simple one-dimensional problem which is a simplification of the loading seen by a constituent

wire of a wire rope. It is shown that the reduction potential depends closely on the importance of the sliding front propagation (see Figure 1). Sorting the reduced basis and controlling its size using an appropriate downsizing algorithm is also crucial to recover optimality with respect to SVD [7].

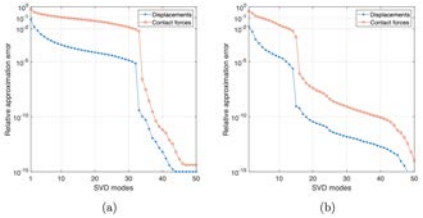

Figure 1: SVD analysis of sliding displacement and frictional force for a one-dimensional problem when the sliding front propagation is important (a) and less significant (b)

Finally, the LATIN-based multiscale mixed DDM with PGD [6] is applied to a two-dimensional problem representative of the targeted application. It is shown to what extent the DDM coarse problem and the introduction of the PGD, which builds reduced basis per subdomain, are able to effectively represent the multiscale content of the problem.

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Numerical analysis of a non-clamped dynamic thermoviscoelastic contact problem

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Keywords: Dynamic contact, non-clamped contact, thermoviscoelastic material, non-monotone friction law, finite element method, error estimate, numerical simulations.

We analyze a non-clamped dynamic viscoelastic contact problem involving thermal effect. The precise description of the problem is following.

A viscoelastic body, in its reference configuration, occupies a bounded domain $\Omega \subset \mathbb{R}^d$, $d = 2, 3$ with a Lipschitz boundary Γ, consisting of two parts Γ_N and Γ_C, such that $\Gamma_N \cap \Gamma_C = \emptyset$. Let ν denote the unit outward normal vector to the boundary Γ. The part of the boundary Γ_C is subject to contact with a foundation. We are interested in a mathematical model that describes the evolution of body displacement, stress, and temperature. Let $T > 0$ be given, and let $[0, T]$ represent the time interval. In what follows, we denote by $x \in \Omega$ and $t \in [0, T]$ the spatial and temporal variables, respectively. We denote by $u = (u_i(x, t))$, $\sigma = (\sigma_{ij}(x, t))$ and $\theta = \theta(x, t)$ the displacement field, the stress field, and the temperature field, respectively, which are assumed to satisfy the following constitutive law (for simplicity, we omit the symbols x and t below)

$$\sigma = \mathcal{A}(\varepsilon(\dot{u})) + \mathcal{B}\varepsilon(u) + C\theta \quad \text{in } \Omega \times (0, T). \tag{1}$$

The symbols $\mathcal{A}$, $\mathcal{B}$ and C in (1) represent the viscosity operator, the elasticity operator, and the heat expansion tensor, respectively.
The process is assumed to be dynamic, hence the equation of motion takes the form

$$\rho\ddot{u} = \mathrm{Div}\sigma + f_0 \quad \text{in } \Omega \times (0, T), \tag{2}$$

where ρ is the density of the material and f_0 is the density of the volume forces. We assume that the traction force is applied at Γ_N, that is,

$$\sigma\nu = f_2 \quad \text{on } \Gamma_N \times (0, T). \tag{3}$$

The normal compliance condition is described by relation

$$-\sigma_\nu = p(u_\nu) \quad \text{on } \Gamma_C \times (0, T), \tag{4}$$

where p_ν is a given nonnegative function which vanishes when its argument is negative. Moreover, we assume that the frictional law takes the following multivalued, possibly non monotone form

$$-\sigma_\tau \in \partial j(\dot{u}_\tau) \quad \text{on } \Gamma_C \times (0, T), \tag{5}$$

where j is a locally Lipschitz function, and ∂j stands for its Clarke subdifferential; see [1] for details. In our model we assume a simplified and linearized version of the energy law that takes the form

$$\rho c_p\dot{\theta} - \mathrm{div}(K\nabla\theta) = c_{ij}\dot{u}_{i,j} + g \quad \text{on } \Omega \times (0, T), \tag{6}$$

where c_p is the heat capacity, K is the thermal conductivity operator, and g represents the heat sources. We assume that the heat transfer between body and foundation is described by the law

$$-K\nabla\theta \cdot \nu = -r(\theta) - h(\|\dot{u}_\tau\|_{\mathbb{R}^d}) \quad \text{on } \Gamma_C \times (0,T). \tag{7}$$

Finally, we impose initial conditions

$$u(0) = u_0,\ \dot{u}(0) = u_1,\ \theta(0) = \theta_0 \quad \text{in } \Omega. \tag{8}$$

Our mechanical problem reads as follows.

Problem $\mathcal{P}_{\mathcal{M}}$**.** *Find the displacement* $u\colon \Omega \times [0,T] \to \mathbb{R}^d$*, the stress* $\sigma\colon \Omega \times [0,T] \to \mathbb{S}^d$ *and the temperature* $\theta\colon \Omega \times [0,T] \to \mathbb{R}$ *that satisfy conditions (1)-(8).*

Problem $\mathcal{P}_{\mathcal{M}}$, in its variational formulation, leads to a coupled system of hyperbolic inclusion for displacement and a parabolic equation for temperature. We provide a fully discrete approximation of the studied problem and find optimal error estimates between the exact solution and the approximate one.

Our result provides a generalization of the one obtained in [2]. In contrast to the model studied there, we deal with the non-clamped body, which introduces an additional difficulty to the problem. Moreover, our result is obtained without any smallness assumptions on the data of the problem, which was the case in [2].

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SESSION 5 (parallel)
Solution algorithms and numerical efficiency
Thursday 6, 11:20 – 13:20

Error analysis of the Hybrid Hyper-Reduction method for frictionless contact problems.

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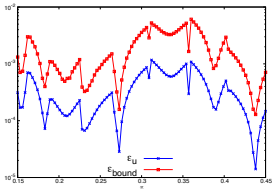
Keywords: Hyper-Reduction; reduced order models; frictionless contact problems; Lagrange multipliers method; Error estimators

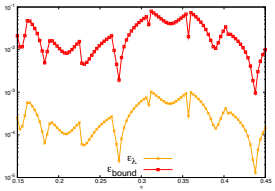
The solution of contact problems is often time consuming because of the non-linearity and non-regularity of the contact constraints. For contact problems treated by Lagrange multipliers (ensuring the Signorini contact conditions to be respected), applying reduced-order modeling remains challenging due in particular to the non-negativity constraint on the Lagrange multipliers [1, 2].

We focus here on the Hybrid Hyper-Reduction (HHR) approach for frictionless elastostatic contact problems proposed in [1, 3]. This approach is based on a reduced integration domain (RID) built through a discrete empirical interpolation method as described in [4]. A reliable reduced dual basis is obtained by restricting the full order dual basis to the RID. The HHR model preserves the saddle-point formulation of the Lagrange multipliers method, and allows the verification of the Signorini conditions inside the RID (without projection). Numerical experiments proved the capacity of the HHR method to provide accurate results both for the displacement and the Lagrange multipliers solutions [1, 3], even for large parametric spaces [3].

In this work, we introduce a priori and a posteriori estimators of the HHR method, providing error bounds to quantify the discrepancy between the HHR and FE (Finite Element) solutions. Thanks to the hybrid formulation of the HHR model, a Cea-like primal bound is obtained on the RID (see Figures 1a and 2a). Indeed, the FE-like dual reduced basis hybrid enables the corresponding bounding term to vanish. Classical Brezzi-like analysis leads to bound the dual approximation error by the primal one (see Figure 1b).



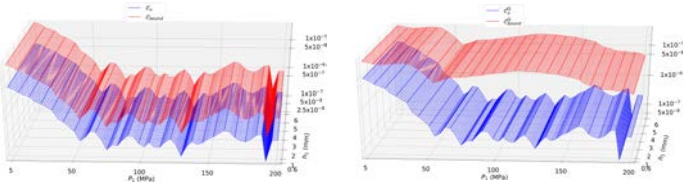
(a) A priori primal error bound



(b) Dual error bounded by the primal error

Figure 1: Error bounds on the RID for the Hertz's half disks test case

Hence, the final a priori error estimators for both primal and dual errors, depend on the distance between the primal FE solution and the HHR primal approximation space only. Furthemore, an a priori primal error bound can be derived on the whole computational domain, following Cea's lemma for Petrov-Galerkin formulation (see Figure 2b).



(a) On the RID. (b) On the whole domain.

Figure 2: A priori primal error bounds for an industrial test case with 2 parameters.

Primal and dual a posteriori error estimators (depending only on HHR solutions) are also introduced to propose certified HHR error bounds, useful in particular for a efficient greedy snapshot selection. These estimators are based on the HHR equilibrium and constraints equations residuals and will be introduced in the light of those already present in the literature for ROM methods on mixed problems [1, 5, 6].

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Contact mechanics and inseparability: towards dictionary-based sparse approximations

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Keywords: model order reduction, contact mechanics, sparse methods

In the context of decision-making and design optimization of mechanical or structural assemblies, reduced contact mechanics models are important for resolving the mechanical behaviour in real time. However, Reduced Order Modelling (ROM) of parametric contact mechanics problems is a recent topic of research. Additional challenges appear in reducing such inequality-constrained minimization problems, such as finding a non-negative reduced basis for contact pressure. Though the traditional ideas of ROM have been applied in this context [1], the local nature of contact and the consequent irreducibility hasn't been addressed sufficiently. The assumption of linear separability does not generally apply to the contact pressure field, thereby limiting the effectiveness of a low-rank basis [2].

In this work, over-complete dictionaries have been used to mitigate the irreducibility effects of frictionless non-adhesive contact problems. The idea is to populate the dictionary with a large number of contact pressure snapshots generated by the high-fidelity (finite element) model. Without attempting to extract a reduced basis, the dictionary is directly used to reconstruct instances by seeking only a few snapshots from the dictionary that approximate the contact pressure field closely. Therefore, the idea of a sparse coefficient vector becomes central to the approximation strategies.

Two strategies are devised using sparse approximation ideas. The first is based on an active-set approach, in which dictionary snapshots are selected greedily. The second one is based on convex-hull approximations, where the idea is to naturally satisfy the constraints [3] by limiting the solution space to a convex region.

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Parallel, High Performance Contact Solvers

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Keywords: Nitsche's method, parallel implementation, software.

Phenomena involving two objects coming into contact are ubiquitous in science and engineering, often forming part of complex multiphysics problem. At the same time algorithms for computational contact mechanics are notoriously difficult to implement and only very few results regarding scalability of the algorithms in parallel are available. Our goal is to develop a scalable and robust implementation for frictional contact problems as an open-source extension to the FEniCSx framework. The motivation for our work is the modelling of contact between components within a jet engine in the context of the thermo-mechanical modelling of the engine. The overarching goal to achieve a high-fidelity simulation of a jet engine in operation including the effects of thermo-mechanics, electromagnetics, and computational fluid dynamics. The problems we are interested in are therefore first and foremost characterized by the requirement of a vast number of degrees of freedom as well as complex geometries, complex material models and complex interactions with other aspects of the model. Figure 1 shows one of our test problems that is both sufficiently simple and, at the same time, representative of some of the features we encounter in realistic geometries.

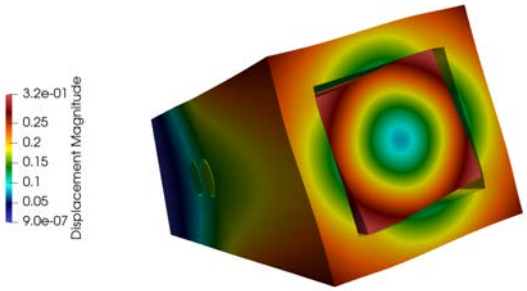

Figure 1: Simple test problem

FEniCSx [1] is the most recent version of FEniCS, an open source software for solving partial differential equations based on finite element methods. One of the most attractive features of FEniCS from the user perspective is the so-called Unified Form Language (ufl) which allows the user to express the PDE in a notation very close to mathematical notation. One major challenge is that integrals on the contact surfaces typically cannot be expressed as a ufl-form. In our implementation, we combine custom integration kernels on the contact surfaces with automatically generated kernels based on ufl-forms for all remaining integrals in order to leverage the convenience of ufl for all remaining aspects of the thermo-mechanical model. Currently, we implement Nitsche's method based on [2, 3] to enforce the contact constraints, but in principle other methods, such as Lagrange multiplier methods, can be implemented in a similar fashion. We investigate the scalability of our implementation in parallel as well as the robustness of the iterative linear solvers we employ.

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Energy conserving contact-impact algorithm using the method of Lagrange multipliers and the explicit central difference time integration scheme

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Keywords: contact algorithm, impact analysis.

The presented work is being conducted as part of the research activities on the protection of public spaces against vehicle ramming attacks. In this context, developments are being implemented in the EUROPLEXUS [1] software so that it can be efficiently used for virtual testing of security barriers (see also [2]). One of the key ingredients for simulating an impact of a vehicle on a barrier concerns the chosen contact-impact algorithm, which is applied to a large number of interactions between the vehicle and the barrier components.

In fact, the field of crashworthiness simulation has benefitted from an active research since decades and has attained a certain maturity, becoming a common engineering practice for various applications, i.e. those concerning passengers' safety. Therefore, there exist several computer codes capable of conducting numerical simulations of most impact and crash situations encountered in practice. The majority of them are based on the explicit central difference time integration scheme, very convenient for a broad field of fast dynamics, combined with state-of-the-art constitutive laws to represent the structural behavior under extreme loading.

For the modelling of the contact-impact interaction, most of the software use the so-called penalty methods, which are very convenient from the perspective of implementation and of computational cost. As stated in [3], the principle of the penalty method implies that the contact impenetrability condition is satisfied only approximately, achieving a more precise fulfilment using stiffer penalty parameters. This makes penalty methods generally not consistent (see [4]) within the variational formulation of the problem and the final matrix form is ill-conditioned.

The contact stiffness related parameters used in penalty methods are determined by experience-based rules of thumb, which are problem dependent and without any sound theoretical framework for their application to an arbitrary contact configuration. For these reasons, the EUROPLEXUS software (see [1], [5]) offers the possibility to use the method of Lagrange multipliers for the modelling of contact constraints, in addition to the penalty method.

Following a standard implementation of Lagrange multipliers method for transient mechanics, the contact is considered as a kinematic constraint activated when a penetration between the surfaces submitted to the contact condition occurs (see [6]). However, this standard approach is analogous to the principle of inelastic shock and leads to a numerical energy dissipation, which can be very significant in some situations. In this communication is presented an alternative implementation of the Lagrange multiplier method (see [7]), which avoids the numerical energy dissipation, by adopting the approach presented in [8] for rigid multi-body collision dynamics. According to [8], instead of applying a kinematic condition between the contact surfaces, the energy conservation is

imposed explicitly. The implementation of the alternative Lagrange multipliers method is very simple and does not impact the computational cost.

In the present communication the difference between the standard and the proposed alternative Lagrange multiplier methods is demonstrated on several numerical examples. It is worth stressing that in many situations the energy dissipated due to the standard contact algorithm is very small and the difference between the two methods is negligible. However, in some cases, the numerical energy dissipation due to the contact algorithm can exceed 10%, which is largely unacceptable for most of contact-impact applications.

One of such examples is the elastic shock of a tube-like vehicle component against a rigid wall. In that case, even though the overall response of the structure rebounding from the rigid wall is very similar for both algorithms, the energy comparison reveals a significant discrepancy between the results (see Figure 1). Namely, in spite of the absence of any dissipative mechanism (the impacting structure is considered elastic and the impacted wall rigid), the standard version of the contact algorithm leads to dissipation of almost 15% of the initial total energy, which is completely avoided by the new algorithm. Beside the effect on the energy conservation, the contact algorithm can also influence the stress distribution, as it will be shown on other examples during the conference presentation.

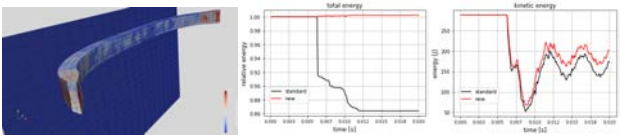

Figure 1: Tube-like vehicle component hitting a rigid wall. The difference between the standard (black) and the new (red) algorithms is clearly observable in terms of the total energy.

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Interior point methods for computing frictional contact problems with hyperstaticity

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Keywords: nonsmooth mechanics, contact mechanics, cone complementarity problem, fixed-point theorem, second-order cone optimization, interior-point method, Jordan algebra.

We are interested in numerically solving the problem of Coulomb's friction with unilateral contact, in dynamics and in quasi-statics. Many methods are already available in the literature to solve this problem [1]. When the discrete reaction forces are uniquely defined, nonsmooth Newton techniques are very efficient and the method of choice to reach high accuracy solutions. In the case of hyperstaticity, for instance, in rigid multi-body systems, robotics or granular materials, nonsmooth Newton methods fail most of the time. The fallback technique is then first order technique for which is sometimes difficult to get tight accuracy. In this work, we propose an interior point method to remedy this problem, getting high accuracy solutions even in the hyperstatic case.

We aim to study in a d-dimensional space a mechanical system of unilateral contact between solid bodies with m degrees of freedom. The dynamical system is discretized in time and space. Two models are then formulated: frictional contact problem ($d = 3$) and friction problem with rolling resistance at contact ($d = 5$). The first one will be mainly presented in this talk.

A triplet of unknowns describes the mechanical system: the generalized velocities $v \in \mathbb{R}^m$, the impulses of n contact points are concatenated in a vector $r := (r_1, \dots, r_n) \in \mathbb{R}^{nd}$ and the relative velocities $u := (u_1, \dots, u_n) \in \mathbb{R}^{nd}$ at these contact points. Let us introduce a cone $\mathcal{K} = \prod_{i=1}^{n} \mathcal{K}_i$, each $\mathcal{K}_i$ is the second-order cone of friction coefficient $\mu_i \in]0, +\infty[$ defined by

$$\mathcal{K}_i := \{x \in \mathbb{R}^d : \|x_{\mathrm{T}}\| \leq \mu_i x_{\mathrm{N}}\},$$

where the subscripts N, T stand for normal and tangential components of the vector. The dual cone of $\mathcal{K}$ is defined by $\mathcal{K}^* := \prod_{i=1}^{n} \mathcal{K}_i^*$, each $\mathcal{K}_i^*$ is given by

$$\mathcal{K}_i^* = \{y \in \mathbb{R}^d : \mu_i \|y_{\mathrm{T}}\| \leq y_{\mathrm{N}}\}.$$

Let us define the vector $\tilde{e}$ concatenated by n unit vectors $e := (1, 0, 0)^\top$ and let $i \in \{1, \dots, n\}$. In the view of the method proposed by De Saxcé [2], we perform a change of variables $\tilde{u}_i = u_i + \mu_i \|u_{i,\mathrm{T}}\| e$, to obtain a reformulation of Coulomb's law, known as a second-order complementarity condition

$$\mathcal{K}_i^* \ni \tilde{u}_i \perp r_i \in \mathcal{K}_i.$$

The new variable $\tilde{u}$ is joined by vectors $\tilde{u}_i$ in order. The Coulomb frictional contact problem is formulated as a conic complementarity problem given by

$$\begin{aligned} &Mv + f = H^\top r, \\ &Hv + w + \varphi(s) = \tilde{u}, \\ &\mathcal{K}^* \ni \tilde{u} \perp r \in \mathcal{K}, \\ &s_i = \|\tilde{u}_{i,\mathrm{T}}\|, \quad i = 1, \dots, n, \end{aligned} \tag{1}$$

where $\varphi : \mathbb{R}^n_+ \to \mathbb{R}^{nd}$ is defined by $\varphi(s) = (s_1, 0^\top_{d-1}, \ldots, s_n, 0^\top_{d-1})^\top$. The given data are: a symmetric positive-definite matrix $M \in \mathbb{R}^{m\times m}$, $f \in \mathbb{R}^m$, $H \in \mathbb{R}^{nd\times m}$ and $w \in \mathbb{R}^{nd}$. Acary et al. [3] showed the existence of a solution to (1) under an assumption. This problem (1) is straighforwardly obtained when the linearized dynamics is discretized. In linearized quasi-statics, it also possible to formulate a problem of this form [4].

