ATMA 2024 Approximation: Theory, Methods and Applications

Lecce, June 11-14, 2024

Speakers

Mirella Cappelletti Montano (University of Bari)
Danilo Costarelli (University of Perugia)
Kai Diethelm (THWS, Germany)
Filomena Di Tommaso (University of Calabria)
Maryam Mohammadi (University of Padova)
Ioan Raşa (Technical University of Cluj-Napoca)

Special session on some open problems in Approximation Theory:
Francesco Altomare (University of Bari)

Organizing Committee

Michele Campiti (University of Salento)
Vita Leonessa (University of Basilicata)
Donatella Occorsio (University of Basilicata)
Maria Grazia Russo (University of Basilicata)

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Maria Grazia Russo (University of Basilicata)
Alvise Sommariva (University of Padova)
Gianluca Vinti (University of Perugia)

Università del Salento — L'Università dei due mari
Dipartimento di Matematica e Fisica "Ennio De Giorgi"
Università degli Studi della Basilicata 1982
BPP Banca Popolare Pugliese
INdAM GNCS
Teoria dell'Approssimazione e Applicazioni (T.A.A.) Unione Matematica Italiana (UMI)
Approssimazione Numerica ed Analitica di dati e di funzioni con Applicazioni ANA&A

Program of the Conference ATMA2024

11 giugno	
The registration will be opened from 15:00 until 18:00 in the Conference Hall	
16:00 – 17:00	**Poster session**
17:00 – 18:00	**Welcome Cocktail**
18:00 –	**Guided Tour in Lecce**

12 giugno	
9:00 – 9:30	**Opening Ceremony**
Chairman: **Roberto Cavoretto**	
9:30 – 10:10	**Danilo Costarelli** *Approximation properties and applications of sampling-type operators*
10:15 – 10:55	**Filomena Di Tommaso** *Multinode Shepard method: theory and applications*
10:55 – 11:25	*Coffee break*
Chairman: **Laura Angeloni**	
11:25 – 11:40	**Lorenzo Boccali,** *Danilo Costarelli, Gianluca Vinti* *Approximation by Max–product Sampling Kantorovich operators: quantitative estimates in Functional Spaces*
11:45 – 12:00	**Marco Cantarini,** *Danilo Costarelli, Gianluca Vinti* *Approximation properties of Sampling Kantorovich operators in Sobolev settings*
12:05 – 12:20	**Rosario Corso***, Gianluca Vinti* *Mean sampling Kantorovich operators*
12:25 – 12:40	**Eleonora De Angelis,** *Danilo Costarelli, Gianluca Vinti* *Convergence and order of approximation for perturbed operators*
12:45 – 13:00	**Ivan Gadjev** *On the constants in Hardy Inequalities in* L_p *and* l_p
13:05 – 13:20	**Vincenzo Schiano Di Cola,** *Marco Berardi, Salvatore Cuomo* *Approximation Techniques for Environmental Modeling: Insights and Applications*
Pausa Lunch	
Chairman: **Alvise Sommariva**	
15:00 – 15:15	**Valerii A. Galkin** *(online)* *Approximations of Topological Structures in Flows Described by Navier-Stokes Equations for Incompressible Fluid*
15:20 – 15:35	**Yuan Xu** *Minimal cubature rules and interpolation on the square*
15:40 – 15:55	**Francesca Acotto,** *Ezio Venturino, Iulia Martina Bulai* *Interaction between individualistic predators and responsive herd of prey*
16:00 – 16:15	**Kaido Lätt,** *Arvet Pedas* *Approximate solution of singular fractional integro-differential equations*
16:20 – 16:35	**Svilen S. Valtchev** *Meshfree Domain Decomposition Methods with Fundamental Solutions for Elliptic Boundary Value Problems*
16:40 – 17:10	*Coffee break*
17:10 – 19:00	**Groups Assemblies (RITA, TAA, ANA&A)**

13 giugno	
Chairman: **Maria Grazia Russo**	
9:00 – 9:40	**Kai Diethelm** *The Approximation of Power Functions with Exponents in (–1, 0) by Sums of Exponentials and Its Applications*
9:45 – 10:25	**Maryam Mohammadi,** *Mohammad Heidari, Stefano De Marchi, Milvia Rossini* *How differential geometry works in the RBF approximation theory!*
10:25 – 10:55	*Coffee break*
Chairman: **Donatella Occorsio**	
10:55 – 11:10	**Jean–Paul Berrut,** *Richard Baltensperger, Malika Jan* *Rational sinc interpolants and point shifts*
11:15 – 11:30	**Alvise Sommariva,** *Marco Vianello* *On unisolvence of unsymmetric random Kansa collocation*
11:35 – 11:50	**Giacomo Elefante,** *Alvise Sommariva, Marco Vianello* *CQMC: Tchakaloff–like compression of QMC integration*
11:55 – 12:10	**Francesco Marchetti**, *Tizian Wenzel, Emma Perracchione* *A machine learning perspective for optimized kernel–based approximation*
12:15 – 12:30	**Gianluca Audone,** *Francesco Della Santa, Emma Perracchione, Sandra Pieraccini* *Variably Scaled Kernels: A Deep Learning Approach to Adaptive Scale Selection*
12:35 – 12:50	**Leokadia Białas-Cież**, *Mateusz Suder* *Evaluating Lebesgue constants by Chebyshev polynomial meshes on cube, simplex and ball*
12:55 – 13:10	**Dimitri Jordan Kenne** *Chebyshev admissible meshes and Lebesgue constants of complex polynomial projections*
Pausa Lunch	
Chairman: **Stefano De Marchi**	
15:00 – 15:40	Open problems conference: **Francesco Altomare,** *Local approximation problems and Korovkin–type theorems*
15:45 – 16:00	**Domenico Mezzanotte,** *Luisa Fermo, Donatella Occorsio* *A global method for Volterra-Fredholm integral equations*
16:05 – 16:20	**Luisa Fermo,** *Domenico Mezzanotte, Donatella Occorsio* *Numerical treatment of mixed Volterra-Fredholm integral equations*
16:20 – 16:50	*Coffee break*
Chairman: **Lucia Romani**	
16:50 – 17:05	**Mikk Vikerpuur,** *Arvet Pedas* *Approximate solutions to linear fractional integro-differential equations*
17:10 – 17:25	**Rumen Uluchev,** *Ivan Gadjev, Parvan Parvanov* *Recent Progress in Weighted L_p-Approximation of Functions by Kantorovich-type Operators*
17:30 – 17:45	**Borislav R. Draganov** *Approximation by Kantorovich sampling operators in variable exponent Lebesgue spaces*
19:45 –	**Social dinner and Musical show (Music and dancing from Salento)**