As we say above, in the literature, there are some applications [5, 6, 7] dealing with problems relevent to (1) for full-rank matrix H. For the case of rank deficiency, the Newton semi-smooth approach does not work with some of the experiments performed. Therefore, we want to find an alternative algorithm that can give a high-precision numerical solution of (1) no matter how the rank of H is.

It is shown that the system (1) can lead to an optimization problem parametrized by s. We want to develop an interior-point method to solve such a problem. The complementary condition in (1) can be rewritten as a form of Jordan product $u \circ r = 0$ [8]. A perturbed formulation of (1) is then given by

$$\begin{aligned} &Mv + f = H^\top r, \\ &Hv + w + \varphi(s) = \tilde{u}, \\ &\tilde{u} \circ r = 2\mu\tilde{e}, \\ &s_i = \|\tilde{u}_{i,\tau}\|, \quad i = 1, \ldots, n, \\ &(\tilde{u}, r) \in \text{int}(\mathcal{K}^*) \times \text{int}(\mathcal{K}), \end{aligned} \tag{2}$$

where the parameter μ is driven to zero. We will show that (2) has a unique solution for all $\mu > 0$. Then, (2) defines a curve of solutions, called the central path, which is bounded and converges to the solution of (1) as $\mu \to 0$. The results of numerical experiments will be presented to validate our approach.

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A monolithic computational method for elasto-dynamics with plasticity and contact based on variational approach.

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Keywords: nonsmooth mechanics, contact mechanics, plasticity, optimization, variational approach.

The objective of this presentation is the modelling and the numerical simulation of the response of elastoplastic structures to impacts. To this end, a numerical method is proposed that takes into account one-sided contact (Signorini condition) and impact phenomena together with plasticity in a monolithic solver, while accounting for the non-smooth character of the dynamics.

The formulation of the plasticity and the contact laws are based on inclusions into normal cone of convex sets, or equivalently, variational inequalities following the pioneering work of [3] and [1], who introduced the assumptions of normal dissipation and of generalised standard materials (GSM) in the framework of associated plasticity with strain hardening. After a standard FEM discretization, a differential variational inequality in terms of measures under the small strain assumption is obtained:

$$\left\{\begin{array}{l} M\mathrm{d}v + B^\top\sigma(t)\mathrm{d}t = f_{\text{ext}}(t)\mathrm{d}t + H(u(t))\mathrm{d}i_{\text{N}} \\ \dot{u}(t) = v(t) \\ \sigma(t) = E(\varepsilon(t) - \varepsilon^p(t)) \\ a(t) = -D\alpha(t) \\ \begin{pmatrix} \dot{\varepsilon}^p(t) \\ \dot{\alpha}(t) \end{pmatrix} \in N_C \begin{pmatrix} \sigma(t) \\ a(t) \end{pmatrix} \\ v_{\text{N}}(t) = H^\top(u(t))v(t) \\ -\mathrm{d}i_{\text{N}} \in N_{T_{\mathbb{R}^m_+}(g_{\text{N}}(t))}(v_{\text{N}}(t) + ev_{\text{N}}^-(t)), \end{array}\right. \tag{1}$$

where u is the displacement, v is the velocity, $\mathrm{d}v$ the acceleration measure, $\mathrm{d}i_{\text{N}}$ the contact impulse measure, σ is the Cauchy stress, ε the strain, ε^p the plastic strain, α the hardening parameter, a the hardening forces, g_{N} the gap function, v_{N} the relative normal velocity at contact and e the coefficient of restitution. The definition of matrices are classical: M is the mass matrix, B the equilibrium matrix, H the contact Jacobian, E the elasticity matrix and D the hardening matrix. The interior of the set $C(\sigma, a)$ is the domain of stress and hardening forces where the material is elastic.

The proposed time–stepping method is an extension of the [2] scheme for nonsmooth dynamics. Using the relation $x_{k+\theta} - x_k = \theta(x_{k+1} - x_k)$, the discretized equations can be formulated in terms of

variables approximated at $t_{k+\theta}$

$$\left\{\begin{array}{l} M(v_{k+\theta}-v_k)+h\theta B^\top\sigma_{k+\theta}=h\theta f_{\text{ext},k+\theta}+\theta H p_{\text{N},k+1} \\ S(\sigma_{k+\theta}-\sigma_k)-h\theta B v_{k+\theta}=h\theta z_{k+\theta}, \\ D^{-1}(a_{k+\theta}-a_k)=h\theta y_{k+\theta} \\ \theta v_{\text{N},k+1}=H^\top v_{k+\theta}-(1-\theta)v_{\text{N},k} \\ -\begin{pmatrix} z_{k+\theta} \\ y_{k+\theta} \\ v_{\text{N},k+1}+ev_{\text{N},k}\end{pmatrix}\in N_{C\times\mathbb{R}^m_+}\begin{pmatrix}\sigma_{k+\theta}\\ a_{k+\theta}\\ p_{\text{N},k+1}\end{pmatrix} \end{array}\right. . \tag{2}$$

where $z=-\dot{\varepsilon}^p$ and $y=-\dot{\alpha}$, and $p_{\text{N},k+1}$ is the approximation of the impulse measure over the time–step. This finite-dimensional variational inequality at each time–step is well-posed as it can be recast as a well-posed saddle point problem:

$$\begin{array}{ll} \min_{v,\dot{\varepsilon}}\max_{\sigma,a} & \frac{1}{2}(v-v_k)^\top M(v-v_k)-\frac{1}{2}(\sigma-\sigma_k)^\top S(\sigma-\sigma_k)-\frac{1}{2}(a-a_k)^\top D^{-1}(a-a_k) \\ & \quad +h\theta\sigma^\top\dot{\varepsilon}-h\theta f^\top_{\text{ext},k+1}v \\ \text{s.t.} & B v=\dot{\varepsilon} \\ & \theta v_{\text{N}}=H^\top v-(1-\theta)v_{\text{N},k} \\ & \begin{pmatrix}\sigma\\ a\\ v_{\text{N}}+ev_{\text{N},k}\end{pmatrix}\in C\times\mathbb{R}^m_+. \end{array} \tag{3}$$

and can be solved by optimisation methods for convex quadratic programs, providing an interesting alternative to the return mapping algorithm. Furthermore, the discrete energy balance shows that spurious numerical damping can be removed and the scheme is practically unconditionally stable.

The presentation will be completed by numerical illustrative examples of impacts on metallic structures made of beams.

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SESSION 5B (parallel)
Modelling of friction joints under dynamical loading, Multi-scale approaches, Multifield problems with contact constraints, Contact in biomechanics
Thursday, 11:20 – 13:20

Variability of vibration response in friction-damped structures due to non-unique residual tractions: Computation of bounds

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Keywords: Nonlinear vibrations, Friction damping, Uncertainty quantification.

The residual traction of stuck contacts refers to the distribution of static friction force, and its tangential component value is non-unique under different loading histories. Due to this hysteretic phenomenon, friction-damped structures exhibiting partial slip behavior at their contacts may have a large response variability during steady state forced vibrations [1-3]. It has been shown in [4, 5] that the resonant vibration level can vary by 20 to 300 percent due to non-unique residual tractions, and the bounds are always of high importance from the engineering point of view.

In this work, we present a new approach based on the single-nonlinear-mode theory for the prediction of conservative frequency response envelopes. First, the bounds of the amplitude-dependent modal parameters are computed. Interval arithmetic is then applied to forward propagate the uncertainty from modal parameters to the frequency response. Response bounds for a given harmonic excitation are synthesized by using the closed-form expression as

$$\Omega_{1,2} = \sqrt{\tilde{\omega}^2\left(1-2\xi^2\right) \pm \sqrt{\left(\tilde{\omega}^2\left(1-2\xi^2\right)\right)^2 - \tilde{\omega}^4 + \frac{\left|\left(\hat{\boldsymbol{\psi}}_{\mathrm{nl}}^1\right)^H \hat{\mathbf{f}}_{\mathrm{ex}}^1\right|^2}{a^2}}} \,. \tag{1}$$

Here, $\Omega_{1,2}$ represent the synthesized frequency values. $\tilde{\omega}$, ξ, $\left(\hat{\boldsymbol{\psi}}_{\mathrm{nl}}^1\right)^H \hat{\mathbf{f}}_{\mathrm{ex}}^1$, and a are the nonlinear modal parameters of natural frequency, damping ratio, forcing and amplitude, respectively. In Eq. (1), optimum values of modal parameters, whose derivation is analytically performed, are used to obtain conservative envelopes.

The developed approach is demonstrated and validated against the forced responses numerically obtained by both the Harmonic Balance Method and the Direct Time Integration. As a benchmark, a planar structure imitating the friction-damped shrouded turbine blades is constructed (see Figure 1).

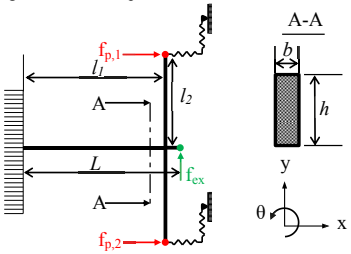

Figure 1: The benchmark with two point contact elements at the flange tips

Figure 2 shows the conservative upper and lower frequency response bounds as well as the non-unique steady state responses computed with the Harmonic Balance Method (HBM) and the Direct Time Integration (DTI). It is clear that the variability is quite large in both frequency and amplitude axes. The upper and lower bounds correctly predict the range of all possible non-unique solutions throughout the entire frequency range. It is also worth mentioning that the synthesis step of the response bounds is free of extra computational burden, because the computation of response bounds is based on analytical developments and uses the modal parameter bounds already available from the nonlinear modal analysis step.

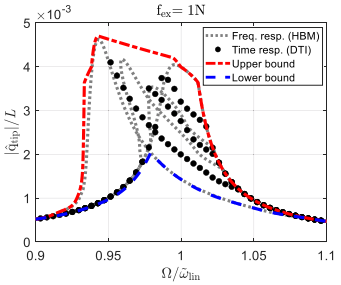

Figure 2: Response variability and the bounds

For the planar structure with two point contact elements, we also used the hypothesis that the limiting cases with barely sticking contacts give the bounds of modal parameters. The results show that the hypothesis gives accurate and relevant results. For a more general case containing more than two contacts, an optimization algorithm could be used to compute the bounds of modal parameters, which would involve more computational effort. This is a prospective future work.

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Competing dry friction contact models for underplatform dampers

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Keywords: dry-friction damping, contact parameters, underplatform dampers, turbine blades

Contact mechanics problems presented by Underplatform Dampers (UD), devices used in aircraft engines, to prevent blade fatigue failure caused by vibration, a major problem affecting engine reliability, are discussed. UDs are small, simple and reliable elements that, inserted between two adjacent blades, are in sliding frictional contact with the underside of the platforms at the base of the airfoil. An example of UD in the context of blades is shown in Fig. 1a from [1].

After recalling that today's highly sophisticated software codes allow the treatment of both the linear part of the calculation (that of blades and disks) and the nonlinear part, i.e., the interfaces where contact occurs in relative motion, in this case the UDs, it is illustrated why the treatment of nonlinear contacts is where the greatest difference occurs among alternative approaches in use today.

The case of UDs is particularly elusive because during the oscillation cycle the contacting surfaces experience resultant forces varying in magnitude and position, under alternating displacements at varying velocities. Given the large number of scientific contributions, the discussion here is limited to examining only two sharply divergent lines of development.

A first line, developed over time at Imperial College, [2, 3] is discussed: typically with a symmetrical wedge damper, then with two opposite flat surfaces, whose contact pressure distribution is determined by the FEM through a fine distribution of stick-slip elastic contact elements, Fig. 1c.

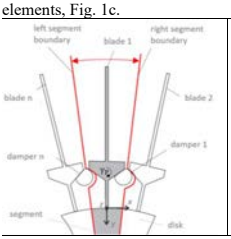

Fig. 1a: asymmetrical dampers and blades, from [1] adapted

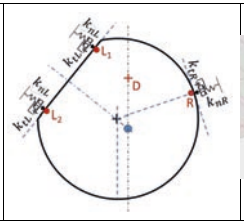

Fig. 1b: asymmetrical damper, and its simplest contact model

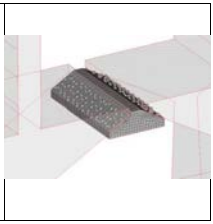

Fig. 1c: symmetrical wedge damper, from [3]

Which, however, would imply, in practical engineering cases, an ad hoc detection of the shape of each individual contact pair. Contact parameters are assumed based on measurements on a separate alternating sliding pin-type test-rig, which a tangential contact stiffness per unit area of k_t = 60,000 N/mm^3, and a friction coefficient μ = 0.6. Normal contact stiffness k_n=k_t. The validation of the model is done by comparing experimental vs. simulated results for the FRF. It turns out that even assuming, quite optimistically, a perfect alignment between the two opposing platforms, important FRF discrepancies appear. To correct them, [3] employs very refined experimental and numerical techniques to reproduce in the model the real surfaces due to machining.

The second line of development, which is being pursued in the AERMEC laboratory of the Politecnico di Torino, is based on different concepts. This includes a special test-rig that allows direct measurement of the forces exchanged between the damper and adjacent platforms [4], further improved [5] to allow for higher contact pressures and more controlled contact areas. Relative kinematics at contacts is detected by laser measurements, while load cells measure directly forces exchanged between damper and platform, allowing tangential contact stiffness and friction coefficient to be directly measured on the damper. In particular, the tangential contact stiffness is determined to be about 1500 N/mm^3, which clearly shows a problem compared to the other values mentioned above. In this method, we do not calculate contact pressures, but the resultants of contact forces on surfaces. Moreover, cross check of experimental vs simulated results is now made by comparing the damper hysteresis cycle, rather than the FRF.

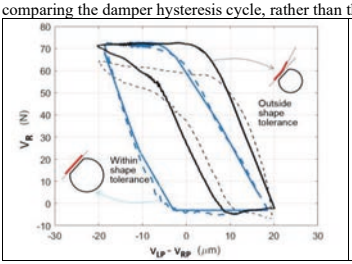

Fig. 2: experimental (dashed lines) and numerical (solid lines) force-displ. cycles for model in Fig. 1b

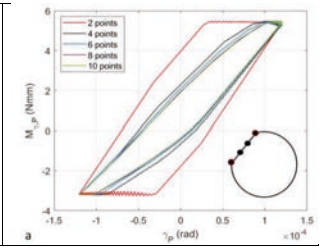

Fig. 3: simulated cycles with increasing contacts on the full flat damper surface

Fig. 2 shows that already with the model of Fig. 1b, with two elements at both ends of the flat contact cross-section, the experimental (full line) and simulated (dashed line) cycles of vertical platform displacement v_{LP}-v_{RP} and vertical right contact force V_R match well, in a regular case as well as in the case of non-planarity of the contact surface due to a machining out-of-tolerance [6].

Fig. 3 supports the concept that a very limited number of contact elements are required on a cross section of the damper to achieve the correct hysteresis cycle (onset of full slip).

The conclusion is threefold. The geometric model adopted must be supported by a data acquisition system consistent with it, in any case very accurate. The experimental-simulated validation of results through FRF cannot match that damper hysteresis loops. The model based on a limited number of contact points has a greater potential for blade-damper coupled optimization.

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Homogenization based two-scale modelling of fluid-saturated porous media with self-contact in micropores

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Keywords: Unilateral contact, Porous media, Multiscale modeling, Homogenization, Fluid-structure interaction, semi-smooth Newton method.

We consider fluid-saturated poroelastic structures characterized by unilateral self-contact at the pore level of the periodic microstructure. The contribution is devoted to the two-scale modelling based on the homogenization [1] and related numerical methods. The unilateral frictionless contact interaction is considered on matching pore surfaces of the elastic skeleton. Depending on the deformation due to applied macroscopic loads, the self-contact interaction alters the one between the solid and fluid phases. For the closed pore microstructures (see Fig. 1, left), a nonlinear elastic model is obtained at the macroscopic scale, since the pressure, being constant for each pore can be eliminated. An efficient algorithm for two-scale computational analysis is based on alternating micro- and macro-level steps. A dual formulation of the pore-level contact problems in the local representative cells $Y_s \subset Y =]0,1[^2$ provides actual active contact sets $\Gamma_c^*(x) \subset \Gamma_c$ which enables to compute consistent effective elastic coefficients at particular macroscopic points $x \in \Omega$, [3]; the local problems for an increment of displacements $\boldsymbol{u}(y,x)$ and the contact Lagrange multipliers $\lambda(y,x)$, $y \in Y_s$ attain the following form involving operators $\mathcal{A}, \mathcal{B}$, and $\mathcal{G}$ arising from the elasticity and stress, and the gap-function constraint

$$\begin{aligned} \langle \mathcal{A}\boldsymbol{u},\, \boldsymbol{v}\rangle_{Y_s} + \left\langle \mathcal{B}\breve{\boldsymbol{\sigma}}^{\rm mic},\, \boldsymbol{v}\right\rangle_{Y_s} + \langle \mathcal{G}^*\lambda,\, \boldsymbol{v}\rangle_{Y_s} = 0\ , \quad \forall \boldsymbol{v} \in \widetilde{\mathbf{H}}^1_{\#}(Y_s)\ , \\ \min\{-\langle \mathcal{G}(\boldsymbol{u}-\breve{\boldsymbol{s}}),\, \vartheta\rangle_{\Gamma_c} \mid \langle \lambda,\, \vartheta\rangle_{\Gamma_c}\} = 0\ , \quad \forall \vartheta \in \mathrm{C}^*_+(\Gamma_c)\ , \end{aligned} \tag{1}$$

where C^*_+ denotes the self-dual cone of $\mathrm{C}_+(\Gamma_c) = \{\vartheta | \vartheta \geq 0\}$ associated with the non-penetration condition. The finite element discretized contact problem (1) can be written as a nonsmooth equation which can be solved by the semi-smooth Newton method [2].

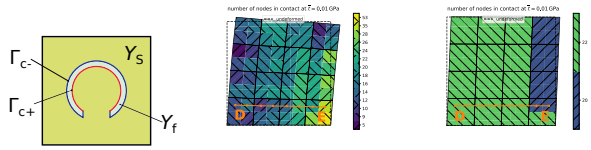
At the macroscopic level, a sequential linearization leads to an incremental equilibrium problem in domain Ω which is constrained by a projection arising from the homogenized contact constraint. The "two-scale active contact set" $\Sigma_\Gamma = \{(x,y) \in \Omega \times \Gamma_c |\ y \in \Gamma_c^*(x) \subset \Gamma_c\}$, such that the macroscopic displacement increment $\boldsymbol{u}^M \in U_0(\Omega)$ and the Lagrange multiplier $\lambda^M \in H(\Sigma_\Gamma)$ satisfy

$$\int_\Omega \left(\mathbb{D}^E \boldsymbol{e}_x(\boldsymbol{u}^M) + \widehat{\boldsymbol{P}}^* \lambda^M\right) : \boldsymbol{e}_x(\boldsymbol{v}) = \int_\Omega \boldsymbol{f}\cdot\boldsymbol{v} - \int_\Omega \breve{\boldsymbol{\sigma}} : \boldsymbol{e}_x(\boldsymbol{v})\ , \quad \forall \boldsymbol{v} \in U_0(\Omega)\ , \tag{2}$$

with the out-of-balance at the right hand side , and the projection accounting for the kinematic constraint,

$$0 = \max\{-\lambda^M, \widehat{\boldsymbol{P}} : \boldsymbol{e}_x(\boldsymbol{u}^M) + \breve{s}\} \quad \text{a.e. in } \Sigma_\Gamma\ , \tag{3}$$

where $\mathbb{D}^E$ is the tangent elastic modulus associated with the actual contact sets $\Gamma_c^*(x)$ and, in the non-smooth equation, the second argument $\widehat{\boldsymbol{P}} : \boldsymbol{e}_x(\boldsymbol{u}^M) + \breve{s}$ expresses the "two-scale" non-penetration condition. The Uzawa algorithm, or the semi-smooth Newton method can be employed. The newly proposed two-scale contact algorithm which comprises alternating micro-problems (1) and the macro-problem



cell Y dry pores fluid-filled pores

Figure 1: Reference cell Y (left). Macroscopic problem with "bending" deformation: in Ω, displayed number of contact nodes at microlevel for dry and fluid-filled pores.

Figure 2: Contact tractions on Γ_c^* at the microscopic level. Dry pores (above) and fluid-filled (bottom) cells Y at macroscopic locations along line D-E, see Fig. 1

(2)-(3) have been implemented in the SfePy finite-element code. It provides better convergence rates than the earlier version of the two-scale algorithm reported in [4] which involves the macro-problem formulated as an equilibrium equality with totally frozen local contacts. The newly proposed algorithm enables detachment of active contact parts near to the contact area boundary, $\partial\Gamma_c^*(x)$. Numerical examples of 2D deforming structures are presented in Figures 1 and 2, comparing the macroscopic and microscopic responses for microstructures with dry and fluid-filled pores.

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Multi-scale analysis of contact of rough surfaces through FEM/BEM code coupling

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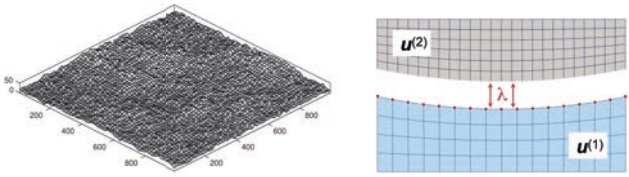
Keywords: Multi-scale approach, Rough surface, Fractal surface, Mortar method, Contact mechanics

The study of contact problems related to rough surfaces is a vigorously researched field that has practical applications in both engineering and physics. However, determining contact variables like traction distributions, contact stiffness, and the actual contact area is challenging due to the intricate geometrical complexity of the interface. The fractal feature of the surface spans a few orders of magnitude of length scales, further complicating the impact on materials' deformation and stress states [1]. Consequently, there is a significant difficulty in understanding the evolution of the contact domain and determining these variables, making it a challenging problem that still needs to be actively explored. Therefore, understanding the connection between the topological features of surface roughness and the consequent nonlinear constitutive relation at the interface is of prime importance for researchers today.

Several semi-analytical micromechanical contact theories relying on the statistical distribution of the elevation of the asperities and their radii of curvature have been proposed in the last decades. In [2], a new numerical formulation that operates on multiple scales has been introduced. In this approach, the micro-scale problem involving roughness was solved using the Boundary Element Method (BEM) to pre-compute the gap-pressure relation [3]. This gap-pressure relation was then fitted into an analytical function to be used as a contact constitutive law to solve the macro-scale problem using the Finite Element Method (FEM). While this approach allows to incorporate the effect of surface roughness into FEM models of contact problems, it comes with limited accuracy in the high and low separation regime [2], i.e., at very low and very high contact pressures.

Our current work explores a fully coupled implementation of the two scales to overcome the shortcomings of the previous methods through a consistent coupling of both the micro and the macro scale within a single simulation framework. At the micro-scale, a linear elastic frictionless normal contact between a rigid rough indentor and an elastic half-space is solved. For the topology of the indentor, any statistically representative microscopically rough surface can be provided as input, also variable with the position along the contact surface of the macro-scale finite element model. This approach allows testing any surface roughness topology without making assumptions on the surface height distribution. In our examples, the Random Mid-point Displacement (RMD) method [4] is used to generate a rough surface using its fractal parameters. A sample surface generated using the RMD method is shown in Figure 1a. The empirical relations given in [5] can be used to get the fractal parameters of the surface given its statistical parameters to define a rough surface uniquely. The BEM is used to solve the contact problem at the micro-scale, as it has the ability to handle a very fine discretization of the interface without the need to discretize the bulk volume. The macro-scale model is solved using FEM, where the contact interface is discretized using mortar elements, see Figure 1b. In this approach, the interface's relative normal displacement, as computed by the macro-scale finite element model, is passed as far-field displacement



(a) A sample rough surface generated using the RMD method (b) Mortar method at the contact interface

Figure 1: Discretization at the micro-scale and the macro-scale

to the micro-scale model, which is called at each node of the mortar elements within the macro-scale framework. At every mortar node, the normal contact traction is calculated using the BEM algorithm, and the interface stiffness is approximated by a finite-difference approach. These parameters calculated at the micro-scale are then passed back to the macro-scale model.

In this presentation, we will outline our approach for the multi-scale coupling realized through FEM/BEM code coupling. We will show various qualitative and quantitative examples to demonstrate the validity and usefulness of the proposed multi-scale approach as well as discuss its advantages and limitations.

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A thermodynamically consistent computational framework for brittle crack propagation along contact interfaces

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Keywords: Brittle fracture, Mortar contact, Coupled unilateral constraints, Hierarchical base.

Brittle crack propagation is present in various engineering applications, from the design of lithium-ion batteries to the assessment of safety-critical structures. One such example is brittle crack propagation in irradiated graphite bricks in a nuclear reactor, where one of the challenges is associated with handling contact interaction between different parts of the core's assembly. As a result, to model such systems, the computational framework has to handle, along with other nonlinearities, coupled unilateral constraints emerging from the crack propagation and from the contact interaction.

Computational studies have been conducted regarding the influence of contact between crack surfaces on the crack propagation, e.g. using the XFEM approach. These models are often employed to investigate the influence of fretting fatigue on the nucleation of a crack on one side of the contact interface and its propagation towards the interior of the body [1]. However, in the mentioned above nuclear industry applications, the cracks are not observed to close, while cases when the trace of the crack front traverses one side of a contact interface have to be addressed to assess the influence of the contact traction on the evolving crack morphology. Therefore, in this work, contact is considered only between separate bodies and not between the crack surfaces, see Figure 1.

A thermodynamically consistent numerical framework based on configurational mechanics for modelling brittle crack propagation has been proposed in [2]. In the present work, this framework has been extended to account for contact conditions [3]. In the proposed framework, crack propagation is described by means of the Arbitrary Lagrangian-Eulerian (ALE) formulation: material kinematic variables, $\mathbf{X}$, that describe crack surface increment are coupled with spatial kinematic variables, $\mathbf{x}$, that

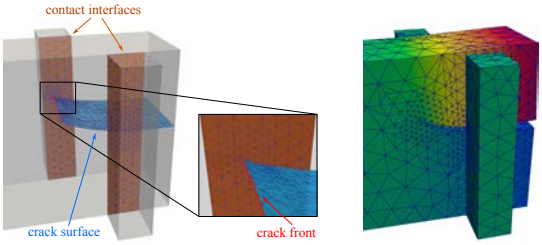

Figure 1: Crack propagation along contact interfaces: material (a) and spatial (b) configurations.

describe the elastic body deformation. Configurational forces, $\mathbf{G}^{\partial\Gamma_{\mathrm{cr}}}$, which drive the crack propagation in the material domain, are considered only at the crack front, $\partial\Gamma_{\mathrm{cr}}$, and are dual to material velocities $\dot{\mathbf{W}}$. The equilibrium equation for the crack front reads:

$$\dot{\mathbf{W}} \cdot (g_{\mathrm{cr}} \mathbf{A}^{\partial\Gamma_{\mathrm{cr}}}/2 - \mathbf{G}^{\partial\Gamma_{\mathrm{cr}}}) = \mathbf{0}, \tag{1}$$

where g_{cr} is the Griffith's energy, $\mathbf{A}^{\partial\Gamma_{\mathrm{cr}}}$ is the vector that determines the crack front extension and $g_{\mathrm{cr}}\mathbf{A}^{\partial\Gamma_{\mathrm{cr}}}/2$ denotes the material resistance force.

The crack propagation is based on the Griffith's criterion that can be understood as a unilateral constraint and can be expressed via the active set method with a complementarity function which is similar to the one proposed for frictionless contact constraints [4]:

$$C_{\mathrm{cr}}(\rho, \dot{W}_K) := \rho - \max\left(0, \rho - c_{\mathrm{cr}} \mathbf{A}^{\partial\Gamma_{\mathrm{cr}}} \cdot \dot{\mathbf{W}}\right), \tag{2}$$

where c_{cr} is the complimentary parameter and ρ is the Lagrange multiplier function which enforces the crack front balance equation (1).

At the same time, contact conditions between two deformable bodies are considered, where traditionally one of them is master and the other one slave and the fields of interest corresponding to each body are denoted by superscripts $^{(1)}$ and $^{(2)}$, respectively. Similarly to the crack propagation problem, contact constraints are enforced via the the complementatiry function proposed in [4]:

$$C_{\mathrm{con}}(\lambda, \mathbf{x}, \mathbf{X}) := \lambda - \max\left(0, \lambda - c_{\mathrm{n}} g_{\mathrm{con}}\right), \text{ with } g_{\mathrm{con}} := -\mathbf{N} \cdot (\mathbf{x}^{(1)} - \mathbf{x}^{(2)}), \tag{3}$$

where g_{con} is the gap function and $\mathbf{N}$ is the normal to the slave surface in the material domain. A mortar contact formulation [5] is employed for contact surfaces which are not expected to be traversed by the crack front, while a special case of mortar contact, where "shadowing" of contact elements originally placed on top of each other is preserved, is employed on the surfaces traversed by the crack.

The proposed framework is implemented by means of finite elements in a fully implicit formulation within the open-source software MoFEM [6]. The development has been verified and validated against numerical and experimental results. Furthermore, the effect of the contact traction on the crack front material forces is investigated.

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An Embedded Approach for Fluid-Structure-Contact Interaction Problems and application to the aortic flow.

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Keywords: Contact mechanics, Fluid-Structure Interaction, Aortic Flow.

Cardiovascular diseases remain one the leading causes of morbidity and mortality worldwide.
Computational models provide a unique opportunity to investigate the mechanisms leading to the initiation and the progressive development of vascular pathologies such as aneurysms and atherosclerosis. These computational models provide detailed data, in terms of fluid dynamics and stress distribution, that complement in vivo and in vitro testing, and that can be used in a predictive sense to support clinical practice and medical device manufacturing.
In contrast to in vitro studies, CFD is more convenient for studying model sensitivity with respect to boundary conditions and material constitutive law and parameters.
In this work, we employ a localized version of the L^2-projection for handling the fluid-structure volumetric coupling and a variant of the mortar method for coupling the surfaces of the structures in contact. The fluid-solid coupling is enforced with a Lagrange multiplier over the entire overlapping region.
The proposed method allows for the efficient and accurate resampling of fields across the fluid and the solid domains, which may feature very complex geometries. The choice of discretization techniques and parallel algorithms ensures convergence, efficiency, flexibility, and accuracy without requiring a priori information on the relation between the different meshes.
The discretization of the large deformation contact problem is based on a mortar approach for obtaining a well-posed and computationally feasible variational formulation. We note that the related literature usually employs a penalty method for modeling the contact arising among the leaflets during the closure of the valve. Such a method requires properly tuning nonphysical parameters, i.e., the penalty constant, which introduces additional errors.
We present numerical results for various benchmark problems and employ the proposed approach to simulate the dynamics of the aortic flow and the bio-prosthetic aortic valves.

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SESSION 6 (parallel)
Solution algorithms and numerical efficiency, General papers
Thursday, 14:20 – 16:20

RATTLE for mechanical systems with frictional unilateral constraints

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Keywords: RATTLE, nonsmooth mechanics, frictional contacts.

In 1983, Andersen [1] proposed the RATTLE integrator as an extension of the SHAKE algorithm [2]. The RATTLE integrator is a well-established method for simulating mechanical systems with bilateral constraints. However, there is a need for higher-order integration schemes for simulating nonsmooth mechanical systems with frictional contact, see e.g. Figure 1. This talk presents an extension to the RATTLE integrator that addresses this issue within the framework of nonsmooth contact dynamics [3, 4, 5]. The proposed scheme can simulate impact-free motions, such as persistent frictional contact, with second-order accuracy and prohibits penetration by unilateral constraints on position level.

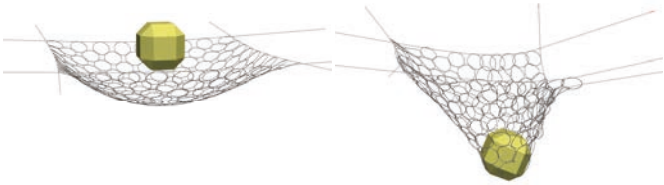

Figure 1: Simulation of rockfall protection ring nets with frictional contact.

Consider a finite-dimensional mechanical system whose state is described by the generalized coordinates $\mathbf{q}(t) \in \mathbb{R}^n$ and by the generalized velocities $\mathbf{u}(t) \in \mathbb{R}^m$, which are both functions of time t. The relation between positions and velocities is a consequence of the kinematics of the system and is generally of the form

$$\mathrm{d}\mathbf{q} = \dot{\mathbf{q}}(\mathbf{q},\mathbf{u})\,\mathrm{d}t \quad \text{where} \quad \dot{\mathbf{q}}(\mathbf{q},\mathbf{u}) = \mathbf{B}(\mathbf{q})\mathbf{u} + \boldsymbol{\beta}(\mathbf{q}) \quad \text{and} \quad \mathbf{B}(\mathbf{q}) \in \mathbb{R}^{n\times m}\,. \tag{1}$$

At a point $\mathbf{x} \in \mathbb{R}^f$ in a closed, convex, and non-empty set C, the normal and tangent cones are defined by $\mathcal{N}_C(\mathbf{x}) = \{\mathbf{y} \in \mathbb{R}^f \,|\, \mathbf{y}^{\mathrm{T}}(\mathbf{x}^* - \mathbf{x}) \leq 0,\, \forall \mathbf{x}^* \in C\}$ and $\mathcal{T}_C(\mathbf{x}) = \{\mathbf{v} \in \mathbb{R}^f \,|\, \mathbf{v}^{\mathrm{T}}\mathbf{y} \leq 0,\, \forall \mathbf{y} \in \mathcal{N}_C(\mathbf{x})\}$, respectively. The dynamics of a mechanical system subjected to both unilateral and bilateral constraints can be described by the measure differential inclusion

$$\mathbf{M}(\mathbf{q})\,\mathrm{d}\mathbf{u} = \mathbf{h}(\mathbf{q},\mathbf{u})\,\mathrm{d}t + \mathbf{W}(\mathbf{q})\mathrm{d}\mathbf{P} \tag{2a}$$

$$0 = \mathbf{g}(\mathbf{q})\,, \qquad 0 = \dot{\mathbf{g}}(\mathbf{q},\mathbf{u})\,, \tag{2b}$$

$$-R_N^k \in \mathcal{N}_{\mathbb{R}_0^+}\big(g_N^k(\mathbf{q})\big)\,, \qquad -R_N^k \in \mathcal{N}_{\mathcal{T}_{\mathbb{R}_0^+}\left(g_N^k(\mathbf{q})\right)}\big(\xi_N^k(\mathbf{q},\mathbf{u}^-,\mathbf{u}^+)\big)\,, \tag{2c}$$

$$\boldsymbol{\gamma}_F^k(\mathbf{q},\mathbf{u}) \in \mathcal{N}_{B(\mu R_N^k)}(-\mathbf{R}_F^k)\,, \qquad \boldsymbol{\xi}_F^k(\mathbf{q},\mathbf{u}^-,\mathbf{u}^+) \in \mathcal{N}_{B(\mu R_N^k)}(-\mathbf{R}_F^k)\,, \tag{2d}$$

using the notation introduced by Moreau [3] in 1988. Herein, the equality of measure (2a) relates the change in velocity $\mathrm{d}\mathbf{u}$ to the forces acting on the system. Non-constraint forces, such as spring forces, gyroscopic terms, and dashpot forces, are assumed to be non-impulsive and can be conveniently grouped together as $\mathbf{h}(\mathbf{q}, \mathbf{u})$. The force measure $\mathbf{W}(\mathbf{q})\mathrm{d}\mathbf{P}$ in (2a) includes the ideal constraint forces from both unilateral and bilateral constraints, as described by equations (2b) through (2d). The percussion measure $\mathrm{d}\mathbf{P} = \mathbf{R}\,\mathrm{d}\mu$ captures the combined effects of both impulsive and non-impulsive forces. The bilateral constraints at position and velocity levels are enforced by equation (2b). Meanwhile, equation (2c) incorporates Signorini's law and a mixture of Signorini's law at velocity level and Newton's impact law in normal direction. Finally, equation (2d) describes Coulomb's friction law and its combination with a Newton-type impact law in tangent direction.

To solve the normal cone inclusions that arise in the contact laws (2c) and (2d), we use the fact that for two vectors $\mathbf{x} \in \mathbb{R}^f$ and $\mathbf{y} \in \mathbb{R}^f$ the following are equivalent relations

$$\mathbf{y} \in \mathcal{N}_C(\mathbf{x}) \quad \Leftrightarrow \quad \mathbf{x} = \mathrm{prox}_C(r\mathbf{y} + \mathbf{x}) \quad \forall r > 0\,, \tag{3}$$

where prox_C is the proximal point function to the closed convex non-empty set $C \subset \mathbb{R}^f$, see [4, 6].

With the normal cone inclusions (2c) and (2d) replaced by their corresponding implicit equations given by (3), we are able to generalize the classical RATTLE scheme [7] to frictional unilateral constraints.

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An algebraic domain decomposition strategy for the solving contact problems

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Keywords: Sub-structuring methods, domain decomposition methods, parallel computing

Contact problems refer to modeling of the interaction between two or more elastic bodies when they are in contact with each other. While solving such contact problems, we must accurately model the forces the bodies induce at the contact surfaces and compute the resultant deformation due to the interaction between bodies. These problems are quite common in many scientific fields, e.g., mechanical engineering, automotive engineering, robotics, and biomechanics.

The formulation of the contact problems gives rise to variational inequalities, as the contact boundaries are part of the solution and are unknown apriori. Due to these reasons, it becomes essential to employ iterative solution methods to solve contact problems. At each iteration, the algorithm aims to satisfy equilibrium conditions for a given contact boundary. In many cases, the iterative schemes are computationally intensive and heavily rely on the geometric information of the problem.

To address these challenges, we present a domain decomposition-based solution strategy for solving contact problems in a fully algebraic manner. In this work, we employ a dual Lagrange multiplier-based mortar Finite Element method for discretization. In addition, we also apply a Householder reflection to transform the local coordinate system such that the linear contact constraints can be transformed to point-wise constraints [1]. Once the algebraic system is constructed, we need to solve a sequential quadratic programming problem. To this aim, we employ a sub-structuring domain decomposition-based strategy that decomposes the domains in non-overlapping subdomains. Using these non-overlapping subdomains, we decouple the unknowns into the interior of the subdomains and the ones that are on the interfaces and the contact boundary (also called skeleton). By employing this decomposition, we eliminate the interior unknowns and construct a smaller subproblem on the skeleton which is subjected to inequality constraints. Once we have constructed a smaller constrained optimization problem, we can solve it using projection-based methods [3, 2]. This robust approach allows us to solve large-scale problems using distributed computing environment.