14 giugno	
Chairman: **Vita Leonessa**	
9:00 – 9:40	**Ioan Rasa**, *Ana-Maria Acu (online)* *Positive linear operators, convexity and approximation* (online conference)
9:45 – 10:25	**Mirella Cappelletti Montano,** *Francesco Altomare, Vita Leonessa* *Representation formulae for strongly continuous semigroups in terms of integrated means and related approximation processes*
10:25 – 10:55	*Coffee break*
Chairman: **Clemente Cesarano**	
10:55 – 11:10	**Gheorghe Bucur** *Multiplicative Characteristic Functions*
11:15 – 11:30	**Ileana Bucur** *Multiplicative Characteristic Functions*
11:35 – 11:50	**Harun Karsli** *On wavelet type Chlodovsky Operators and their Bézier-type variants*
11:55 – 12:10	**Geno Nikolov** *Bounds for the Extreme Zeroes of Jacobi Polynomials*
12:15 – 12:30	**Francesco Esposito** *Holomorphic* L^2 *signals of several complex variables*
12:35 – 12:50	**Domenico Vitulano,** *Vittoria Bruni, Silvia Marconi* *A Formal Approximation of CNN Filters*
12:55 – 13:10	**Conference closure**

List of speakers

General Conferences

Mirella Cappelletti Montano, *Francesco Altomare, Vita Leonessa* *Representation formulae for strongly continuous semigroups in terms of integrated means and related approximation processes*	June 14, 9:45–10:25
Danilo Costarelli *Approximation properties and applications of sampling-type operators*	June 12, 9:30–10:10
Kai Diethelm *The Approximation of Power Functions with Exponents in (–1, 0) by Sums of Exponentials and Its Applications*	June 13, 9:00–9:40
Filomena Di Tommaso *Multinode Shepard method: theory and applications*	June 12, 10:15–10:55
Maryam Mohammadi, *Mohammad Heidari, Stefano De Marchi, Milvia Rossini* *How differential geometry works in the RBF approximation theory!*	June 13, 9:45–10:25
Ioan Rasa, *Ana-Maria Acu (online)* *Positive linear operators, convexity and approximation* (online conference)	June 14, 9:00–9:40

Open problems conference:

Francesco Altomare *Local approximation problems and Korovkin–type theorems*	June 13, 15:00–15:40

Talks

Francesca Acotto, *Ezio Venturino, Iulia Martina Bulai* *Interaction between individualistic predators and responsive herd of prey*	June 12, 15:40–15:55
Gianluca Audone, *Francesco Della Santa, Emma Perracchione, Sandra Pieraccini* *Variably Scaled Kernels: A Deep Learning Approach to Adaptive Scale Selection*	June 13, 12:15–12:30
Leokadia Białas-Cież, *Mateusz Suder* *Evaluating Lebesgue constants by Chebyshev polynomial meshes on cube, simplex and ball*	June 13, 12:35–12:50
Jean–Paul Berrut, *Richard Baltensperger, Malika Jan* *Rational sinc interpolants and point shifts*	June 13, 10:55–11:10
Lorenzo Boccali, *Danilo Costarelli, Gianluca Vinti* *Approximation by Max–product Sampling Kantorovich operators: quantitative estimates in Functional Spaces*	June 12, 11:25–11:40
Gheorghe Bucur *Multiplicative Characteristic Functions*	June 14, 10:55–11:10
Ileana Bucur *Multiplicative Characteristic Functions*	June 14, 11:15–11:30
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Rosario Corso, *Gianluca Vinti* *Mean sampling Kantorovich operators*	June 12, 12:05–12:20

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Yuan Xu *Minimal cubature rules and interpolation on the square*	June 12, 15:20–15:35

Posters

Ecem Acar
Approximation by Truncated Bivariate Favard-Szász-Mirakjan Operator of Max-product Kind

Paolo Erminia Calabrese, Rosanna Campagna, Costanza Conti
An algorithm for bivariate HP-splines regression

Francesco Dell'Accio, Federico Nudo
Defeating the Runge phenomenon on different domains though a mixed interpolation-regression method

Döne Karahan Dınsever
Korovkin type approximation of q-conformable fractional linear positive operator

Anna Lucia Laguardia, Domenico Mezzanotte
Nyström methods for FIE's based on RBF functions

Thomas Mejstrik, Vladimir Yu. Protasov, Ulrich Reif
Common invariant cones of sets of matrices

Woula Themistoclakis, Marc Van Barel
A new kernel for uniform approximation in RKHS

Approximation by Truncated Bivariate Favard-Szász-Mirakjan Operator of Max-product Kind

Ecem ACAR
Department of Mathematics and Science Education, Harran University

The purpose of this paper is to introduce nonlinear bivariate Truncated Favard-Szasz-Mirakjan operators of max-product kind. Then, we give an error estimation for the bivariate Truncated Favard-Szasz-Mirakjan operators of max-product kind by using a suitable generalizition of the Shisha-Mond Theorem. There follows an upper estimates of the approximation error for some subclasses of functions. Additionally, shape-preserving properties of these new operators have been studied.

References

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Interaction between individualistic predators and responsive herd of prey

Acotto Francesca, *Venturino Ezio*
Department of Mathematics “Giuseppe Peano", University of Turin

Bulai Iulia Martina
Department of Chemical, Physical, Mathematical and Natural Sciences, University of Sassari

The choice to live in group is one of the strategies that some prey use to defend themselves against predator attacks and is therefore classified among behavioral defense mechanisms. For example, we can refer to the savannah biome, focusing on big herbivores and some of their predators. The predator hunting on herding prey can be modeled by observing that the individuals most likely to be affected by the attack are those on the herd perimeter.

The benefits that the aggregation brings for prey increase with the size of the herd. A large herd of big herbivores can discourage hunting attempts by predators, who are frightened, fearing that they in turn will be struck back and injured. This happens especially in cases where other structural elements of passive defense are present, such as strong horns, in addition to the size.

To model this feature, we depart from the Holling type II functional response reformulated to account for prey herding. We consider a denominator of the Beddington-DeAngelis form and a function $\Phi(N, \alpha)$ in the numerator, denoting by N the number of prey per spatial unit as a function of time and $\frac{1}{2} \leq \alpha < 1$ the shape index, instead of N^{α}. This function behaves as N for small values of N and as N^{α} for large values of N, in agreement with the fact that as the members of the herd decrease they each tend to interact individually with the predators. Further, in the denominator, we use a binary-value parameter that allows us to take or not take into account the prey response to predator attacks, depending on the prey species considered.

The equilibrium points that the model admits are the origin, the predator-only equilibrium, the predator-free equilibrium, and two, one or no coexistence points. At the coexistence equilibrium, the model shows that prey and predators thrive respectively in larger and smaller numbers than in the previous two-population systems for individualistic predator and herding prey. The first three equilibria are unconditionally feasible, while coexistence is conditionally feasible. Only the coexistence and predator-only equilibrium can be locally asymptotically stable. Transcritical bifurcations from the first of these last two equilibria to the second one are possible. In addition, saddle-node bifurcations have been numerically identified. Furthermore, there are combinations of parameter values for which simultaneously these two equilibria are admissible and locally asymptotically stable. In these cases, the equilibrium toward which the system converges depends only on the initial conditions. This bistability has been numerically explored using bSTAB.

References

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Local approximation problems and Korovkin-type theorems

Francesco Altomare
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The starting point of the lecture is a result due to P. P. Korovkin (1953) which states that, given a real interval I, a linear subspace E of real-valued functions on I containing the functions $\mathbf{1}(t) := 1$, $e_1(t) := t$ and $e_2(t) := t^2$ $(t \in I)$ and a sequence $(L_n)_{n\geq 1}$ of positive linear operators on E, then for every compact subintervals K of I and for every bounded function $f \in E$ which is continuous on each point of K, one gets

$$\lim_{n\to\infty} L_n(f) = f \text{ uniformly on } K,$$

provided that the same limit formula holds true for the three above mentioned functions $\mathbf{1}$, e_1 and e_2.

This result contains, as a very special case, the most renowned theorem of Korovkin which concerns the case where I is compact, $E = C(I)$ and $K = I$.

As it is well-known, this last theorem originated a plenty of extensions, generalizations and applications which are documented in several monographs as well as in hundred of papers (see, e.g., [1] - [5] and the references therein).

The above mentioned more general theorem of Korovkin seems to be potentially useful to investigate, even for the most well-studied approximation processes, some local approximation problems for bounded locally continuous functions defined on not necessarily compact domains (for which the more famous special case cannot be applied).

For these reasons very recently ([2] - [4]) we started a series of investigations whose main aim is to ascertain to what extent this result can be extended and generalized and, in particular, whether the functions $\mathbf{1}$, e_1 and e_2 can be replaced by other test functions or whether there is the possibility to obtain similar results in multidimensional as well as in infinite dimensional settings even considering, as a limit operator, a general positive linear operator $T : E \to F(X)$.

The talk will be devoted to present some of the main results obtained along these directions together with some applications and open problems.

References

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[2] F. Altomare, *On positive linear functionals and operators associated with generalized means,* J. Math. Anal. Appl. 502 (2021), no. 2, Paper No. 125278, 20 pp.

[3] F. Altomare, *Korovkin-type theorems and local approximation problems,* Expo. Math. 40 (2022), no. 4, 1229-1243.

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[5] F. Altomare and M. Campiti, *Korovkin-type Approximation Theory and its Applications,* de Gruyter Studies in Mathematics 17, W. de Gruyter, Berlin, New York, 1994.

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[1]

Variably Scaled Kernels: A Deep Learning Approach to Adaptive Scale Selection

Gianluca Audone, *Francesco Della Santa, Emma Perracchione, Sandra Pieraccini*
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Variably Scaled Kernels (VSKs) methods have gained popularity in the context of meshfree approximation, due to their adaptability to local data characteristics, offering improved accuracy and flexibility compared to traditional fixed-scale kernel methods. Their effectiveness though is dependent on the choice of a scaling function which can be thought of as a continuous version of the shape parameter. Previous research suggests that a scaling function mimicking the target function can improve approximation results, but this claim lacks rigorous theoretical evidence.

This work addresses these issues by providing theoretical justification for the claim, demonstrating using the Lebesgue function that a scale function reflecting the target function's behaviour leads to enhanced approximation accuracy. We also propose a user independent way of choosing the scaling function by training a discontinuous neural network to learn the "optimal" scaling function directly from data.

Our results confirm the theoretical findings: the learned scaling function closely resembles the target function, leading to improved approximation performance in various scenarios. This data-driven approach for scale function selection offers a user-independent, adaptive solution for enhancing the accuracy and flexibility of VSKs in meshfree approximation tasks.

References

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[2] H. Wendland, *Scattered Data Approximation*, Cambridge University Press, 2004.

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Rational sinc interpolants and point shifts

Jean-Paul Berrut
Department of Mathematics, University of Fribourg

Richard Baltensperger
University of Applied Sciences and Arts Western Switzerland, Fribourg

Malika Jan
Lycée-Collège de la Planta, Sion

The interpolation of functions with steep gradients is greatly improved by putting more points in the vicinity of these gradients. In pseudospectral methods, a conformal map of the domain is used for that purpose and classically introduced into the polynomials replacing the functions appearing in the differential equation. The exponential convergence is conserved, when the interpolation/collocation points are zeros or extrema of orthogonal polynomials. However, because of the use of the chain rule, this leads to complicated differentiation matrices.