Finally, we will present some numerical results to demonstrate the performance of our method using several numerical examples. We will test our solution strategy for solving the contact problem with hyperelastic material with multiple bodies in two and three dimensions.

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Novel approach for accurate identification of the real contact area for numerical modelling of triboelectric nanogenerators

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Keywords: Surface roughness; real contact area; contact virtual work, triboelectric nanogenerator

Triboelectric nanogenerators (TENG) are modern devices that use repeated cycles of contact between suitably chosen surfaces to transform mechanical energy into electrical energy, see Fig. 1(a). TENG have attracted significant attention in recent years as autonomous clean energy harvesters, where various sources of mechanical energy can be used: from human motion (wearable textile systems for charging miniature medical sensors) to ocean waves (large-scale networks for "blue energy" harvesting) [1].

Since tribo-charges appear only in zones of the real contact between two surfaces, the surface roughness and the external load have a significant effect on the output performance of TENG [2]. Numerical simulation of TENG requires multi-physical coupling of contact mechanics and electrostatics, and is further complicated if the surface roughness is taken into account. Recently we developed a computational framework capable of solving such problems using open-source library MoFEM [3], where the surface roughness measurements are projected on the finite element mesh, and the deformable-to-rigid-body contact problem is solved [4], see Fig. 1(b)-(c). The aim of the present work is to increase the accuracy of the real contact area identification by using a novel contact formulation.

First, we compute the contact area based on contributions from Gauss points of surface elements. We extend the formulation proposed in [5] and consider the values of the gap g and Lagrange multipliers λ interpolated also at Gauss points. The virtual work on the variation of the displacement field $\mathbf{u}$ reads:

$$\int_{\Gamma^c} W^c_{\mathbf{u}}(\mathbf{u},\lambda,\epsilon)\cdot\delta\mathbf{u}\,d\Gamma \approx \sum_{k=1}^{n^{\text{gp}}} w_k J_k \begin{cases} \lambda_k \dfrac{\partial g(\mathbf{u}_k)}{\partial \mathbf{u}_k}\cdot\delta\mathbf{u}_k, & \lambda_k + \epsilon g(\mathbf{u}_k) \le 0 \\ 0, & \lambda_k + \epsilon g(\mathbf{u}_k) > 0, \end{cases} \tag{1}$$

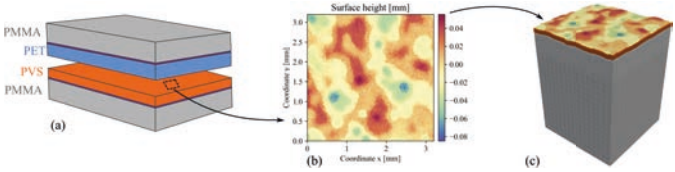

Figure 1: (a) schematic of the contact-separation TENG; (b) example of surface roughness measurements; (c) projection of the surface morphology on finite-element mesh for the contact problem setup.

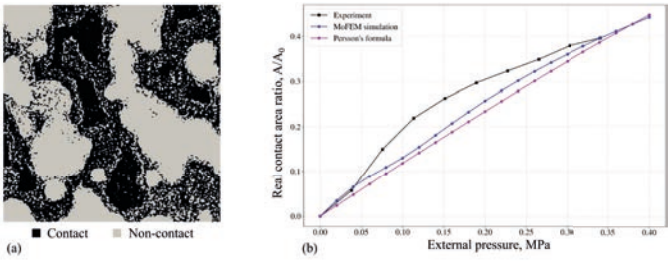

Figure 2: (a) real contact area morphology for the surface element, Fig 1(a)-(b), under external load of 0.4 MPa, (b) evolution of the real contact area with the increasing load: comparison with the experiment.

where ϵ is the augmentation/complimentary parameter, w_k is the weight associated with k-th Gauss point and J_k is the value of the Jacobian. Accordingly, the real contact area is computed as:

$$A^c \approx \sum_{k=1}^{n^{\text{gp}}} \begin{cases} w_k J_k, & \lambda_k + \epsilon g(\mathbf{u}_k) \leq 0 \\ 0, & \lambda_k + \epsilon g(\mathbf{u}_k) > 0. \end{cases} \tag{2}$$

Furthermore, we use a novel contact formulation based on the approximation of Lagrange multipliers in Raviart-Thomas space. Such formulation, along with Gauss-point-based evaluation of quantities (1) permits utilisation of hierarchical shape functions, providing stability and allowing for p-refinement.

The resulting morphology of the real contact area is presented in Fig. 2(a), while the evolution of the contact area is demonstrated in Fig. 2(b), where our results are compared with predictions of Persson's analytical formula [4] and experimental results obtained using interference reflection microscopy. While the results of the numerical simulation are closer to the experimental data than the analytical formula, the remaining discrepancy between the experiment and simulation is the topic of our ongoing work.

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Improving performance of augmented Lagrangians

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Keywords: Quadratic programming, augmented Lagrangian, active and free sets.

SMALSE-rho and SMALSE-M (Semi-Monotonic Augmented Lagrangian method for Separable and Equality constraints) algorithms are efficient tools for solving the quadratic programming problems with equality constraint and simple bound [1]. These algorithms consist of an outer loop for update of parameters rho or M, approximation of the Lagrange multipliers for equality constraint and MPRGP (Modified Proportioning with Reduced Gradient Projection) algorithm [3] used as an inner solver for problems with penalized equality constraint and simple bound. These algorithms are implemented for massively parallel usage into our in-house PERMON library [4] based on PETSc. SMALSE terminates if norms of the equality constraint violation and norm of the projected gradient are sufficiently small compared to the norm of the right hand side multiplied by the relative tolerance. MPRGP in general terminates, if the norm of the projected gradient is less than the norm of the violation of the equality constraint multiplied by SMALSE M-parameter. Parameter M is fixed for SMALSE-rho while penalty rho increases depending on the augmented Lagrangian growth. In SMALSE-M depending on the augmented Lagrangian growth, M decreases while rho is fixed. The larger penalty rho accelerates an outer loop, while larger parameter M accelerates an inner one. The presentation deals with the new theoretically supported SMALSE-rho,M variant increasing both M and rho parameters and reducing both numbers of outer and inner iterations. The performance of SMALSE versions can be further essentially improved by enhancing the information on the free set of current iterates into the reorthogonalization of equality constraints [2]. The presentation deals with the efficient parallel implementation requiring no transformation of the Lagrange multipliers and no assembling and no factorizing the whole product of matrices with equality constraints reflecting changed free set, but just their small submatrix dealing with rows affected by simple bound. All these improvements will be demonstrated by large-scale numerical experiments with contact problems solved using the FETI (Finite Element Tearing and Interconnect) method.

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Beam-inside-beam contact

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Keywords: Beams, Contact, Friction.

Beam finite elements are dedicated to efficiently predict the mechanics of slender constituents. Contact formulations for beams typically aim to repel beam finite elements from each other in case they penetrate each other [1, 2]. This is for instance useful for mesostructural models in which the mechanical deformation of each fiber or strut is described by a string of consecutive beam elements.

In this presentation however, a so-called beam-inside-beam contact formulation is discussed for beams with both circular and elliptical cross sections [3]. The beam-inside-beam contact formulation aims to keep a string of consecutive beam elements embedded within another string of consecutive beam elements. Because the presented work accounts for large rotations and large deformations, a variety of applications may be considered. These include slender instruments and sensors to investigate pipes and tubes, but also slender medical instruments inside the human body. Examples are hearing aids that must be precisely inserted in the cochlea, or guide wires that have to find their way through the vascular system.

Figure 1: The beam-inside-beam contact frameworks keeps a string of consecutive beam elements embedded within another string of consecutive beam elements. Left: initial configuration, center: intermediate configuration, right: final configuration.

The framework is first discussed for frictionless contact. A smooth center line approximation is introduced so that the surface approximation of the series of consecutive beam elements is smooth as well. Potential contact is considered for (the rigid) cross sections of the inner string of beam elements at predefined axial locations (see Fig. 2). This enables the treatment of non-local contact: in other words, contact between the inner beams and the outer beams can occur all through the model.

Figure 2: Potential penetration is considered for cross sections of the inner string of beam elements.

Subsequently, the frictional case is considered [4]. Whereas in conventional node-to-surface contact formulations, the sliding distance can 'simply' be quantified by tracking how far a slave node has slid over the master surface, this is not possible for the beam-inside-beam contact scheme (nor for beam-to-beam contact schemes for that matter). As a solution to this issue, the sliding distance is expressed in terms of contact points of the current increment and contact points of the previous increment, where the locations of the current contact points are considered in the configuration of the previous increment.

More details of the beam-inside-beam contact scheme, such as advantages and limitations, will be discussed in the presentation.

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Physics-informed neural networks for contact mechanics

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Keywords: Physics-informed neural networks, contact mechanics, mixed-variable formulation, inequality constraints, NCP functions

Machine learning models usually require an extensive amount of simulation or experimental data, which may be challenging to acquire due to the complexity of simulations and the cost of experiments. Also, data scarcity can cause data-driven techniques to perform poorly in terms of accuracy. This is particularly true when using real-world observations that are noisy or datasets that are incorrectly labeled, as there is no physical control mechanism to validate the predictions. To tackle this problem, physics-informed neural networks (PINNs) have been developed. PINNs integrate boundary or initial boundary value problems and measurement data into the neural network's loss function to compensate for the lack of sufficient data and the unknown behavior of data-driven techniques [1]. Based on sensor data of a physical object, PINNs can be used in hybrid digital twins of civil engineering structures [2]. In the field of computational mechanics, PINNs can serve as a forward solver, as data-enhanced forward model, as inverse solver for parameter identification, and as fast-to-evaluate surrogate model.

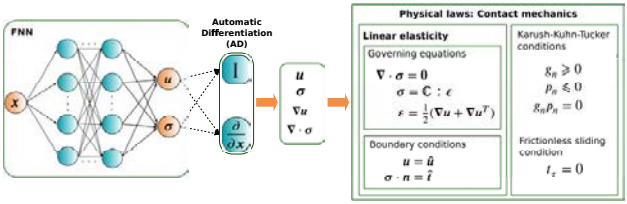

Figure 1: Physics-informed neural networks for contact mechanics in the case of linear elasticity.

In this contribution, we extend the concept of physics-informed neural networks to solve elementary problems of contact mechanics. We start with the governing equations of linear elasticity in a mixed-variable formulation in which displacements and stresses are taken as independent quantities. In comparison to pure solid mechanics, contact problems involve additional constraints, namely *Karush-Kuhn-Tucker (KKT)* type inequalities (see Fig. 1). The simplest way to enforce KKT conditions is to use an adopted *sign*-function to distinguish active or inactive contact regions. As an alternative, an adopted Sigmoid function with finite gradients can be used, but this requires a rather well-tuned softness parameter. Moreover, a nonlinear complementary function, namely the *Fischer-Burmeister NCP-function*, provides an elegant way to reduce the three inequality constraints into a single equality.

For our PINN implementation, the mixed-variable formulation has the advantage that only first-order derivatives of the network input are required. An appropriate output transformation allows us to enforce displacement and stress boundary conditions as hard constraints, i.e., they are exactly fulfilled by the

augmented network output. Further boundary conditions, e.g., the KKT conditions are enforced as soft constraints by adding corresponding contributions to the overall loss function.

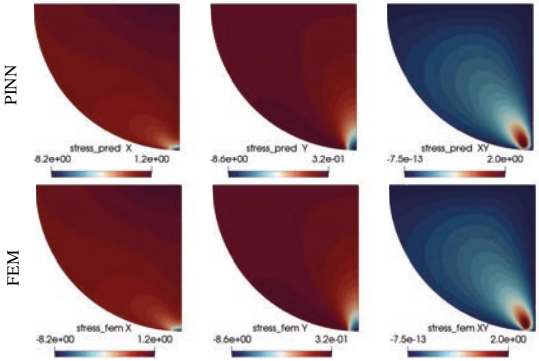

Figure 2: Comparison of stresses computed by PINN and FEM for a 2D Hertzian contact problem.

As a specific example we consider the 2D Hertzian contact problem of an elastic half-cylinder and a rigid plane [3]. For this setup, four distinct PINN application cases are demonstrated. In the first use case, we deploy PINNs as a pure contact mechanics solver to validate our approach (see Fig. 2 for a comparison to FEM). One of the key features of PINNs is to be capable of involving "external" (measurement or simulation) data very easily. Thus, in the second use case, we enhance our PINN model with displacement and stress values obtained by FEM to improve the prediction accuracy. In the third use case, the PINN is used to solve an inverse problem, namely, the prescribed load on the half-cylinder is identified from FEM data. As fourth and final example, the load (external pressure) is considered as a network input to construct a fast-to-evaluate surrogate model for different pressure values.

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SESSION 6B (parallel)
Discrete element methods for contact, Constraints enforcement methods
Thursday, 14:20 – 16:20

Simulating tribocharging of flowing granular materials with patchy particles

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Keywords: DEM simulations, Granular Materials, Tribocharging.

When two objects are rubbed against each other, they charge because of the triboelectric effect. It occurs as well between insulators as between metals whether they are made of the same material or not. This effect is predominant in granular materials which show significant charging due to the large rate of collisions between their constituents [1,2]. The charge of individual particles is surprisingly determined by their size as in binary mixtures, large particles tend to charge positively while small ones charge negatively. Neither this effect nor the specific charging it creates in binary mixtures have a fully detailed explanation. Moreover, reproducible electrostatic measurements are difficult to perform.

In this presentation, we consider the existence of acceptor and donor sites (also called patches) of charges at the surface of the particles [3] and show that the experimental measurement of binary mixtures' charging can be retrieved using numerical simulations. We provide a rational explanation based on the distribution of those sites on the particles' surface. Our results demonstrate that the charging of objects by rubbing is unavoidable and could be explained if patches were present on the surface of those objects.

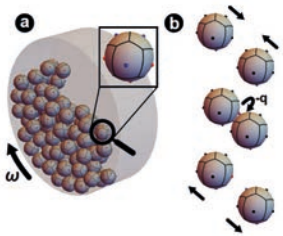

Figure 1: Numerical simulation of the triboelectric effect in flowing granular materials. (a) Snapshot of a numerical simulation to reproduce tribocharging with a close-up view of a typical patchy particle that is covered with sites that can either give a certain number of charges (donor patches, in red) or receive charges at contact with another patch (acceptor patches, in blue). (b) Sketch of the charge transfer mechanism between two particles.

To simulate tribocharging, we defined patches at the surface of spherical particles using a Voronoi tessellation (see Figure 1). These patches could be donors or acceptors of charges. We used a site donor probability p and a uniform distribution to assign the sites' nature. Once the particles are in motion and touch each other, charge transfer occurs between the patches. The Discrete Element Method (DEM) is used to compute the dynamics of the granular material in a rotating drum. It consists in computing every possible force on each spherical particle at a given time step and then apply Newton's Second Law of Motion to displace them to the following time step. In addition to the classical contact forces, Coulomb forces between the paches are considered.

We placed electrically neutral and equal in size particles in a rotating drum and measured the total transferred charge over time while the drum is rotating. The total transferred charge increases rapidly until saturation. Therefore, the patches are charging, inducing the formation of agglomerates as observed experimentally [4]. We reproduced also numerically the charging of binary mixtures with small and large grains charging with an opposite charge. One more time, this result is coherent with experimental observations [5].

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An Improved Normal Compliance Method for Non-Smooth Contact Dynamics

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Keywords: Unilateral constraint, Normal Compliance, Friction, Newmark methods, Non Smooth Contact Dynamics, Semi-Smooth Newton method, Primal-Dual Active Set.

Nowadays, the comprehension of granular media and their behaviors has become a significant concern for multiple industries. The scientific community, which exhibits a particular interest in researching these materials, is confronted not only with the complex character of the interactions governing their dynamics but also with the ever-increasing and recurring demand to simulate them. In front of these challenges, tools adapted to model granular dynamics have emerged, mainly grouped under the term of Discrete Element Method (DEM) [1]. Granular dynamics is then governed by Newtons second law of motion combined with a regular contact model. In particular, DEM simulations can be computationally expensive due to the explicit time schemes small time steps. Later, other discrete element based strategies have been developed (see, e.g., [1] and the references therein). Among those works, we mention the Non-Smooth Contact Dynamics (NSCD) approach [2] which (roughly speaking) takes into account the frictional contact interactions collectively during a time step and handle the non-regular dynamics equations through the Non-Linear Gauss-Seidel algorithm (NLGS).

In this work, we are interested in a discontinuous (with bounded variation) Moreau second-order sweeping process modeling the contact dynamics of rigid particles. Recall that sweeping processes [4] are particular differential inclusions governed by the normal cone of a (possibly nonconvex) moving set. The contact law is modeled through the Moreau-Yosida regularization [3] of the unilateral condition. It seems that the Moreau-Yosida regularization with parameter α (which encompasses the usual Moreau-Yosida method with $\alpha = 2$) is an appropriate tool to develop a regular contact model (normal compliance). The α-Moreau-Yosida regularization seems to be an appropriate tool to find find a regular contact model (normal compliance) which preserves the kinetic energy of the system while preserving the non-interpenetration of the contact.

The discrete Improved Normal Compliance (INC) method is considered suitable for ensuring energy conservation in adequation with the continuous framework. To solve the nonlinearity issue, a combination of the Newmark method [5] and the Primal-Dual Active Set (PDAS) method using complementarity of the various contact models [6] will be employed. The aim of this approach is to compare the efficacy of this improved method with other regular (DEM) and non-regular (NSCD) numerical modeling techniques.

The main aim of the present work is to improve an implicit regularization method for which energy conservation and non-penetration are quite similar to NSCD-NLGS along with a suitable computational cost. Several numerical experiments are reported for verification and validation purposes, and also to evaluate the efficiency and assess the performances of Newmark-PDAS-INC method compared to other numerical methods (DEM, NSCD-NLGS).

We assume that a particle q among N_q particles is described by the position of its center of gravity and its rotation, denote by $\mathbf{q}$ the generalized coordinate describing its position ($\mathbf{q} \in \mathbb{R}^{\bar{\mathbf{d}} \times \mathbf{N_q}}$, where $\bar{d} = 6$ in 3D and $\bar{d} = 3$ in 2D). Due to the possible shocks between particles, we introduce the generalized velocity $\dot{\mathbf{q}}$ as a function of bounded variation and its associated differential measure $d\dot{\mathbf{q}}$. According to the fundamental principle of rigid dynamics, the equations of motion formulated in terms of differential measures can be written as

$$\mathbb{M}d\dot{\mathbf{q}} = \boldsymbol{F}(\mathbf{t}, \mathbf{q}, \dot{\mathbf{q}})\mathbf{dt} + \mathbf{d\Lambda} \tag{1}$$

where $\mathbb{M}$ is the generalized mass matrix; $\boldsymbol{F}$ is external forces; $d\boldsymbol{\Lambda}$ is a non negative real measure representing reaction forces and impulses between particles at contact. Given a time discretization$(t_i)_i$ of the interval $[0, T]$ and setting $\Delta t := t_{i-1} - t_i$ we easily derive from 1 the following discrete approximation:

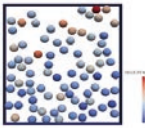
$$(P) \begin{cases} \mathbb{M}\dfrac{\dot{\mathbf{q}}^{n+1} - \dot{\mathbf{q}}^{n}}{\Delta t} = \boldsymbol{F} + \boldsymbol{\Lambda}^{n+1} \\ \boldsymbol{q}(0) = \boldsymbol{q}_0,\ \dot{\boldsymbol{q}}(0) = \boldsymbol{v}_0 \end{cases} \tag{2}$$

In order to almost conserve the energy and to respect the energy balance of the continuous case, we replace the α-normal compliance contact distance $([D_\nu^{n+1}]_+)^\alpha$ by Improved Normal Compliance value

$$\tilde{D}_\nu^{n+1} = \frac{([D_\nu^{n+1}]_+)^\alpha - ([D_\nu^n]_+)^\alpha}{\alpha(D_\nu^{n+1} - D_\nu^n)},$$

where $D_\nu = D_\nu(q_i, q_j)$ represents the signed distance from particle i to particle j, and $\alpha > 2$. We then obtain the normal stress value $\boldsymbol{\Lambda}^{n+1}$ of the Improved Normal Compliance condition in the discrete case: $\boldsymbol{\Lambda}^{n+1} = c_\nu[\tilde{D}_\nu^{n+1}]_+\boldsymbol{\nu}^{n+1}$ with $c_\nu > 0$ and $\boldsymbol{\nu}$ the normal.