To avoid the latter, the first two authors have suggested in 2001 to use a linear rational barycentric interpolant introduced in [1] instead of the usual polynomial one when solving a differential equation on an interval. Differentiation formulas as simple as those of the interpolating polynomial [2] lead to systems that are themselves as simple as those of the classical polynomial pseudospectral method [3]. The effect of the maps is impressive [4,5].

The first author has studied the effect of conformal shifts in the (Fourier) periodic case [6].

In the present work we treat the approximation on the infinite line, replacing the sinc interpolant with a limit of linear rational sinc ones, and we show that, here as well, the exponential convergence is conserved with the conformal map. We also demonstrate through numerical examples how point shifts greatly improve the interpolant's accuracy for the approximation of functions with steep gradients. Moreover, since we start with equispaced points instead of Chebyshev ones, the precision is even better than with the linear rational interpolant at conformally shifted Chebyshev nodes mentioned above.

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Evaluating Lebesgue constants by Chebyshev polynomial meshes on cube, simplex and ball

Leokadia Białas-Cież, Mateusz Suder
Faculty of Mathematics and Computer Science, Jagiellonian University in Kraków

We will show that Chebyshev-type polynomial meshes can be used, in a fully discrete way, to evaluate with rigorous error bounds, the Lebesgue constant (i.e. the maximum of the Lebesgue function), for a class of polynomial projectors on cube, simplex and ball, including interpolation, hyperinterpolation and weighted least-squares. Several examples will be shown. Moreover, we will present some optimal admissible meshes for ball and simplex, based on Chebyshev nodes and we will compare them with other recently studied point sets by giving numerical evaluations, using the covering radius related to the Dubiner distance.

The talk will be based on two recent papers [1] and [2].

References

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ATMA 2024 Approximation: Theory, Methods and Applications
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Approximation by Max-product Sampling Kantorovich operators: quantitative estimates in Functional Spaces

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In [3], we started to face the problem of convergence for the so-called max-product sampling Kantorovich operators K_n^χ based upon generalized kernels in the setting of Orlicz spaces L^φ, establishing a modular convergence theorem for the approximation of non-negative functions defined on both bounded intervals $[a, b]$ and on the whole real axis. This non-linear (more precisely, sub-additive) version of Kantorovich sampling operators, obtained by replacing the series (or sum for finite terms) of the linear case [1] by the supremum (or maximum for finite terms), denoted in the literature (see, e.g., the monograph [2]) by the symbol $\bigvee$, has been introduced in [4]. Here, the authors studied their approximation properties, including convergence results and quantitative estimates, in the continuous setting and in L^p-spaces, $1 \leq p < +\infty$. Since, as it is well known, the latter functional spaces, among many others, can be considered particular cases of Orlicz spaces, the treatment in this general setting allows us to develop a unifying theory that covers a wide class of functions, including the not-necessarily continuous ones, that are very common in applications of signal and image processing.
In the present talk, we will show a recent study on the order of approximation of K_n^χ in L^φ via a quantitative estimate established using the Orlicz-type modulus of smoothness, introduced by means of the modular functional defined on the space if the approximation of functions $f : \mathbb{R} \to \mathbb{R}_0^+$ is considered. This result allows us to obtain the qualitative order of approximation for functions belonging to suitable Lipschitz classes. Furthermore, we also present an analogous study in the compact case $[a, b]$, where, due to technical reasons, it is preferable to work with K-functionals [5] in place of moduli of smoothness in order to obtain an upper bound for the modular of the error of approximation.
Finally, some specific examples of kernels of K_n^χ for which the main presented results can be applied, will be taken into account.

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Multiplicative Characteristic Functions

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We consider two arbitrary non empty sets X, Y and a map

$$D : X \times Y \to \mathbb{R}.$$

On the set X we consider the uniform (respectively pointwise) topology associated to this duality. Similar topologies are considered on the set Y. We show that X is relatively compact w.r. to the uniform topology iff Y is relatively compact w.r. its uniform topology.

Shmulyian–Eberline result is obtained for the pointwise topologies

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Multiplicative Characteristic Functions

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Multiplicative Characteristic Functions

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Very often, in Mathematics, the object we are dealing with is decomposed into other objects much simpler to understand. In this paper we introduce the concept of “multiplicative characteristic function” on an arbitrary set X and we give a general representation theorem of any function $f : X \to [0, 1]$ as a uniform limit of finite products of such type of functions.

This result may be fruitfully used in measure theory, distribution theory. Some consequences in uniform approximation of continuous functions are presented here.

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An algorithm for bivariate HP-splines regression

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A bivariate regression model is presented as an extension of univariate Hyperbolic-Polynomial penalized splines (HP-splines) [1]. The univariate basis requires the selection of a suitable frequency parameter, for which a data-driven procedure is available [2]. The proposed approach combines the alternating construction of univariate HB-spline basis functions along both coordinates and the tensor product structure to capture interactions between the two dimensions [3]. First results are provided and compared with state-of-art smoothers [4], according to the selected smoothing parameters. Numerical results are promising for further investigations and application to Gaussian surface approximation.

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Approximation properties of Sampling Kantorovich operators in Sobolev settings

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In this talk, we will analyze some approximation results of Kantorovich-type sampling operators in the context of classical and fractional Sobolev spaces settings. In particular, in the fractional case, we will show some approximation properties for a new class of Sobolev spaces based on the well-known Gagliardo fractional Sobolev spaces and a class of recently introducted Sobolev spaces by Feng and Sutton.

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Representation formulae for strongly continuous semigroups in terms of integrated means and related approximation processes

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The aim of the talk, which is based on the paper [1] and [2], is to present some representation formulae for strongly continuous semigroups on Banach spaces, in terms of limits of integrated means with respect to some given family of probability Borel measures and other parameters. Estimates of the rate of convergence by means of the rectified modulus of continuity and the second modulus of continuity are also provided.

The cases where the representation formulae hold true pointwise or uniformly on compact subintervals are discussed separately. In order to face them different approaches have been used: the former case has been studied by using purely functional-analytic methods, the latter one by involving methods arising from Approximation Theory. To this purpose, a suitable and very general sequence of positive linear operators, acting on continuous function spaces on an arbitrary real interval, has been introduced.

In the talk, we also discuss in detail the approximation properties of such a sequence in the context of weighted spaces of continuous functions, with respect to wide classes of weights. In particular, pointwise estimates and weighted norm estimates of the rate of convergence are presented, together with a weighted asymptotic formula.

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ATMA 2024 Approximation: Theory, Methods and Applications
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Mean sampling Kantorovich operators

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As well-known, generalized sampling operators and sampling Kantorovich operators are able to approximate continuous signals and even L^p-signals in the latter case. Anyway, in the situation of a signal affected by a noise, these operators are not very efficient to approximate the cleaned signal (i.e., filtered by the noise) when the parameter goes to infinity. In order to solve this problem, we introduce a new type of operator, which we call the mean sampling Kantorovich operator, we study its approximation properties and made a comparison with the classical sampling Kantorovich operator in dealing with noised signals.

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Approximation properties and applications of sampling-type operators

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In this talk, I will present an historical overview concerning the main approximation results related to the classical sampling theory, starting from the celebrated *Sampling Theorem* of Whittakker, Kotel'nikov and Shannon, and its various generalizations. We analyse the main reasons that brought back the introduction of the sampling-type operators, showing both classical results and some recent development of such a theory. In particular, we discuss the pointwise, the uniform, and the L^p convergence, for the *generalized and Kantorovich sampling operators* ([1,2,4]), showing also their saturation order ([1,5]). Furthermore, we will also present some extensions of the mentioned families of operators, including the very recent *Steklov sampling operators* ([3]). In conclusion, I will also present a real world application in the setting of biomedical diagnoses based on image analysis ([6]), in which the application of the Kantorovich sampling operators (in the multivariate form) plays a central role.

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Convergence and order of approximation for perturbed operators

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This talk aims to present some results regarding the convergence and the order of approximation for a class of sampling Kantorovich operators perturbed by multiplicative noise. By using the convergence result for continuous functions with compact support (see [1]) and a density approach, we establish the convergence of these operators in the general setting of modular spaces (see [2]). This framework allows us to apply the results to many classical cases such as Orlicz spaces, L^p-spaces and other spaces generated by modulars without integral representation. Specifically, in the case of Orlicz spaces, we further study estimates of the order of approximation in terms of the modulus of smoothness (see [3]). As a direct consequence, we can deduce similar estimates in L^p-spaces ($1 \leq p < +\infty$), and we are able to achieve sharper estimates than the previous ones, by using the properties of the modulus of smoothness. In the final part of the talk, we furnish an estimate in the space of uniformly continuous and bounded functions.

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Defeating the Runge phenomenon on different domains though a mixed interpolation-regression method

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The constrained mock-Chebyshev least squares approximation is a recently introduced method that operates on a grid of equidistant points, aiming to eliminate the Runge phenomenon. The implementation of the idea behind this approximation method involves interpolating the function exclusively on the subset of nodes closer to the Chebyshev–Lobatto node set of a suitable order and using the remaining nodes to enhance the accuracy of the approximation through a simultaneous regression. The main goal of this work is to discuss various extensions of the constrained mock-Chebyshev least squares approximation on different domains and its generalization through the interpolation on zeros of orthogonal polynomials, leveraging their inherent favorable properties.

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ATMA 2024 Approximation: Theory, Methods and Applications
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Multinode Shepard method: theory and applications

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In this talk, we provide an introduction to the Multinode Shepard methods and discuss their various applications. These methods are commonly used for interpolating scattered data in both two and three-dimensional domains [1]-[5], as well as on the surface of a sphere. Additionally, we explore how the Multinode Shepard method can be applied to the numerical solution of elliptic partial differential equations (PDEs) [6]-[8] and numerical integration [9].

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ATMA 2024 Approximation: Theory, Methods and Applications
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The Approximation of Power Functions with Exponents in $(-1, 0)$ by Sums of Exponentials and Its Applications

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In this talk, we will describe and compare various approaches for approximating the function $f_\alpha : (0, T] \to \mathbb{R}$, $t \mapsto t^{\alpha-1}$, with some $\alpha \in (0, 1)$ by sums of exponential functions, i.e. by expressions of the form

$$(*) \qquad \tilde{f} : (0, T] \to \mathbb{R}, \quad t \mapsto \sum_{\ell=1}^{L} w_\ell \exp(a_\ell t)$$

with suitably chosen values w_ℓ and a_ℓ ($\ell = 1, 2, \ldots, L$).

The primary application of such methods is in the construction of efficient algorithms for the numerical computation of Riemann-Liouville fractional integrals and for the numerical solution of Caputo-type fractional differential equations. Indeed, when comparing this concept to conventional approaches, one can see that by exploiting important features of the functions of the form $(*)$, both the memory and the run time requirements can be reduced significantly.

Keeping in mind this concrete application, we shall point out which features of our approximation process are particularly significant. An important aspect in this context is that we need to clarify in what sense it is reasonable to approximate the unbounded function f_α by bounded functions like $\tilde{f}$.

Clearly, to obtain an accurate approximation of the given function f_α by such a function $\tilde{f}$, it is necessary to choose the parameters w_ℓ and a_ℓ carefully. By looking at the construction underlying the sum-of-exponentials approach, we will derive certain strategies for finding suitable choices for these parameters. It turns out that this question can be looked at from various significantly different viewpoints each of which gives some unique insight, and so only a combination of all these specific perspectives allows to provide a complete picture.