Figure 1 fives some numerical results concerning a two-dimensional system with 81 particles. Each particle has a random trajectory and velocity with respect to time without friction and without gravity.



$\Delta t = 10^{-4}$	Energy	Error	Penetration
DEM	16.7219349	$1.4866 \times 10^{+1}$	1.4798×10^{-2}
NSCD	6.7248957	4.8171×10^{-9}	3.2928×10^{-4}
INC ($\alpha = 2$)	6.6738305	7.5935×10^{-3}	8.8043×10^{-3}
INC ($\alpha = 3$)	6.7248927	4.8092×10^{-7}	6.4869×10^{-2}

Figure 1: 2-Dimension system with 81 particles and numerical comparison bewteen different methods.

It turns out that the INC method implemented are at least just as much relevant as the NSCD-NLGS method and much better than DEM, regarding energy conservation properties and non penetrations, as they display physically realistic behaviour.

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Comparative analysis of experimental and numerical results for viscoelastic indentation of thin layers

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Keywords: Viscoelasticity, indentation, thin films.

Viscoelasticity is a complex and fascinating area of research in applied mechanics that has garnered increasing attention in recent years due to the prevalence of soft materials in industrial engineering and biological systems. Due to the time-dependent behavior viscoelastic materials exhibit, it is important to develop experimental techniques to accurately characterize them on a mechanical level. Dynamic mechanical analysis (DMA) is currently considered the gold standard for characterizing viscoelastic material. DMA involves testing samples in various ways and sweeping a temperature interval to construct the master curve of the viscoelastic modulus (see Figure 1a). However, DMA has some limitations, such as requiring sample extraction and testing at different temperatures, which may not be feasible for many biomechanical problems where sample availability and thermal heating can be problematic [1, 2].
In recent years, indentation tests have emerged as a promising alternative to DMA: a rigid punch indents a sample, and the evolution of the process in terms of force or penetration is measured to retrieve creep and relaxation properties. It is important to highlight that, differently from DMA, indentation tests do not require sample extraction and can be performed at a single temperature.
When testing thin films, particular care has to be paid in the correlation used to recover the material properties. Indeed, a variety of numerical studies have been recently proposed to analyze the indentation process, yet they present a strict limitation, as they are based on the half-space assumption, for the contact regions are assumed much smaller than the thickness of the solids in contact. Clearly, for thin layers, this can lead to a significant misinterpretation of the phenomenon, as shown for sliding contact problems [2,3].
The study aims to determine the degree of correspondence between numerical and experimental outcomes, obtained using MACH-1, Biomomentum) (see Figure 1b), and assess the reliability of numerical simulations in predicting the results of various samples.
Specifically, the study paves over a Boundary Element (BE) approach [4,5] to investigate the indentation process also when the solids in contact are thin layers. In particular, the BE methodology is based on the following integral formulation, explicitly accounting for the time and space domain:

$$u(x,t) = \int_{-\infty}^{t} d\tau \int d^2x' \, J(t-\tau) G(x-x') \, \dot{\sigma}(x',\tau) \, , \tag{1}$$

where $G(x)$ is the spatial Green's function, $G(x)$, $\dot{\sigma}(x,t)$ is the time derivative of the stress, and $J(t)$ is the so-called creep function, which obeys the causality principle, i.e. $J(t) = 0$ for $t < 0$. It is important to notice that the Green's function $G(x)$ satisfies the boundary conditions imposed on the viscoelastic substrate, thus addressing the problem to correctly take into consideration the influence of the viscoelastic slab thickness.
In summary, this study presents a significant opportunity to explore the potential of indentation tests

as an alternative method for characterizing viscoelastic materials and their applications in various fields. By comparing experimental and numerical results of indentation tests, the study aims to provide a better understanding of the indentation process of thin layers and assess the reliability of numerical simulations in predicting the results of various samples. The study also highlights the importance of numerical assessment of indentation in other problems where the damping offered by viscoelastic hysteresis is fundamental, such as viscoelastic vibrations and soft grasping in robotic devices.

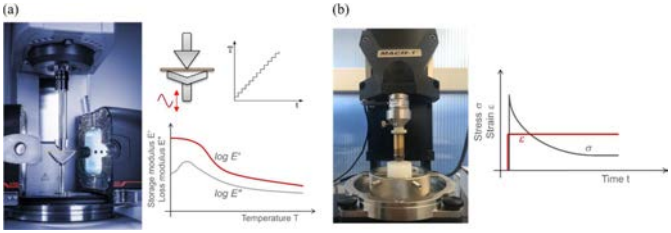

Figure 1: Experimental characterization: (a) DMA, (b) Indentation Test

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SESSION 7
Friction and wear
Thursday, 16:40 – 18:00

Breakdown of Reye's theory in single-asperity wear

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Keywords: Adhesive wear, Ductile fracture, Plasticity, Debris volume, Frictional work.

Building on an analogy to ductile fracture mechanics, we investigate the energetic cost of debris particle creation during adhesive wear. Macroscopically, Reye proposed in 1860 that there is a linear relation between frictional work and wear volume at the macroscopic scale. Earlier work suggested a linear relation between tangential work and wear debris volume also exists at the scale of a single asperity [1], assuming that the debris size is proportional to the micro contact size multiplied by the junction shear strength. However, recent works revealed deviations from linearity at the microscopic scale [2, 3]. These deviations can be rationalized with fracture mechanics and imply that less work is necessary to generate debris than what was assumed. Here, we postulate that the work needed to detach a wear particle is made of the surface energy expended to create new fracture surfaces, and also of plastic work within a fracture process zone of a given width around the cracks. Our theoretical model, validated by molecular dynamics simulations, reveals a super-linear scaling relation between debris volume (V_d) and tangential work (W_t): $V_d \sim W_t^{3/2}$ in 3D and $V_d \sim W_t^2$ in 2D. We also address the logical follow-up questions as to how to reconcile this non-linear scaling at the micro-level with Reye's theory, i.e., the well-documented linear scaling at the macroscale.

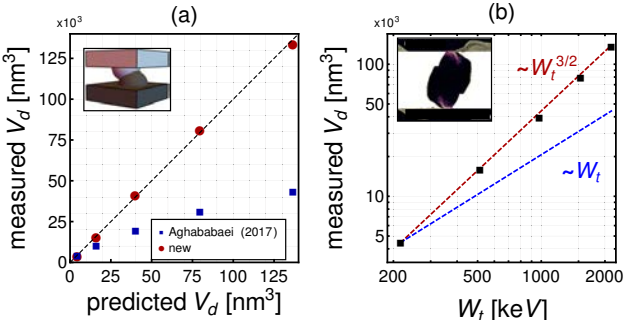

Figure 1: Tangential work (W_t) v. debris volume (V_d) in single-asperity wear [3]. (a) Comparison of results in [1] to new results [3] (inset depicts MD model geometry, $\sim 10^7$ atoms). (c) Verifying the new scaling (inset represents debris particle).

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Railway wheel wear calculation: comparison of local and global applications of Archard's law

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Keywords: Wear laws, multibody simulation, wheel-rail contact.

The prediction of the wear of railway wheels by means of reliable numerical codes is a key point to enable the introduction of predictive maintenance strategies, with considerable benefits in terms of costs and operation safety. The worn profile of railway wheels is commonly computed from the outputs of dynamic simulations, by means of wear laws that express the worn material as a function of the quantities related to the contact conditions [1]. Most wear laws can be applied in either global or local form. In the first case, the total worn volume is evaluated directly from the contact forces and creepages, and then the profile shape is obtained by spreading the worn material along the contact patch according to an a priori distribution. Conversely, local approaches calculate the distribution of the wear depth over the contact patch starting from the local values of tangential pressure and sliding speed.

The present paper aims to point out the main differences between the local and global application of the Archard's wear law, and to give further insights into the wear algorithm implemented in the wear module of the Simpack commercial multibody code. For this purpose, the worn profile is determined on the right wheel of the front wheelset of a Aln663 diesel railcar, modelled in Simpack. The MB model of the vehicle includes 13 bodies, namely one coach, two bolsters, two bogie frames, four wheelsets and eight axle-boxes. All bodies feature six degrees of freedom (d.o.fs), except for the axle-boxes, which only feature a rotational d.o.f with respect to the wheelsets. The nonlinearities of both primary and secondary suspension stages are modelled in detail. The track is modelled without irregularities, and it features 18 curves with radius from 300 m to 2000 m with step of 100 m and a tighter curve with radius of 280 m. The wheel-rail normal contact forces are calculated with a semi-Hertzian approach based on Kik and Piotrowski's work [2], while the tangential forces are evaluated with Kalker's FASTSIM algorithm [3]. Further details on the MB model can be found in [4]. The worn profile shape is calculated with global and local applications of the Archard's wear law, by means of in-house dedicated routines written in MATLAB and of the Simpack wear module, based on a global approach. According to Archard's wear law, the amount of worn material is proportional to the normal load and sliding distance, through a proportionality coefficient evaluated from experimental maps depending on the values of sliding speed and contact pressure. In the present paper, the wear coefficient is extracted from the wear map proposed by the Swedish Royal Institute of Technology (KTH), with application of reduction factors accounting for the effects of natural lubrication [5]. The KTH wear map accounts for four wear zones, namely two mild zones, a severe wear zone and finally a catastrophic wear zone, which is entered when the contact pressure is above a threshold value, typically assumed equal to 80% of the hardness of the softer contacting body. The global algorithm implemented in MATLAB spreads the total worn volume obtained from the global application of Archard's wear law according to an elliptic distribution, thus assuming that the wear depth is proportional to the Hertzian pressure. On the other hand, the MATLAB routine performing the local calculation of wear includes the FASTSIM algorithm [3] for the identification of the adhesion and slip zones on the contact patch.

The wear depths calculated by the global and local algorithms after a single multibody dynamic simulation are shown in Figure 1, which also displays the wear depth extracted from the outputs of

the Simpack Wear module. The three wear routines predict a higher wear on the wheel flange (lateral coordinates in the range between -40 mm and -20 mm), whereby the contact occurs when the vehicle a curved track section. In fact, the lateral creepage and the spin coefficient tend to increase in curves. The wear depth calculated by the MATLAB global routine is close to the one predicted by the Simpack Wear module. Slight differences may be due to different strategies used to smooth the wear depth. Because of the good agreement between the two wear routines, it can be concluded that the Simpack Wear module is extremely likely to rely on an elliptical distribution to spread the worn volume along the profile. The Simpack Wear module seems to estimate the contact pressure without applying filters to the normal load signal. This can cause localized wear peaks when wear is evaluated using the Archard's law, which predicts a step transition towards a catastrophic wear regime based on the contact pressure value. The local algorithm predicts two wear depth peaks, because it can account for local peaks of contact pressure and sliding speed, while the global algorithms can only rely on average values of sliding speed and contact pressure on the whole patch. Eventually, the wear peaks can produce numerical instabilities in the computation when the evolution of the wheel profile shape is calculated with an iteration of dynamic simulations, launched with worn profiles.

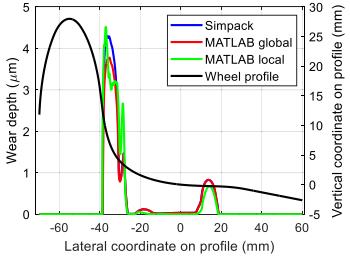

Figure 1: Wear depth along the wheel lateral coordinate calculated with the three algorithms (solid lines) and original wheel profile (dotted line).

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A Novel Single Pass Unbiased Frictional Contact Algorithm

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Keywords: unbiased contact, predictor-corrector, stick-slip, friction, segment-to-segment, computational contact mechanics

The development of engineering systems involves a critical phase of analysis of virtual designs with the models being subjected to boundary conditions that emulate real-world scenarios. In systems with multiple components e.g., vehicles, industrial machinery etc., the mechanical assembly that holds the structure involves physical contact between its body parts. These components undergo motion as well as deformation during the operational phase due to the load transfer from physically interacting parts in the system. These contacts pose a nonlinear problem as the area over which the loading tractions act is not known apriori. Numerically such problems can be solved using finite elements where the solution methods attempt to predict the traction at the elements on the boundaries of the interacting discretized geometries. This involves two stages - evaluation of normal forces due to the interacting elements where the interpenetration between the contacting surfaces has to be prevented and then the relative tangential motion, opposed by frictional tractions needs to be computed.

In this work, a novel tangential traction formulation is presented which is based on the classical predictor-corrector approach to resolving the non-linear response of the frictional surfaces. The updating trial traction bears a resemblance with the loading and unloading path of pressure and rate-dependent elastic-perfectly plastic material. The most often used node-to-surface-based contact methods are prone to inaccuracies due to sharp contacts. While the updating trial traction has been used in the master-slave strategy for the segment-to-segment approach [1] it has inherent biasing due to the choice of the master or slave surface. Researchers have introduced two-pass approaches to overcome biasing where the tractions are calculated on one surface in one pass followed by the same calculation on the other surface [2]. However, such repeated calculations not only add up the cost of computation, but they also don't maintain the necessary equilibrium of tangential tractions on opposite surfaces.

This work presents a midplane-based approach for interacting facets of the opposite surfaces where the trial tractions can be calculated in an unbiased manner in a single pass. This reduces the computational cost associated with dual passes while also eliminating the biasing inherent in the choice of the surface. Here, trial traction calculations are described that can be computed on a common interface basis for the interacting surfaces while considering the arbitrary slip between the two surfaces from the initial point of contact such that both contacting points might change (see Fig. 1). The trial traction is calculated by considering the relative slip in incrementing timesteps and is stored in the common interface basis which is being defined in the average directions of the parametric lines of both surfaces at contact points.

$$\mathbf{t}_{n+1}^{trial} = \mathbf{t}_n + \epsilon_t\left[\left({}^1\xi^{\alpha}_{\mathrm{P}_{n+1}} - {}^1\xi^{\alpha}_{\mathrm{P}_n}\right)\ {}^1\boldsymbol{a}_{\alpha_{n+1}} - \left({}^2\xi^{\alpha}_{\mathrm{Q}_{n+1}} - {}^2\xi^{\alpha}_{\mathrm{Q}_n}\right)\ {}^2\boldsymbol{a}_{\alpha_{n+1}}\right] \tag{1}$$

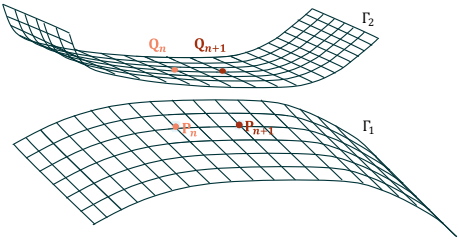

Fig. 1 Contact points for two interacting surfaces

In the discrete description of surfaces, the contacting facets would also have a relative angular motion. To take this into account, this work also introduces a methodology to consider the relative rotation between the two surfaces while accounting for the gradual transition in the stick-slip state of friction. Here, a rotation opposing moment is introduced, the magnitude of which increments with the rotation of the two surfaces as

$$\mathrm{M}_{n+1}^{trial} = \mathrm{M}_n + \epsilon_\theta(\theta_\Delta^{n+1} - \theta_\Delta^n)$$

Similar to the restriction on the updating trial traction, the updating moment is restricted to a threshold which is defined as, $\mathrm{M}_{thresh} = \frac{2}{3}\mu \mathrm{F_N}\sqrt{\frac{\mathrm{A}}{\pi}}$, where $\mathrm{F_N}$ is the normal contact force and A is the area of contact.

An ensemble of numerical simulations is used to probe the performance of the presented methodology for frictional contact.

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Effect of frictional weakening in fretting wear

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Keywords: contact, friction, wear, smoothing, implicit wear-cycle integration

Fretting wear induced by repetitive reciprocal tangential motion in contact interfaces is an important phenomenon for numerous engineering systems including aerospace, automotive and marine components as well as medical devices and electrical contacts. Wear simulation in such systems requires predictive wear models which in turn can rely on frictional models. Numerous wear experiments report a strong variation of wear performance for different material pairs, under various speeds, pressures and sliding distances. Because of this complexity, several families of wear models could be constructed relying on increasing number of parameters, which could include material parameters (elasto-plastic and thermal ones), contact pressure and frictional resistance, their variations and history, surface roughness and its evolution, environment, ratio of tangential amplitude to contact size, and others. Nevertheless, quite often the study of wear is carried out using relatively simple wear criteria based on the dissipated energy, which in turn is based on the simplest Coulomb friction law. In this study, we use an energy based wear model coupled to friction models of increasing complexity. The key objective of the study is to understand the effect of such an enhanced frictional rheology on wear profile in gross slip and partial slip for the general case of bi-material contact interfaces.

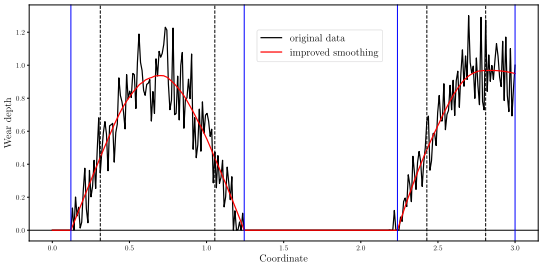

Figure 1: Modern worn profile before and after wear-volume-preserving smoothing which also prevents the worn profile from nonphysical spreading.

We use a finite-element computational framework which relies on few relatively novel techniques. The first one is an implicit integration scheme that we use to update the worn profile after every macro-cycle [1]. It has to be highlighted that contrary to more classical techniques for implicit wear update at every Newton's iteration loop, the implicit integration of wear over the entire fretting-cycle is relatively

novel and has not been yet fully explored. It allows us to get rid of classical instabilities of explicit integration schemes and to decrease the effective number of computed macro-cycles. The second original technique is the smoothing procedure, which targets to properly evaluate the spatial distribution of dissipated energy over every cycle (see Fig. 1). This new smoothing procedure, contrary to a classical Gaussian smoothing [2], prevents unphysical spreading out of worn profile and preserves the overall dissipated energy. These algorithms are implemented in in-house finite element software [3, 4] which is equipped with standard remeshing algorithms for wear propagation and with a field transfer algorithm needed for material models with history variables (visco-elastic and elasto-plastic ones). In addition, a boundary unfitted technique combining the mortar method with the X-FEM is also available and permits to avoid remeshing [5]. Three frictional models are studied: (C) Coulomb's friction law, (S) a slip-weakening friction with a recovery and (V) a velocity-weakening friction. Models (S) and (V) are compared to reference result obtained with model (C) in terms of morphology of worn surface for a cylinder-on-plane configuration for partial and gross slip. An example of slip-weakening friction from static to kinetic friction μ_s to μ_k which also includes a recovery is given by

$$\mu = \mu_s + \begin{cases} (\mu_d - \mu_s)(1 - \exp(-d/d_0)), & \text{if slip } |v| > 0, \\ (\mu_d - \mu_s)(1 - \exp(-d_{\max}/d_0)) \exp(-(t - t')/t_0), & \text{if stick } |v| = 0, \end{cases} \tag{1}$$

where $d_{\max}$ is the maximal slip distance reached in the last slip at $t = t'$, d_0 is a characteristic slip distance and t_0 is a characteristic time which controls return to the static friction at rest.