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Approximation by Kantorovich sampling operators in variable exponent Lebesgue spaces

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Let $f : \mathbb{R} \to \mathbb{R}$ be a locally Lebesgue integrable function and $\chi : \mathbb{R} \to \mathbb{R}$. Let $\{t_k\}_{k\in\mathbb{Z}}$ be a sequence of real numbers such that $t_k < t_{k+1}$ and $\lim_{k\to\pm\infty} t_k = \pm\infty$, as $\theta \le \theta_k := t_{k+1} - t_k \le \Theta$, $k \in \mathbb{Z}$, with some constants $\theta, \Theta > 0$. Bardaro, Butzer, Stens and Vinti [1] introduced the Kantorovich-type sampling operators

$$(S_w^\chi f)(x) := \sum_{k\in\mathbb{Z}} \frac{w}{\theta_k} \int_{t_k/w}^{t_{k+1}/w} f(u)\, du\, \chi(wx - t_k), \quad x \in \mathbb{R}, \quad w > 0,$$

provided that the series is convergent.

The main subject of the presentation is a direct estimate and a matching two-term strong converse estimate of the rate of approximation of these operators in variable exponent Lebesgue spaces. The estimates are stated in terms of moduli of smoothness. They enable us to establish that the approximation process $\{S_w^\chi\}_{w>0}$ possesses the saturation property and to describe its saturation rate and class as well its trivial class. In addition, a Voronovskaya-type estimate for S_w^χ is included.

It is essential that the Hardy-Littlewood maximal operator is bounded in variable exponent Lebesgue spaces whose exponent satisfies certain assumptions.

A secondary goal is reducing the assumptions on the kernel of the operator. That leads to certain auxiliary results, which should be known but rarely, if ever, stated; so their explicit formulation might be helpful.

The results have recently appeared in [2].

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CQMC: Tchakaloff-like compression of QMC integration

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We present a method for Tchakaloff-like compression of Quasi-Monte Carlo (QMC) volume or surface integration. The key tools of the algorithm are Tchakaloff-Davis-Wilhelmsen theorems on the so-called "Tchakaloff sets" for positive linear functionals on polynomial spaces, and Lawson-Hanson algorithm for NNLS. Such compressed formulae preserve the approximation power of QMC up to the best uniform polynomial approximation error of a given degree of the integrand, but using a much lower number of sampling points.

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Holomorphic L^2 signals of several complex variables

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We build on work by P. Bouboulis & S. Theodoridis, [2], and pursue their program of recovering the kernel methods (as employed in signal analysis and machine learning theory) from *real* RKHS and kernels, to the *complex* domain. We solve the maximum problem

$$\sup \Big\{ \sum_{j=1}^{p} \big| f(z_j) \big|^2 \, : \, \|f\|^2 \leq E \Big\}$$

in the complex RKHS of holomorphic L^2 functions $f : \Omega \to \mathbb{C}$, for any bounded domain $\Omega \subset \mathbb{C}^n$ and any finite set of points $z_1 \, , \, \cdots \, , \, z_p \in \Omega$. The result is applied to the space $L^2 H(\mathbb{B}^n)$ of holomorphic L^2 functions on the unit ball $\mathbb{B}^n \subset \mathbb{C}^n$. The problem of producing sampling expansions starting from complete orthonormal systems $\{\phi_\nu\}_{\nu \geq 0} \subset L^2 H(\Omega)$ is taken up by refuting [based on counterexamples, such as the Bergman kernel $K(z, \, \zeta)$ for the unit ball $\Omega = \mathbb{B}^1$] K. Yao's hypothesis (cf. [4]) that

$$\phi_\nu(z) = c_\nu \, K(z, \, \zeta_\nu)$$

and instead by approximating each ϕ_ν uniformly on Ω by a linear combination of reproducing kernels. The means to said approximation are provided by the Faber-Kaczmarz-Mycielski algorithm $\mathcal{A}(h)$ learning (cf. [3]) from the data $\big\{ \big(\zeta_k \, , \, \phi_\nu(\zeta_k) \big) \big\}_{k \geq 0}$ and producing an approximating sequence $\big\{ \big(\phi_\nu \big)_k \big\}_{k \geq 0} \subset L^2 H(\Omega)$.

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Numerical treatment of mixed Volterra-Fredholm integral equations

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This talk deals with the numerical treatment of the following mixed Volterra-Fredholm integral equation

$$(I + \mu V K)f = g,$$

where $\mu \in \mathbb{R} \setminus \{0\}$, f is the unknown function, g is a given right-hand side, I is the identity operator, V is the Volterra operator given by

$$(Vf)(y) = \int_{-1}^{y} h(x,y) f(x)(y-x)^{\rho}(1+x)^{\sigma} dx, \qquad \rho, \sigma > -1,$$

with h an assigned kernel, and K is the Fredholm operator defined as

$$(Kf)(y) = \int_{-1}^{1} k(x,y) f(x)(1-x)^{\alpha}(1+x)^{\beta} dx, \qquad \alpha, \beta > -1,$$

with k a known kernel.

A method of Nyström type is presented to approximate the solution of the equation in the case when the given functions may have algebraic singularities at $y = \pm 1$. The integral operator is approximated by a Gauss-product mixed rule and a theoretical analysis of the method is carried out in suitable weighted spaces equipped with the uniform norm. Numerical tests are given to show the good performance of the procedure.

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On the constants in Hardy Inequalities in L_p and l_p

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The behavior of the smallest possible constants $d(a,b)$ and d_n in classical Hardy inequalities

$$\int_a^b \left(\frac{1}{x}\int_a^x f(t)dt\right)^p dx \le d(a,b) \int_a^b [f(x)]^p dx$$

and

$$\sum_{k=1}^{n} \Big(\frac{1}{k}\sum_{j=1}^{k} a_j\Big)^p \le d_n \sum_{k=1}^{n} a_k^p.$$

is discussed.

In the case $p = 2$ the exact constant $d(a,b)$ and the exact rate of convergence of d_n are established and the extremal function and "almost extremal" sequence are found.

For $2 < p < \infty$ the exact rate of convergence of $d(a,b)$ and d_n and "almost extremal" function and sequence are found.

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Approximations of Topological Structures in Flows Described by Navier-Stokes Equations for Incompressible Fluid

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Classes of exact solutions of the Navier—-Stokes equations for incompressible fluid flows are obtained. Invariant varieties of flows are highlighted and the structure of solutions is described. It is established that the typical invariant domains of such flows are rotation figures, in particular homeomorphic to torus, forming the structure of a topological bundle, for example in a ball, a cylinder and generally in complexes composed of such figures. The structures of these flows obtained by approximation by the simplest 3-D vortex unsteady flows are investigated. Classes of exact solutions of the Navier–Stokes system for an incompressible fluid in bounded domains of space $\mathbb{R}_3$ based on the superposition of the above topological bundles are distinguished.

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On wavelet type Chlodovsky Operators and their Bézier-type variants

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This paper deals with construction and studying wavelet type Chlodovsky operators and their Bézier variants by using the compactly supported Daubechies wavelets of the target function f. By using the Chanturiya modulus of variation we estimate the rate of pointwise convergence of $(WC_{n,\alpha}f)(x)$ at those $x > 0$ at which the one-sided limits $f(x+)$, $f(x-)$ exist.

It is clear that our wavelet type operators include at least the classical version of the Chlodovsky operators and the Kantorovich form. Hence our results extend some of the previous results on Chlodovsky, Chlodovsky Bézier, Chlodovsky-Kantorovich Bézier operators presented in [3], [4] and [5].

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Chebyshev admissible meshes and Lebesgue constants of complex polynomial projections

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We construct admissible polynomial meshes on piecewise polynomial or trigonometric curves of the complex plane, by mapping univariate Chebyshev points. Such meshes can be used for polynomial least-squares, for the extraction of Fekete-like and Leja-like interpolation sets, and also for the evaluation of their Lebesgue constants.

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ATMA 2024 Approximation: Theory, Methods and Applications
Lecce, June 11-14, 2024

Nyström methods for FIE's based on RBF functions

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The poster deals with the numerical approximation of 2D Fredholm linear integral equations of the type

$$f(x,y)-\mu\int_S k(x,y,s,t)f(s,t)dsdt = g(x,y), \qquad (x,y)\in S,$$

where S is the square $[-1,1]^2$, g and k are known functions defined on S and S^2, respectively, μ is a fixed real parameter and f is the unknown in S.

For this kind of equation several methods were proposed, based on piecewise polynomial approximation (see for instance [1], [4]) or on global approximation methods using tensorial operators (see [5], [3]). All these methods require that the known functions can be evaluated at fixed grids in $[-1,1]^2$.

Here we propose numerical methods of Nyström type for solving FIEs where k and g are known only on scattered data. In particular we compare two Nyström methods, one based on a cubature formulas introduced in [6], which uses the RBF functions, the other based on the near-optimal meshless cubature formula introduced in [2].Some numerical tests show the better performance on the second one.

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ATMA 2024 Approximation: Theory, Methods and Applications
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Approximate solution of singular fractional integro-differential equations

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In [2], the unique solvability of singular fractional differential equations was studied. More recently (see [1]), the numerical solution of singular fractional integro-differential equations with constant coefficients was investigated.

For the Banach space of q times continuously differentiable functions u on $[0,T]$ we use the notation $C^q[0,T]$, $q \in \mathbb{N}_0 = \{0,1,2,\dots\}$, $C^0[0,T] = C[0,T]$. By $L^1(0,1)$, we denote the Banach space consisting of real or complex-valued functions φ defined on the interval $(0,1)$ such that $\|\varphi\|_{L^1(0,1)} = \int_0^1 |\varphi(x)|dx < \infty$.

In this work, we consider singular fractional integro-differential equations of the form

$$(D_0^\alpha M^\alpha u)(t) = \sum_{k=1}^{l} b_k(t)(D_0^{\alpha_k} M^{\alpha_k} u)(t) + b(t)(V_\psi u)(t) + f(t), \quad 0 < t \le T. \tag{1}$$

Here the multiplication operator M^α, $\alpha \in \mathbb{R} = (-\infty,\infty)$, is defined as $(M^\alpha u)(t) = t^\alpha u(t)$ $(0 < t \le T)$ for $u \in C[0,T]$,

$$(V_\psi u)(t) = \int_0^t \frac{1}{t}\psi\left(\frac{s}{t}\right) u(s)ds = \int_0^1 \psi(x)u(tx)dx, \quad 0 \le t \le T, \quad u \in C[0,T],$$

i.e. $V_\psi : C[0,T] \to C[0,T]$ is a cordial Volterra integral operator [3] with core $\psi \in L^1(0,1)$, $\alpha, \alpha_k \in \mathbb{R}$ and

$$q < \alpha \le q+1, \quad \alpha > \alpha_k \ge 0, \quad b_k, b, f \in C^q[0,T], \quad k = 1,2,\dots,l, \quad q \in \mathbb{N}_0. \tag{2}$$

In equation (1) the fractional differential operator D_0^α, of the order $\alpha \in [0,\infty)$, is defined as the inverse of the Riemann-Liouville integral operator $J^\alpha : C[0,T] \to C[0,T]$ given by (in the following Γ denotes the Euler gamma function)

$$(J^\alpha u)(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} u(s)ds, \quad u \in C[0,T], \quad t > 0, \ \alpha > 0; \quad J^0 = I,$$

on the space $J^\alpha C[0,T]$, i.e. $D_0^\alpha v = (J^\alpha)^{-1} v$, where v belongs to the range $J^\alpha C[0,T]$ of J^α, $\alpha \ge 0$.

We first present some results about the unique solvability of equations of the form (1). Next, we introduce a scheme based on piecewise polynomial collocation to find the numerical solution of such equations and analyse the convergence and the convergence order of the proposed method. We also give the results of numerical experiments.