Elastic and elasto-plastic material models are studied, however damage accumulation responsible for fretting fatigue is neglected, the focus is set up on wear modeling. From computational contact mechanics' point of view, this study addresses the heuristic stability of wear profiles and performance of smoothing technique within surface-to-surface and node-to-segment contact discretizations. From the physical point of view, novel results are obtained for the frequency effects on the wear within enhanced frictional rheology.

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SESSION 8
Contact detection algorithms
Friday, 9:20 – 11:00

Novel framework for modelling contact between curved surfaces in large-scale Finite Element simulations

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Keywords: Curved surfaces, Gregory Patches, Finite Element Method (FEM)

Engineered products often contain parts in contact along curved interfaces. Modelling such interfaces with faceted Finite Element (FE) meshes and traditional contact mechanics algorithms (e.g., penalty or augmented Lagrangian methods) works well for matched meshes with little to no relative movement. However, obtaining accurate solutions for non-matching meshes with large sliding behaviour requires fine mesh discretizations that can be computationally prohibitive for large models.

Mesh faceting can lead to modelling challenges, such as fringes on stress distributions resulting in artificially high stresses. Figure 1 shows the pathology when the white sphere is pressed and rotated against the red block. Various approaches exist to improve this problem such as penetration removal, offset algorithms and geometry-based contact, but they still present limitations affecting accuracy, stability, computational efficiency, or validity beyond small scale deformation.

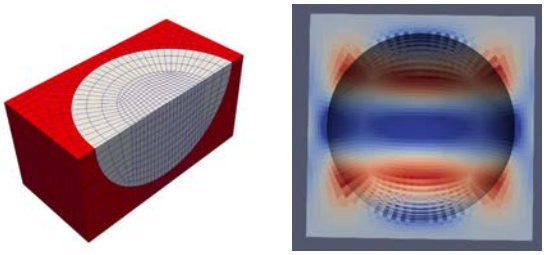

Figure 1. Resultant stress fringes for non-matching using traditional augmented Lagrangian contact

In this work, an approach for surface reconstruction using node normals is adopted to alleviate these limitations, and the technique is implemented using SIERRA Mechanics [1], a FEM software for large-scale multiphysics simulations developed at Sandia National Laboratories. Using a node-face contact formulation, the face reconstruction was done with both Nagata [2] and Gregory [3] patches

and compared for accuracy, performance, and robustness. The developed algorithms show great promise in a variety of test problems, with better accuracy across many applications and faster convergence in non-linear implicit quasistatic analyses.

This presentation will showcase how the new algorithm was developed and the details behind the final implementation. In addition to the patch reconstruction, different closest point projection algorithms based on gradient descent methods and the Nelder-Mead algorithm [4] were implemented. It is also shown how the use of simple problems from the literature influenced the choice of both the closest point projection algorithm and the patch type used. These simulations were only possible once the algorithm had been implemented within SIERRA, which led us to different default options compared to those originally considered based on results of small-scale problems.

For usability, a curvature metric [5] was implemented that picks the most appropriate side for face reconstruction in the node-face interaction. The results comparing analytical solutions from the literature [6] as well as large-scale simulations are shown to demonstrate the advantages of this new approach.

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Strategy to address two-dimensional pointwise concave contact problems

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Keywords: curve-to-curve contact, concavity, local contact problem, master-to-master.

Contact detection is a very important computational task for different fields, from computer graphics to computational contact mechanics. In the last, traditionally, one of the curves/surfaces is discretized into a set of points that are paired with their projections onto the other curve/surface, a procedure called Closest Point Projection (CPP) [1]. Problems of non-uniqueness arise when dealing with concave bodies [2].

In the context of pointwise contact, usually a valid approximation for non-conformal contact with small deformations, an approach not requiring prior discretization of any boundary was proposed for curve-to-curve [3] and surface-to-surface [4] contacts. In the curve-to-curve case, one searches for a pair of points that satisfies orthogonality relations necessary for the minimization of the objective function

$$f(x_1, x_2) = \frac{1}{2}\big(\Gamma_A(x_1) - \Gamma_B(x_2)\big) \cdot \big(\Gamma_A(x_1) - \Gamma_B(x_2)\big) = \frac{1}{2}\big(d(x_1, x_2)\big)^2, \tag{1}$$

which gives the squared distance between material points of two planar curves Γ_A and Γ_B, parameterized by x_1 and x_2 coordinates, respectively.

Such search for pairs of contact points is referred to as Local Contact Problem (LCP). A peculiarity of the approach is that for intersecting boundaries, which is the case when a penalty-like method is employed to enforce contact, the contact pairs are not actual minimizers of f, but saddle points [5]. It is possible to convert the LCP into a minimization problem [6], enabling the use of more robust optimization methods to solve the LCP. However, the approach is restricted to convex bodies.

In this work, we investigate properties of the objective function in equation (1) when the bodies do not intersect each other. It is supposed that a barrier-like method is used to enforce contact while preventing intersections. The bodies may contain concavities, but it is supposed that all curves are composed of strictly convex segments. The convex segment is said to be a concave boundary if the outward normal of the body $\boldsymbol{n}_{\text{ext}}$ points to the same direction of the normal of the segment $\boldsymbol{n}_{\Gamma}$, and it is said to be a convex boundary otherwise (Figure 1).

(a) (b)

Figure 1: Convex segments representing (a) convex boundary (b) concave boundary.

We first show that, for non-intersecting configurations, only minimizers of f can effectively

contribute to contact. Then, we proceed to a discussion of uniqueness of contact pairs in two cases (Figure 2). The first one is the contact of convex bodies. We prove by geometric considerations that, even though the minimizer of f is not necessarily unique, the contact pair is unique.

(a) (b)

Figure 2: Contact cases for (a) convex boundaries (b) convex and concave boundaries.

The second case is the contact between convex and concave boundaries. A univariate restriction of f is obtained by projecting the concave boundary onto the convex boundary. The properties of the restricted function are deeply related to geometric properties of the curves. The relations are encapsulated into the function

$$g(x_1) = R_B\big(x_2(x_1)\big) - R_A(x_1) - d(x_1), \tag{2}$$

where $R_A(x_1)$ is the curvature radius of the convex boundary at point x_1, $R_B\big(x_2(x_1)\big)$ is the curvature radius of the concave boundary at point x_2 with projection x_1, and $d(x_1)$ is the distance between the two points. The function g can be used to split the curves into regions, thus isolating the minimizers of f. With that, all plausible contact points are expected to be found.

Examples will be presented to demonstrate the behavior of the proposed strategy for detecting pointwise contact between general curves.

Acknowledgments

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aHigh-Fidelity Stress Fields in Contact Problems using Beam, Plates, and Shells Layer-Wise Models

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Keywords: CUF, finite element analysis, Beams, Plates, Shells

Structural analysis often involves contact mechanics, as mechanical systems typically feature static or dynamic contact between components. Examples of contact include meshing gears, forming processes, and simulating indentation, with impact analysis of structures being a crucial engineering application of contact. With composite laminated materials being increasingly used, it is essential to study the impact of contact, as it can result in localized damage and delamination, which can severely reduce the mechanical properties of a structure. The numerical modeling of contact is a current challenge in computational mechanics [1]. Over the last few decades, various techniques of varying complexity have emerged, e.g., node-to-node contact [2-3] and node-to-surface [6-7], and surface-based [8-9]. Such techniques are, then, coupled with structural mechanics models and the Finite Element Method (FEM). For high-resolution results concerning stress and strain fields, 3D FEM is often required; however, the computational overhead of 3D can be prohibitive in many cases, e.g., laminated structures. This work proposes using 1D and 2D models as alternatives to 3D. The Carrera Unified Formulation (CUF) is used to derive such reduced models based on higher-order expansions of the displacement field and ensure 3D-like accuracy for all the components of the stress and strain vectors. The study focuses on normal, frictionless contact using a penalty method to enforce the contact constraints. With a view to the accuracy and the computational time required for analysis, different numerical models, such as higher-order CUF and 3D finite element models, are presented and compared. The results indicate that Layer-Wise CUF models require at least an order of magnitude fewer degrees of freedom and computational time than 3D finite element analysis and can furnish very accurate results regarding stress distribution.

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Robust and generic contact detection strategy using tandem traversal of Bounding Volumes Hierarchies and spatio-temporal intersection

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Keywords: Computational contact mechanics, Bounding Volume Hierarchies, computational geometry

In the general context of computational mechanics, contact simulation is a complex and essential component in a large majority of diverse mechanical problems. In particular, high fidelity simulations of material and structure problems require robust methods in the three main phases of contact simulation which are detection, formulation and resolution. This work first focuses on contact detection, whose robustness is imperative to avoid convergence problems and false results. Besides the robustness, contact detection also has to deal with a wide variety of hypothesis, material behaviors, and physical phenomena related to all kinds of contact problems. Each problem leads to different difficulties and detection requests, requiring case-specific tools and techniques. For example, detection for auto-contact problems is difficult because each mesh entity is near to its topological neighbors in the mesh, while they shouldn't be considered in contact. Another difficult case is contact of thin elements, because standard static intersection tests can fail to detect any contact. These two cases therefore lead to more computations.

The detection phase consists in finding all pairs of elements from two meshes which are in contact, by performing a given test (minimal distance, volume intersection...) on each pair of elements to evaluate if it is in contact or not. Looping directly over all pairs of mesh elements isn't an option since it represents both too many and too heavy tests to compute. In the standard node-to-face discretization, one often performs a contact detection based on a warning distance [1, 2], which is the distance below which a slave node and a master face are considered in contact. To avoid looping over all possible pairs of contact elements, a hierarchical data structure (Octree, KD-tree, BVH...) is built over the entities on one side of the contact and greatly accelerates the search [3].

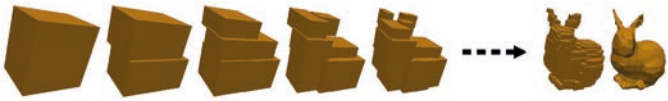

Figure 1: Iterative approximation with a Bounding Volume Hierarchy

This work aims to generalize this procedure, that is to develop a generic, robust and fast tool to filter the mesh elements to test (global detection), and a generic and adaptive API for the local tests to perform on the mesh elements (local detection). Such a contact detection tool is a major help in the development of contact simulation for industrial applications. We develop a detection pipeline based on Bounding Volumes Hierarchies (**BVH**) (see Figure 1), which are well-known data structures used to solve a wide variety of computational geometry problems [4]. Meshes are iteratively approached by

an increasing number of simplified geometric primitives (such as Axis Aligned Bounding Boxes) in a hierarchical data structure (a binary tree) to perform a fast detection prediction without false negative. Such structures are studied for more than 30 years in the computational contact mechanics and the video game communities.

The approach we have developed leads to many optimizations of the global detection pipeline. One of them is based on a tandem traversal algorithm described by Yang and Laursen [5] to search two BVHs alternatively in a contact detection. With this approach, a BVH is built on each side of the contact, and the search for contact elements uses both BVHs at the same time in order to benefit from the hierarchical structure on both sides of the contact. We bring this approach a bit further by searching both BVHs simultaneously. This contrasts with the standard approach using a single hierarchical data structure on one single side of the contact.

In addition, complex detection operations have been developed. Vlack, Tachi and Cameron describe the necessity of developing spatio-temporal intersection tests in contact detection [6, 7], which prevent from missing contact occurring exclusively between two instants of the simulation (see Figure 2). The use of spatio-temporal intersection tests is needed to define specific tests for the Bounding Volumes (global detection) and between the mesh elements (local detection), but also to develop a robust detection procedure in pathologic cases. Indeed, once the contact is detected and the corresponding constraints are added to the simulation, slipping nodes stay very near to the interface, and a space-time intersection based detection is not suitable anymore. Therefore, a specific procedure switching to normal projections is needed to follow and update the contact pairs.

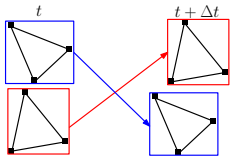

Figure 2: Contact between these two triangles can't be detected by looking at t or $t + \Delta t$ because it occurred exclusively between these two instants

This work will be presented with various examples to illustrate the performances of each approach. It is developed in an external C++ template library integrated in the FEM solvers *Manta* and *Z-set*.

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A refined algorithm for hierarchical face clustering and contact detection for segment-to-segment contact search between three-dimensional deformable bodies with irregular surface meshes

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Keywords: Contact search, contact detection, hierarchical clustering, bounding volumes.

Contact search is one of the key processes of contact simulation in structural analysis and is potentially the most expensive. In the case of large deformations where contact regions may evolve dramatically throughout the analysis, the process would have to be executed at every load- or time-step or even at every iteration, which means that it could easily dominate the computing time. Contact search can generally be divided into two stages, namely global and local search, which are also known as 'spatial search' and 'contact detection' respectively. The former is associated with identifying pairs of surface discretisation units (nodes, segments, or both) which are potentially in contact, whereas the latter is associated with the determination of actual contacting pairs based on criteria usually set by the adopted contact constraint enforcement method. Over the past decades, various spatial search algorithms have been developed to drastically reduce the number of operations compared to the rudimentary 'brute force' algorithm, many involving bounding volume hierarchies and hierarchical face clustering [1]. An effective hierarchic search algorithm allows for unnecessary branches to be pruned as early as possible to forego unnecessary operations. Several hierarchic search algorithms have been developed over recent years for node-to-segment [2] and segment-to-segment [3] contact. Many of these algorithms use binary trees where each non-leaf group in the hierarchy has exactly two child groups. However, notably in adaptive analysis, three-dimensional surfaces may feature localised regions of increased mesh density due to the presence of stress or geometric discontinuities. For segment-to-segment contact, the use of binary trees for such surfaces may result in the formation of additional hierarchical branches at these localised regions which do not contribute much to the accuracy of the spatial search yet introduce additional computational cost. Figure 1(a) demonstrates this issue, where the left-hand side (LHS) of the upper surface region is expected to have the same contact status as the right-hand side (RHS) depending on the value of the uniform gap g_u, and hence going through all its hierarchical branches may not be necessary.

The current work aims to overcome the aforementioned issue, drawing upon the assumption that, for two reasonably continuous surface regions which are potentially in contact, the localised regions of denser mesh will likely be subject to the same contact status as that of the adjacent areas of coarser mesh. Accordingly, a novel feature of the proposed method is its capability to generate clusters with a more even size distribution across each surface region in order to minimise unnecessary additional hierarchical branches at these localised regions of denser mesh. In the proposed method, rather than clustering adjacent pairs of units as per a binary tree (units being segments or clusters depending on the hierarchical level), cluster seed nodes are designated automatically based on two main criteria: (1) each seed node is approximately at a half-cluster distance away from the borders of adjacent clusters; and (2) adjacent clusters should share as many border nodes as possible. The result of adopting such criteria for the automatic selection of seed nodes is a clustering as shown in Figure 1(b), where the LHS and RHS of the upper surface region will have clusters of more similar size as well as the same number of hierarchic levels. Although other choices of bounding volumes are possible, this work adopts spheres since these can be easily checked for overlap. The proposed

clustering method is most importantly applied to the lowest levels of the hierarchy, with the remaining higher levels being binary, but it can be extended to higher levels as well depending on the irregularity of the mesh.

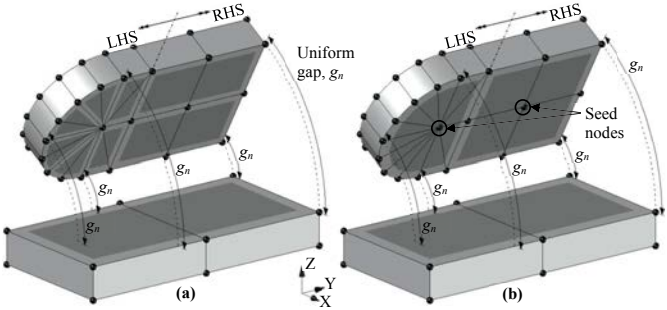

Figure 1: (a) Surface regions with binary clusters (shaded); (b) surface regions with clusters from the proposed method.

As for the contact detection method proposed in the current work, the spherical bounding volume is further applied in a fashion akin to the pinball algorithm but with the sphere externally enclosing the solid element instead of being embedded in it. The spherical bounding volume applied this way has a dual function in the sense that it is used for both establishing the distance between the paired segments before evaluating the gap at each Gauss point, and ascertaining the relative position of the counterpart segment with respect to the domain of the body to which the reference segment belongs. The proposed method is superior to approaches in which the spherical bounding volume is generated over individual segments alone in that it would typically result in a smaller distance between the segment face and the boundary of the enclosure in the direction away from the domain of the body, hence providing a more refined overlap detection. Non-convergence in the iterative closest-point algorithm can also be simultaneously avoided by first evaluating the angle between the average normal vectors of the paired segments. The proposed algorithms for both spatial search and contact detection have been implemented in ADAPTIC [4] which is used to demonstrate their exceptional benefits over existing techniques for contact search between surfaces discretised with highly irregular meshes.

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SESSION 9
Discretization techniques
Friday, 11:20 – 13:20

Virtual element methods and higher order penalty-based Node-to-Segment contact

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Keywords: Virtual element method, node-to-segment contact.

The virtual element method is a generalization of the finite element method for polytopal shapes and arbitrary convergence order. Almost a decade has passed since its first introduction [1], and it has been used for many applications where finite element methods dominated, especially those where geometrical versatility of the elements was welcome. One such case is contact, where a whole novel approach for Node-to-Node contact has been proposed based on this method for both 2D [2] and 3D [3].

Whereas new approaches are certainly valuable contributions to the state-of-art, investigating the use of new tools for approaches that, despite their flaws, have stood up to the test of time, can also lead to unexpected results and insights.

In this case, by approaches the authors refer to the penalty-based Node-to-Segment (NTS) method. This contact scheme can be found in literature as early as 1976 [4], representing a departure from Node-to-Node schemes where the contact location was required *a priori*. Although more modern (and mathematically sounder) approaches are now available (*e.g.*, Mortar methods), the Node-to-Segment is still widely found in commercial finite element analysis software and employed by the industry due to its low computational cost and sufficient accuracy for the intended purposes, justifying the interest in penalty-based formulations, which does not add degrees of freedom to the system.

A modern detailed formulation of NTS can be found in [5]. The method is inherently flawed by its asymmetrical treatment of the contact surfaces, this affects its capacity to provide a discrete contact contribution to the problem's weak formulation consistent with the adjacent continuum discretization, which becomes evident in its patch test performance [6]. In the last years, some modifications of the method have been proposed such that the patch test is passed for first-order interpolations, *e.g.*, virtual node schemes [7] and, more recently, an improved area regularization scheme [8]. Nevertheless, these approaches focus only on first order elements and guarantee consistency of the contribution only for uniform contact pressure, on other cases (*e.g.*, Hertz contact) the solution is still inconsistent. For quadratic polynomial onward, a new source of incompatibility on the treatment of the surfaces comes into play: the different weights associated with extremities and in-edge nodes when computing the equivalent nodal load for a distributed load over the segment.

This work presents another outlook on improving the stresses from Node-to-Segment contact schemes, for first and higher order elements. Assuming the method's incompatibility as something cannot not be fixed without losing the simplicity which makes it attractive, the authors aim at reducing the consequent oscillations in the stresses at element scale. This is done not by changing the contact scheme, but the formulation of contact-adjacent elements from finite to virtual.

The reduction of intra-element oscillations in the stresses is a consequence on how stress results are visualized with the virtual element methods, *i.e.*, by taking the polynomial projection of the solution. This projection effectively acts as a filter on these oscillations. This filtering effect may vary with the

characteristics of the element used in the contact region, polygonal elements with over 5 sides have shown the best results.

In Figure 1, one can see a detail of the minimum principal stress on contacting cylinders in a plane-strain linear elastic model, such that the minimum stress according to Hertz's theory should be -669.

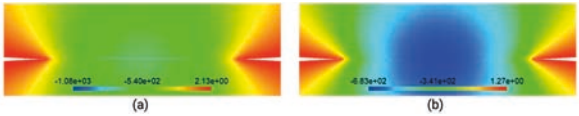

Figure 1: Minimum principal stress (σ_3) for contacting cylinders in plane strain linear elasticity with (a) quadratic triangular finite elements and (b) quadratic polygonal virtual elements

In the conference, a discussion and more examples of this filtering behavior will be provided, showing its potential benefits while post-processing stresses stemming from contact interactions.

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Use of nonsymetric unilateral cinematic constraints to solve Coulomb contact/friction problem

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Keywords: Dirichlet, Lagrange, Coulomb.

We remind the reader of Dirichlet boundary condition and Lagrange multiplier:
The calculation of static structures is based on the resolution of differential equations associated with boundary conditions, and in most cases requires the use of a numerical approach, wich is based on the search for solution fields belonging to a reduced admissible space V_r.
In the conservative case, it translates to finding the minimum of the total potential energy of the structure among all the kinematically admissible fields of displacements belonging to V_r,
i.e. verifying a priori Dirichlet conditions, non-homogeneous in the most general case, on a part $\partial\Omega 1$ of the boundary of the studied domain:

$$\min_{q \in V_r} \left(\frac{1}{2} q^T K q - q^T F \right)$$

$$V_r = \{ q : Aq = b \ \ on \ \partial\Omega_1 \}$$

Admissibility of displacements requires the introduction of bond strengths F_l such as

$$F + F_l = Kq$$

Given that dq verifies $Adq = 0$, necessarily Fl is of the following form:

$$F_l = -A^T \lambda$$

Where λ designates the vector of the Lagrange multipliers, which introduce additional unknowns. Thus, we solve for the following matrix system:

$$\begin{pmatrix} K & A^T \\ A & 0 \end{pmatrix} \begin{pmatrix} q \\ \lambda \end{pmatrix} = \begin{pmatrix} F \\ b \end{pmatrix}$$

The solution trivially satisfies the Dirichlet conditions. These conditions are said to be associated because the binding forces $A^t \lambda$ are dual to the cinematic constraint.

Let's now consider a structure loaded on the one hand by a known loading F and on the other hand by a loading whose intensity is controlled by an inhomogeneous kinematic constraint of the form Aq = b:

$$Kq = F + \lambda G^T$$

Inspired by the previous associated formulation, we add λ to the unknowns of the problem, and we use the formalism of Lagrange multipliers. The system to be solved therefore becomes:

$$\begin{pmatrix} K & -G^T \\ A & 0 \end{pmatrix} \begin{pmatrix} q \\ \lambda \end{pmatrix} = \begin{pmatrix} F \\ b \end{pmatrix}$$

Although similar, the multiplier is no more a Lagrange multiplier for it does not solve a minimization problem, and the matrix form is not symmetric.

The non-symmetry is unavoidable for the mechanical system is not conservative. The relationship is not associated because the reaction is no longer dual to the constraint. We can give the physical interpretation:
- In an associated constraint, the verification of the relation is ensured by the introduction of binding strengths, also called reactions. The minimization of the mechanical energy of the system enforces the form of the reaction. It is the transpose of the constraint.
-In a non-associated constraint, the bond strengths are arbitrary but serve the same to verify the constraint.

Application to control: we define a non-associated condition whose primal part is the cinematic constraint and the dual part the control force vector. It is added to the mechanical stiffness of the system then we solve the total linear system. The load multiplier is directly solution of the problem.

Application to Coulomb friction: One points out the definition of the contact-friction of Coulomb: The condition of contact in the normal direction is written:

$$D_n^T q \geq 0$$

Either there is a positive play, or there is contact and two reactions: a positive normal reaction R_n and a tangential reaction R_t linked by the relation:

$$|R_t| < \mu |R_n|$$

By adding the two multipliers λ_n and λ_t to the unknowns of the problem, the contact-friction is therefore described by the following two unilateral conditions:

- Normal condition: $\begin{pmatrix} 0 & D_n & -\mu D_t \\ D_n^T & 0 & \end{pmatrix} \begin{pmatrix} q \\ \lambda_n \end{pmatrix} \geq \begin{pmatrix} F \\ 0 \end{pmatrix}$
- Tangential condition: $\begin{pmatrix} 0 & D_t \\ D_t^T & 0 \end{pmatrix} \begin{pmatrix} q \\ \lambda_t \end{pmatrix} \geq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Which gives the complete asymmetric system: $\begin{pmatrix} K & D_n & -\mu D_t & D_t \\ D_n^T & 0 & 0 \\ D_t^T & 0 & 0 \end{pmatrix} \begin{pmatrix} q \\ \lambda_n \\ \lambda_t \end{pmatrix} \geq \begin{pmatrix} F \\ 0 \\ 0 \end{pmatrix}$

This linear system with unilateral conditions can be solved for example by an active set method taking in account the non positivity of K due to free modes or post buckling behavior.

A fixed point scheme is then required to determine the unknown direction D_t.That one is updated during the iterations, either using the displacement direction, or using the tangential reaction. The latter is given by $(\lambda_t - \mu \lambda_n) D_t$, while the normal reaction is given by $\lambda_n D_n$.

In 2D, the direction space is discrete, so the search can be merged in the active set search, and an exact solution is achievable in a finite number of iterations.

In 3D the direction space is continuous, and a convergence criteria must be used.

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Towards an Embedded Mesh Approach for Isogeometric Boundary Layers in Contact Mechanics

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Keywords: Embedded mesh methods, isogeometric analysis, finite element method.

Although much work has been invested in developing robust discretization methods for computational contact mechanics within the classical finite element method (FEM), some crucial drawbacks still remain, i.e., the non-exact representation of curved contact surfaces and the limitation to C^0 continuity at element boundaries. The latter leads to a non-continuous field of normal vectors, which requires special treatment during the contact simulation and increases the complexity of the overall contact algorithm. To overcome these drawbacks, contact approaches based on isogeometric analysis (IGA) have been developed over the last decade. These use a NURBS-based mesh to represent complex contact surfaces smoothly and can achieve arbitrary continuity at the element boundaries, eliminating the need for special treatment of the contact normals [1]. However, for contact problems, the high regularity of NURBS meshes in the whole domain of the body is not beneficial since it does not improve the spatial convergence rate due to the reduced regularity of such problems (in the energy norm, the maximum expected order of convergence is $O(h^{3/2})$ for a polynomial order of $p \geq 2$). Therefore, we propose to use the NURBS interpolation only on the contact boundary, where the discontinuity appears, to represent the contact surface smoothly.

Our proposed approach combines the advantages of IGA and FEM by separating the discretization of the body's contact interface and its interior. The contact interface, represented by a NURBS curve (2D) or surface (3D), is extruded towards the domain's interior to create a NURBS boundary layer mesh. Moreover, the inner bulk volume is discretized in the reference configuration with a Cartesian finite element mesh based on Lagrangian shape functions. The selection of a Cartesian finite element mesh is advantageous since this type of mesh simplifies the overall meshing process and exhibits natural compatibility with many aspects of parallel computing. Inevitably, some Cartesian finite elements, known as cut elements, intersect with the inner surface of the IGA boundary layer mesh. These cut elements are integrated using a tessellation approach [2]. Figure 1 illustrates the previously described approach for an exemplary two-dimensional contact problem.

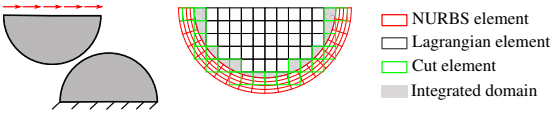

Figure 1: A two-dimensional unilateral contact problem (a), in which one of the bodies is discretized following the proposed approach (b).

The overlapping meshes are coupled by an appropriate embedded mesh method. For the time being, a discrete Lagrange multiplier space along the lines of classical mortar methods will be defined on the inner surface of the IGA boundary layer mesh. While this approach will deliver good results under certain assumptions, we still expect possible stability issues in other scenarios due to the well-known violation of the inf-sup condition [3].

This talk addresses the first steps towards stable discretization schemes for embedded mesh tying constraints between a tailored NURBS boundary layer for contact treatment and an overlapping standard finite element grid. Furthermore, a number of different qualitative and quantitative examples are presented in order to show the numerical properties of the proposed approach.

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HHT-α and TR-BDF2 scheme for dynamic contact problems

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Keywords: contact problem, Nitsche's method, augmented Lagrangian method, finite elements, elastodynamics, time-marching schemes.

One of the main tools for numerical simulation for solid mechanic problems is the finite element method. In the industry, contact problems are omnipresent, and many traditional methods cannot provide acceptable solutions in the context of elastodynamics: they are too influenced by parasitic oscillations or do not conserve energy. A difficulty is that this type of problem has a non-linear boundary condition on the displacement field. The main existing methods for discretizing the Signorini contact conditions are the method of penalization, mixed/mortar methods, Nitsche's method, or the augmented Lagrangian method. Improvements in the accuracy and numerical robustness of these simulations are always expected by industry and researchers.

Usually, the time-space discretization involves the problems of choosing: (i) the finite element space; (ii) the enforcement of the contact condition, and (iii) the time-stepping scheme.

The idea concerned in this work for treating the non-linear boundary condition is to transfer the constrained optimization problems to an unconstrained problem or a sequence of unconstrained problems. Nitsche's method was introduced for contact problems firstly for frictionless unilateral contact in linear elasticity in [1]. The works on dynamic contact with Nitsche's method begin from frictionless case and with implicit scheme [2, 3]. Different time-marching schemes can then be applied to discretize in time, *e.g.*, [4, 5].

The first implementation of the augmented Lagrangian method (ALM) for contact problems with Coulomb's friction law was established by Alart and Curnier [6] and in recent years, convergence analysis for several reformulations have been realized by Burman [7]. Few references can be found for applications in dynamics cases via the augmented Lagrangian method (and finite element method). We can refer for instance the work of [8].

For structural dynamic problems, methods of time integration for second-order differential equations are required. The first tested scheme in this work, the HHT-α scheme is a classical and representative method for elastodynamic problems proposed by Hilber, Hughes, and Taylor [9]. Another main scheme concerned in this work is the TR-BDF2 scheme (also known as the Bathe scheme [10]) which is an implicit scheme with 2 substeps: the first sub-step uses the trapezoidal rule and the second sub-step uses a three-point backward difference approximation.

In this work, we focus on the evolution of the impact of a linear elastic body and a rigid obstacle. We want particularly to study how to combine time-marching schemes as HHT-α and TR-BDF2 schemes with contact via Nitsche's method, with additionally the mass redistribution method or not. For instance, these two kinds of schemes have not been applied with Nitsche's method or ALM. We present then some simulation results with 1D and 3D benchmarks using different methods. By testing their performances, we are particularly interested in the influence of the numerical parameters, the parasitic oscillation associated with the contact surface due to the discontinuity

in time, and the conservation or not of the total energy for the time-marching schemes. The new combinations applied in this work can eventually improve upon existing methods by providing better accuracy and numerical robustness for non-linear (and non-regular) dynamic problems.

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Application of a posteriori analysis to contact problems

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Keywords: Unilateral contact problems, a posteriori error analysis via stress reconstruction, adaptivity with finite element method.

The finite element method is widely used in computational mechanics to numerically solve problems expressed as partial differential equations. For instance, engineering teams often perform finite element numerical simulations to analyze the behavior of large hydraulic structures. In particular, the stability of concrete dams is characterized by the nonlinear behavior of their weakest zones, that is their interface zones, e.g., the concrete-rock contact in the foundation or the interface between different blocks of the structure. As a simple approximation, the behavior of these zones could be mathematically represented with some unilateral contact conditions with friction. Mathematically, these conditions can be written by decomposing the displacement and surface stress into normal and tangential components on the interface, i.e., $\boldsymbol{u} = u_n\boldsymbol{n} + \boldsymbol{u_t}$ (*displacement*) and $\boldsymbol{\sigma}(\boldsymbol{u})\boldsymbol{n} = \sigma_n(\boldsymbol{u})\boldsymbol{n} + \boldsymbol{\sigma_t}(\boldsymbol{u})$ (*surface stress*). Then, the following complementary condition represents the non-penetration conditions and the absence of cohesive forces

$$u_n \leq 0, \quad \sigma_n(\boldsymbol{u}) \leq 0, \quad u_n\sigma_n(\boldsymbol{u}) = 0, \tag{1}$$

while some suitable conditions involving $\boldsymbol{\sigma_t}(\boldsymbol{u})$ describe the presence of friction (e.g., Tresca or Coulomb friction condition).

In this work, the goal is to extend the results of [1] to a wider range of problems. In particular, we focus on unilateral contact problems with friction in which an elastic object represented by a domain Ω is initially in contact with a rigid foundation. From a numerical point of view, we use finite element discretization with weak enforcement of contact boundary conditions à la Nitsche [2]. This choice enables an easy implementation of the contact conditions in a weak sense without the introduction of additional unknowns such as Lagrange multipliers.

We introduce an *a posteriori* error analysis for contact problems with friction based on equilibrated stress reconstructions following the ideas of [3] and [4]. With this approach, we measure the error with a dual norm of the residual operator, whose a posteriori upper bound does not involve unknown constants. The error control can be performed locally through some a posteriori local estimators or globally through the corresponding a posteriori global estimators. Furthermore, this method provides several advantages: the estimators composing the upper bound distinguish the different components of the error (in the considered problem discretization, linearization, regularization), they can be used to adaptively refine the mesh, and to define some stopping criteria for the nonlinear solver and the automatic tuning of a regularization parameter.

The key idea for constructing the a posteriori estimators is to introduce an equilibrated stress reconstruction that locally satisfies some physical properties of the system. Motivated by the results of [4]

and [5], we propose a practical way to obtain this reconstruction by assembling the solution of problems defined locally on patches around the vertices of the mesh using the Arnold-Falk-Winther mixed element space [6].

This approach enables the development of a fully-automated algorithm that adaptively refines the initial mesh. The obtained results can be compared to those coming from a uniform refinement technique. For instance, the numerical example of [1] shows that, for contact problems without friction, the adaptive approach provides better convergence rates for H^1- and energy norm, see Figure 1.

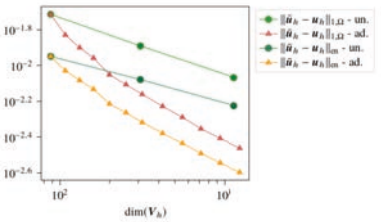

Figure 1: Comparison between uniform approach (*circles*) and adaptive one(*triangles*).

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An arbitrary order contact formulation using Lagrange multipliers from Raviart-Thomas space

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Keywords: deformable-to-rigid-body contact, Raviart-Thomas space, $\mathbf{H}(\mathbf{div})$ space, volume-based Lagrange multipliers.

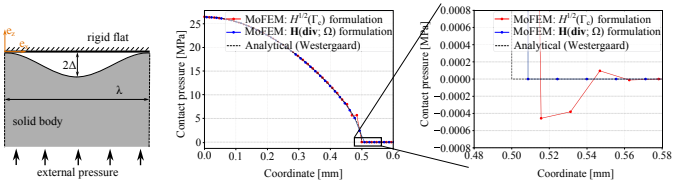
One of the established approaches to enforce contact conditions is via Lagrange Multipliers (LM) field, which is robust and implementation-independent from chosen material models. Consistent approximation of the LM field for h-refinement approaches have been developed and investigated [1, 2, 3]. However, p-refinement approaches are gaining increasing popularity with the standardisation of hierarchical basis functions along with proper choice of basis functions to meet the needs for different functional spaces. For the case of contact problems, the space $H^{-1/2}(\Gamma_{\mathrm{c}})$ is the LMs for contact when displacements are sought in the $H^{1/2}(\Gamma_{\mathrm{c}})$ space. In the present work, a so-called dual approach for modelling contact of a deformable solid coming into contact with a rigid surface is proposed, and is the first step towards a mortar contact formulation. The discrete functional space for LMs, $H^{-1/2}(\Gamma_{\mathrm{c}})$ defined on contact surfaces, Γ_{c}, emerges from the trace of Raviart-Thomas space, $\mathcal{RT} \subset \mathbf{H}(\mathbf{div};\Omega)$, defined in the deformable discretised body domain Ω^h. This enables us to evaluate some terms present in contact formulation both on the boundary and within the volume, via Gauss theorem, providing stability to the discrete solution.

In most of the approaches for solving the contact problem by means of LMs where the basis functions have a support on the contact surface only. However, in the present implementation, LMs are defined on the contact surface and are extended to the interior. Hence, these LMs basis functions have a support both on a boundary element (edge in 2D and triangle/quadrilateral in 3D) as well as its adjacent domain element (triangle/quadrilateral in 2D and tetrahedron/hexahedron in 3D). Construction of basis functions for LMs, i.e. $\Lambda_i \in \mathcal{RT}(\Omega^{\mathrm{h}})$, following work presented in [4], and where construction of basis functions is available for triangels, qadrilaterals, tetraherda and hexahedra.

The model results are compared to more standard contact formulations for first order approximation of LM field in $H^{1/2}(\Gamma_{\mathrm{c}})$ and the Wohlmuth dual space, $\mathcal{W}(\Gamma_{\mathrm{c}})$ presented in [1]. All models are implemented for two types of spatial discretisations for two dimentionalities of ambient space, i.e. triangles and quadrilaterals for a 2D ambient space and tetrahedra and hexahedra for a 3D ambient space. For all LM approximations and spatial discretisations, the models are compared to analytical and numerical results for elastically deformable bodies.

Three versions of the discretisation of the contact problem statement presented in will be considered. More precisely, the first two versions involve standard approaches for choosing the basis functions for approximating contact traction $t_i^{(\mathrm{con})}$ that is the same when choosing the functions being in standard $H^{-1/2}(\Gamma_{\mathrm{c}})$ and the dual shape functions proposed by [1] denoted as $\mathcal{W}(\Gamma_{\mathrm{c}})$ from now on. The third version is the proposed one in this article that involves the choice of basis functions from $\mathcal{RT}$ space.

Preliminary results for a deformable body with a wave contact surface coming into contact with a rigid flat for LMs sought in the $H^{1/2}(\Gamma_{\mathrm{c}})$ and in the the $\mathbf{H}(\mathbf{div};\Omega)$ space are presented in Figure 1.



(a) (b) (c)

Figure 1: (a) schematic of 2D deformable body with a wavy surface (with Δ amplitude and λ wavelength) coming into contact with a rigid flat surface, (b) comparison of analytical solution versus numerical results for LMs sought in $H^{1/2}(\Gamma_c)$ and $\mathbf{H}(\mathbf{div};\Omega)$ spaces and (c) is the zoom-in of (b) in the proximity of the edge of the contact zone.

In Figure 1(c), it can be observed that for $H^{1/2}(\Gamma_c)$ LMs present oscillations and negative values in the vicinity of the end of the contact front. On the other hand, LMs sought in $\mathbf{H}(\mathbf{div};\Omega)$ do not present this discrepancy. The reason for the coordinate miss-match at which traction becomes zero between analytical and the $\mathbf{H}(\mathbf{div};\Omega)$ case is attributed purely to the spatial discretisation. More verification and results are going to be investigated and discussed.

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SESSION 10
General papers
Friday, 14:20 – 16:20

Tube/projectile interaction modeling using finite element simulation

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Keywords: Interface, contact, shrink assembly, finite elements, elastic-plastic, springback.

In artillery, the challenge of loading is to create an efficient shrink fit assembly between the projectile and the gun barrel. The fit is considered optimal if it prevents fallback without creating overconstraints. The assembly mainly lies on the deformations of the driving band, which is a soft-metal ring often made of copper. Most studies on the deformations of the driving band were made during the late twenties [1], [2]. They mostly focused on describing the phenomena occurring after the ignition of charges, to enhance interior ballistics' knowledge using simulations and recovered shells. Recently, some work focused on describing the loading of a projectile [3], also using simulations and experimentations on tank and artillery projectiles. The objective of our work is to go beyond a system scale description. We want to observe and understand the phenomena at the contact interface and within the band. This work is a double approach where we develop finite element and analytical models of the assembly.

Considering the conical surface of the assembly, we use the fundamental principle of statics to derive a first relationship between the penetration force E_{p} (force used to shrink the projectile) and the extraction force E_{ex} (force required to dismantle the assembly). We then define the restitution factor r as:

$$r = -\frac{E_{\mathrm{ex}}}{E_{\mathrm{p}}} = \frac{\mu - \tan(\alpha/2)}{\mu + \tan(\alpha/2)} \tag{1}$$

This factor describes the efficiency of the possible fit only due to cone angle α and Coulomb's friction parameter μ. Figure 1 shows the response surface of r for these two parameters. It is easy to highlight the fact that the extraction force is larger than the penetration force and there are some couples (α, μ) that always create fallback for any penetration force (red area on Figure 1).

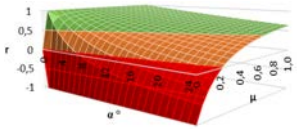

Figure 1: Response surface $r(\alpha, \mu)$.

Using Abaqus software, we developed axisymmetric and 3D finite element models of the assembly. We represented the driving band as an elastic-plastic deformable body that will deform between the rigid projectile body and the tube. Figure 2 shows the distribution of stresses within the band after the first phase of loading for an axisymmetric model. The strain is localised at some points of the band due to the geometric aspects.

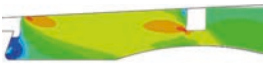

Figure 2: Assembly with von Mises stress colour map at the end of penetration.

Extensive studies of contact strategies have been carried out (hard or soft normal contact shown on Figure 3, geometry influence, meshing influence, …) to obtain an efficient simulation. Moreover, once we had an efficient model, we evaluated the influence of parameters on the fit quality and we identified the friction coefficient as the most important control parameter, just behind the geometry.

According to real cases, the loading can be divided into three consecutive phases: penetration, springback and extraction. The pressure is not uniform along the contact border and varies during the three phases as seen in Figure 4.

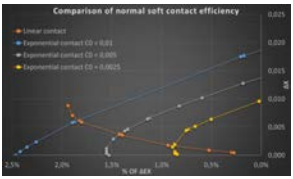

Figure 3: Soft contact laws errors.

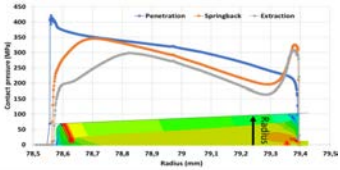

Figure 4: Pressure along the contact for the 3 phases.

Therefore, we have to enhance our first analytical model (1) to take into account this phenomenon. At the contact interface between the cone and the projectile body, we add to the Coulomb's friction model some springs to illustrate the elastic behaviour of material. The full model can be seen on Figure 5.

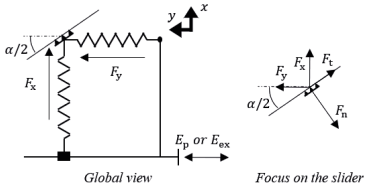

Figure 5: Model with springback.

Thus, we derive equilibrium for the three phases and we obtain new relationship between forces:

$$\frac{E_{ex}}{E_p} = -r\frac{1-\mu\tan(\alpha/2)}{1+\mu\tan(\alpha/2)} \quad (2)$$

We identify a new factor that represents the load inversion:

$$f_{inv} = \frac{1-\mu\tan(\alpha/2)}{1+\mu\tan(\alpha/2)} \quad (3)$$

f_{inv} enhances the prediction of the model. However, we still have at least a 20% error in force prediction, even though we have good accordance in displacement prediction.

All of these models are still in development but they offer a first level of understanding on the phenomena occuring during the loading of an artillery projectile.

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Fast simulations of parametric problem with contact non-linearity.

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Keywords: Contact, pMOR, QDEIM, hyper reduction.

Contact problems demand fine discretization resulting in expensive computations. The requirement for parametric model order reduction (pMOR) emerges owing to computing time constraints. pMOR is the process of reducing the dimension of a full scale model (FSM) while preserving parametric dependencies. In this study, hyper reduction is carried out by approximating contact non-linearity using a variant of discrete empirical interpolation method called QDEIM proposed by Drmac et al [1]. Local and global pMOR techniques are used in this study to build parametric reduced order model.

Following equation can be used to illustrate the FE discretization of two bodies in contact [2]

$$\mathbf{M}(p)\ddot{\mathbf{u}} \ + \ \mathbf{K}(p)\mathbf{u} \ + \ \mathbf{f}_{\mathrm{n}}(\mathbf{u}, p) = \mathbf{f}_{ext} \tag{1}$$

where, p is system parameter, $\mathbf{M}(p) \in \mathbb{R}^{\mathrm{n}\times\mathrm{n}}$ is mass matrix, $\mathbf{K}(p) \in \mathbb{R}^{\mathrm{n}\times\mathrm{n}}$ is stiffness matrix, $\mathbf{f}_{\mathrm{n}}(\mathbf{u}, p) \in \mathbb{R}^{\mathrm{n}}$ is internal nonlinear force vector, $\mathbf{f}_{\mathrm{ext}} \in \mathbb{R}^{\mathrm{n}}$ is external force vector, and $\mathbf{u}$ is a displacement vector. The governing equations (1) are projected onto a suitable low dimensional subspace V leading to a reduced order model of the form

$$\mathbf{M}_r(p)\ddot{\mathbf{u}}_r \ + \ \mathbf{K}_r(p)\mathbf{u}_r \ + \ \mathbf{V}(p)^T\mathbf{f}_{\mathrm{n}}(\mathbf{V}(p)\mathbf{u}_r) = \mathbf{V}(p)^T\mathbf{f}_{ext} \tag{2}$$

where, $\mathbf{M}_r(p) = \mathbf{V}(p)^T\mathbf{M}(p)\mathbf{V}(p)$ and $\mathbf{K}_r(p) = \mathbf{V}(p)^T\mathbf{K}(p)\mathbf{V}(p)$. The relevant quantities in reduced dimension are indicated by subscript r. $\mathbf{V} \in \mathbb{R}^{\mathrm{n}\times\mathrm{k}}$ is basis of subspace, V and it is obtained using proper orthogonal decomposition (POD) [3] of snapshots of the FSM solution. One can compute $\mathbf{M}_r$ and $\mathbf{K}_r$ in offline phase but the reduction $\mathbf{V}(p)^T\mathbf{f}_{\mathrm{n}}(\mathbf{V}(p)\mathbf{u}_r)$ involves a full-scale dimension, n. This bottleneck results in costly computations during reduction of nonlinear systems. To overcome this, hyper-reduction of the nonlinear term must be performed. The nonlinear term is approximated by projecting it on a suitable subspace U, as shown below

$$\mathbf{f}_{\mathrm{n}} \approx \mathbf{U}\mathbf{c} = \hat{\mathbf{f}}_{\mathrm{n}} = \mathbf{U}(\mathbf{P}^T\mathbf{U})^{-1}\mathbf{P}^T\mathbf{f}_{\mathrm{n}}. \tag{3}$$

Here, $\mathbf{U} \in \mathbb{R}^{\mathrm{n}\times\mathrm{m}}$ is a matrix consisting of the basis of subspace, U and it is obtained by using POD of snapshots of nonlinear terms in FSM. $\mathbf{P}$ is obtained by choosing specific columns from the n×n identity matrix using QDEIM algorithm.

A single global basis across a parameter space and interpolating local bases are the two approaches used to construct the pMOR [3]. Let, $p = [p_1, p_2, ..., p_K]$ be a parameter set and $\mathbf{V}_i$ be the local basis corresponding to p_i. In the global approach, local bases are concatenated and a single basis is obtained by performing singular value decomposition (SVD) on the concatenated bases. In second approach, the local bases are interpolated on the tangent space to the Grassman manifolds.

These techniques are demonstrated for the standard Hertzian problem of two cylinders in contact. A harmonic loading of $10sin(0.5t)$ KPa is applied to the top cylinder of 10 mm radius while the bottom cylinder is of 15 mm radius. Considering the symmetry, the cylinders can be modelled as quarter cylinders, as shown in Figure 1a. For simplicity, both the cylinders are assumed to be linear isotropic

with Young's modulus E, Poisson's ratio 0.3 and density $\rho = 1$ tonne/mm^3. For structural discretization, four noded quadrilateral elements are employed, while for contact surface discretization, the node-to-segment contact elements (frictionless) are used. Dynamic implicit analysis is carried out in MATLAB using the Newmark-β method. $E = [1000, 1500, 2000]$ MPa is used as a parameter set to build a pMOR. Using this pMOR model, results are obtained for different $E = 1750$ MPa. Representative results are

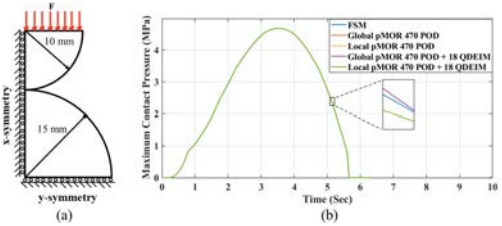

Figure 1: (a) Two Cylinders in Contact, (b) Maximum Contact Pressure Vs Time

presented in Figure 1b, it depicts the maximum contact pressure Vs time, obtained using FSM with 2425 DOF, global and local pROM with 470 POD basis, and global and local pROM with 470 POD + 18 QDEIM basis. L_2 norm error (4) and speed up in computation time are presented in Table 1.

$$L_2 \;\; error = \frac{\|()_{FSM} - ()_{ROM}\|_2 \times 100}{\|()_{FSM}\|_2} \tag{4}$$

Table 1: L_2 error in contact pressure, P_c and speed up factor for different pMOR techniques

		470 POD	470 POD + 18 QDEIM
L_2 error in contact pressure, P_c	Global pMOR	0.0174	0.0232
	Local pMOR	0.0172	0.0231
Speed up factor	Global pMOR	16.94	46.14
	Local pMOR	16.94	47.45

This study shows that, POD and POD+QDEIM based reductions are 17 and 47 times faster than FSM with L_2 error within bound. The indices obtained using QDEIM algorithm turns out to be indices of nodes in contact.

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Identification of contact traction and material parameters for soft bodies

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Keywords: Inverse form, Hyper-elasticity, Calibration, Soft bodies.

Soft bodies can significantly deform under gravity. For that reason, in many applications, they can only by observed in highly deformed configurations. They can also undergo contact interactions with external obstacles or as a result of self-contact. The inability to measure the unloaded (stress-free) configuration, with additional difficulty to measure traction field at contact interfaces, poses problems for modeling and calibration procedures for those systems. To this end, special modeling frameworks have been developed which only used deformed configurations to estimate the material parameters of large-deformation models, see, e.g, fixed-point (iterative) schemes [1, 4] in the context of modeling of breast.

The most common are iterative schemes, which only utilize forward simulations to converge to a desired unloaded configuration and calibrated material parameters. Their advantage is that they can be straightforwardly used with any FE solver, but their drawback is that they are computationally expensive. The computational efficiency can be greatly improved if applying direct *inverse motion* formulations [2] as a basis for the calibration procedure. Those methods are based on inverse kinematics, in which the deformed (current) configuration and loading are known while the undeformed configuration is to be found.

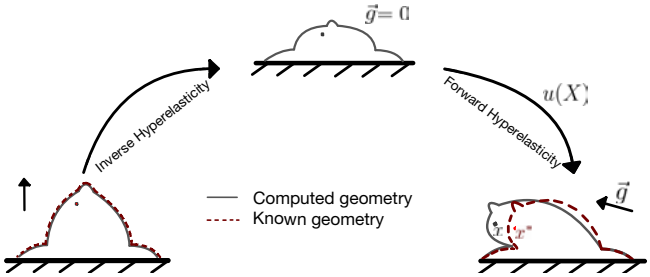
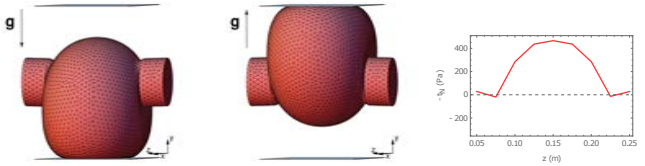


Figure 1: Schematics of the inverse-forward problem used in the identification procedure [3].

In this presentation, the inverse- and forward motion formulations will be combined to solve the parameter identification problem, see Fig. 1. The particular novelty of the presented approach is that both measured loaded configuration can be in contact. The main idea is to parameterize the contact traction field in one of loaded configurations and then identify those parameters together with material parameters, as combined in a single vector ϕ:

$$\phi^* = \underset{\phi}{\operatorname{argmin}}\, f(\phi) = \underset{\phi}{\operatorname{argmin}} \sum_{i \in \text{nodes}} (\mathbb{x}_i(\phi) - \mathbb{x}_i^*)^2. \tag{1}$$

In the identification, we use BFGS optimizer, for which the necessary gradients are obtained with the direct differentiation method of sensitivity analysis. In the presentation I will discuss the performance of the proposed framework, also for the noisy input geometries.



(a) loaded configuration 1 (b) loaded configuration 2 (c) contact traction (section).

Figure 2: Identification of traction field [3]. The unknown traction field (c) is identified together with the Young modulus and Poisson ratio, when having two deformed meshes as inputs (a-b).

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Third Medium Contact Method for Topology Optimization

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Keywords: Topology optimization, Third medium contact, Compliant mechanisms.

Recently Bluhm et al. [1] have effectively created an approach for including contact in topology optimization by adapting third medium contact [2]. The void phase is modeled as a highly compliant nonlinear third medium, which becomes increasingly stiff under ultimate compression. Thus, when ultimately compressed, the third medium can transfer forces between contacting regions of the solid domain. Bluhm et al. [1] have shown promising results using this approach. This has opened the opportunity for developing new topology optimized structures which exploit internal contact.

The present work presents the application of the third medium contact model to generate optimized structures that use internal contact to achieve a design objective. A variety of mechanical systems, including soft robotics and household appliances, as well as biomechanics and structures with inherent safety, may benefit from the results presented in this work and the methods used to obtain them.

Results reveal that structures with internal contact developed by the third medium contact method for topology optimization have promising features. This involves the creation of features that provide internal self-support as a result of deformation, as well as features that decrease internal self-support as a result of deformation. These findings may prove to be valuable in the design of compliant mechanisms since the inclusion of contact expands the solution space, allowing for even better designs to be made.

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Beam lattice metamaterials with internal contact and instabilities

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Flexible and soft mechanical metamaterials with artificially designed microstructures have attracted attention due to their unusual and tunable properties with applications in, e.g., soft robotics and energy harvesting. Moreover, the interest in such materials has been increased hand in hand with the progress in additive manufacturing, enabling the fabrication of the designed microstructures.

The classical mechanical metamaterials include, e.g., auxetic (negative Poisson's ratio) metamaterials [1], metamaterials with vanishing shear modulus, and topological metamaterials [2]. Another exciting group of metamaterials consists of the so-called programmable materials whose properties can be switched by external stimuli. To achieve such a behavior, we propose an architected hexagonal lattice with an additional internal contact mechanism. Careful design and rearrangement of the underlying contact mechanism lead to tunable stiffness, which can be adapted to a specific application.

The design of the proposed metamaterial relies upon a robust and efficient computational tool. The development of such a tool is challenging mainly due to the internal contact and large deformations of the lattice with a possibility to develop instabilities. Therefore, the adopted computational method is based on the recently proposed geometrically exact beam element [3]. Moreover, the element formulation is extended to incorporate the contact internally, leading to a very efficient formulation.

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Still on the Shifted Penalty Method

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The classical penalty method is widely used in Contact Mechanics since it permits the enforcement of contact constraints without variation of the problem size. The main drawback of the method is due to the penetration among the surfaces that affect the solution.
It has been proved that the shifted modification can rapidly reduce the penetration among the contacting surfaces without introducing any additional set of forces or new unknowns into the global stiffness matrix [1].
To evidence the main characteristics of the method, we start from the total potential, Π

$$\Pi \rightarrow min, \;\; g_n \leq 0$$

where g_n is a signed measure of the distance among the bodies (gap), and the inequality constraint enforces the non-penetration of the solid bodies The best definition of the penalty method in given in the Luenberger's book [2], stating that the penalty approximation is accomplished by "adding to the objective function a term that prescribes a high cost for violation of the constraints". Considering the classical example of two elastic bodies, the above problem can be rewritten as

$$\Pi = \Pi_a(\boldsymbol{u}_a) + \Pi_b(\boldsymbol{u}_b) + \Pi_c(\boldsymbol{g_n}) \rightarrow min$$

where $\Pi_a$and Π_b represent, respectively, the total potential of body a and body b; Π_c is the contact-penalty contribution and $\boldsymbol{g}_n$ represents the set of normal gaps. The contact potential, Π_c, is provided by the classical penalty contribution $\Pi_c = \sum_i 1/2\, \varepsilon g_{n_i}^2$, where the penalty parameter, ε, determines the quality of the approximation. The main drawback of this method is because the constraints can be exactly satisfied only for $\varepsilon \rightarrow \infty$, and several methods have been proposed to deal with [3-7]. Nowadays the issue of the optimal choice of the penalty is usually hidden in the commercial codes, with an automatic estimate of the penalty parameter.
The penalty method is characterized by a strong transition between the open and closed states. On the contrary, the shifted penalty method is characterized by a smooth transition among the open and closed state. It consists into a dynamic shift of the application point of the unconstrained potential modification. Hence, contact constraints are apparently enforced in the open state, but in this way the minimum of the potential can be shifted back at the constraint limit. For this scope we introduce a shift, s, as a new variable

$$\Pi = \Pi_a(\boldsymbol{u}_a) + \Pi_b(\boldsymbol{u}_b) + \Pi_c(\boldsymbol{g_n}, \boldsymbol{s}) \rightarrow min$$

The above potential is minimized assuming first a constant value for the shift, which is then updated at every iteration within the newton loop.

$$\Delta\delta\Pi_a(\boldsymbol{u}_a)\Delta(\boldsymbol{u}_a)\delta(\boldsymbol{u}_a) + \Delta\delta\Pi_b(\boldsymbol{u}_b)\Delta(\boldsymbol{u}_b)\delta(\boldsymbol{u}_b) + \Delta\delta\Pi_c(\boldsymbol{g}_n, \boldsymbol{s})\Delta(\boldsymbol{g}_n)\delta(\boldsymbol{g}_n) + \Delta\delta\Pi_c(\boldsymbol{g}_n, \boldsymbol{s})\Delta(\boldsymbol{s})\delta(\boldsymbol{g}_n) = \boldsymbol{0}$$

Indeed, this results into the new contribution $\Delta\delta\Pi_c(\boldsymbol{g}_n, \boldsymbol{s})\Delta(\boldsymbol{s})\delta(\boldsymbol{g}_n)$, but the shift update is simply performed enforcing that the its variation should correspond to the current penetration.

$$\Delta(\boldsymbol{s}) = \boldsymbol{g}_n$$

The results achieved till now are very promising, and there are several new perspectives that should be explored in detail.

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