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A machine learning perspective for optimized kernel-based approximation

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Meshfree kernel methods have proved to be an effective tool in many fields of research. Their effectiveness usually depends on a shape hyperparameter, which is often tuned via cross-validation schemes. In this talk, we present a learning strategy that extends the classical single-parameter framework, and returns a kernel that is optimized not only in the Euclidean directions, but that further incorporates, e.g., kernel rotations. Then, by combining this approach with the usage of greedy strategies, we obtain an optimized basis that adapts to the data. Beyond a rigorous analysis on the convergence of the so-constructed two-layered kernel orthogonal greedy algorithm (2L)-KOGA, the benefits of the presented approach are highlighted on both synthesized and real benchmark datasets.

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ATMA 2024 Approximation: Theory, Methods and Applications
Lecce, June 11-14, 2024

Common invariant cones of sets of matrices

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We present a numerical algorithm to decide whether a finite family of square matrices possesses a common invariant cone. This question arises naturally in the joint spectral radius theory. For example, if a set of matrices possess a common invariant cone, then:
• The invariant polytope algorithm can use "larger" convex hulls, and thus is more efficient [2].
• The lower spectral radius is continuous, and thus can be computed [2].
• The 1-spectral radius equals the spectral radius of an easily constructible matrix, and thus can be computed [1].

Our algorithm is based on a combination of the invariant polytope algorithm and the tree algorithm [2,3,4]. Numerical examples indicate that the algorithm works for a large range of families of matrices.

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ATMA 2024 Approximation: Theory, Methods and Applications
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A global method for Volterra-Fredholm integral equations

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This talk deals with the numerical treatment of Volterra-Fredholm integral equations (VFIEs) of the second kind. In the literature, these equations appear in two forms, namely

$$(I + \mu_1 V + \mu_2 K)f = g,$$

and

$$(I + \mu V K)f = g,$$

where $\mu, \mu_1, \mu_2 \in \mathbb{R} \setminus \{0\}$, f is the unknown function, g is a given right-hand side, I is the identity operator, V is the Volterra integral operator given by

$$(Vf)(y) = \int_{-1}^{y} h(x,y) f(x) (y-x)^{\rho} (1+x)^{\sigma} dx, \qquad \rho, \sigma > -1,$$

and K is the Fredholm integral operator defined as

$$(Kf)(y) = \int_{-1}^{1} k(x,y) f(x) (1-x)^{\alpha} (1+x)^{\beta} dx, \qquad \alpha, \beta > -1.$$

Many mathematical models related to epidemic evolution, physical and biological problems, as well as parabolic boundary integral equations are formulated in terms of these integral equations.

In this context, we consider the case in which the kernels h and k may have algebraic singularities at the endpoints or along the boundary at $x = y$. Moreover, we also assume that the right-hand side may have algebraic singularities at ± 1.

We present a global approximation method of Nyström type to approximate the solution of such equations. Furthermore, we provide conditions that guarantee the stability and convergence of the method and we show some numerical examples to confirm the theoretical expectations.

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How differential geometry works in the RBF approximation theory!

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Given a set of n distinct points $\{x_j\}_{j=1}^n \subset \mathbb{R}^d$ and corresponding data values $\{f_j\}_{j=1}^n$, the radial basis function (RBF) interpolant is given by

$$s(x) = \sum_{j=1}^{n} \lambda_j \phi(\|x - x_j\|), \tag{1}$$

where $\phi(r)$, $r \geq 0$, is some radial function (cf. e.g. [1]). The expansion coefficients λ_j are determined from the interpolation conditions $s(x_j) = f_j$ for $j = 1, \ldots, n$, which leads to a symmetric linear system $A\lambda = f$, where $A = [\phi(\|x_i - x_j\|)]_{1 \leq i,j \leq n}$. The existence of a solution is assured for positive definite RBFs and also for conditionally positive definite RBFs by adding a lower degree polynomial to (1). We can introduce a shape parameter as $\phi(\varepsilon r)$ allowing to scale the basis function ϕ making it flatter as $\varepsilon \to 0$ and spiky as $\varepsilon \to \infty$. It is well-known that the RBF method is increasingly more accurate on steeper gradient surfaces and has difficulty approximating flat functions. The apparent reason is that the flat surfaces are represented by linear combinations of vary small shape parameters ε. But as ε becomes small, so does the condition number. Flat surfaces are parts of planes, cones, or cylinders where the Gaussian curvatures are zero. So it seems that one can choose appropriate RBFs according to the geometric properties of the function to be approximated [2].

In this talk, we first go through differential geometry basics [4]. Then we introduce the fundamental theorem of surface theory which describes the conditions for congruency of two parametrized surfaces. Then RBFs are categorized as surfaces of revolution according to the relation between their Gaussian and mean curvatures with the shape parameter ε. Some discussions are also given on the shape parameter selection of RBFs.

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Bounds for the Extreme Zeroes of Jacobi Polynomials

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The zeros of classical orthogonal polynomials have been a topic of intensive investigation. There are many reasons for this interest, such as the nice electrostatic interpretation of the zeros of the Jacobi, Laguerre and Hermite polynomials, their important role as nodes of Gaussian quadrature formulae, as well as the key role these zeros play in some extremal problems.

Derivation of sharp upper and lower bounds for the extreme zeros of orthogonal polynomials is of particular interest. We shall discuss some recent results about the extreme zeroes of the Jacobi and, in particular, of Gegenbauer polynomials. Typically, comparison of the different estimates does not single out “best bounds” as these estimates depend on two or three parameters. Sometimes preference is given to estimates given by simple expressions which are easier to work with. We are going to present bounds for the extreme zeroes of Jacobi polynomials, some of which are extremely simple, other are rather sharp, and some meet both criteria for simplicity and sharpness.

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Positive linear operators, convexity and approximation

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Positive linear operators are important tools in Approximation Theory. In their study a significant problem is the relationship with convex functions. Basically, this relationship can be expressed in terms of convex stochastic orders. We will present some known facts and new results in this context, as well as some possible new directions of investigation.

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Approximation Techniques for Environmental Modeling: Insights and Applications

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Environmental modeling, particularly in hydrological settings, demands robust and versatile approximation methods capable of addressing the multifaceted nature of environmental challenges. This presentation will look at advances in numerical simulations that can enhance the precision of long-term forecasts by modeling time-dependent processes in environmental systems, notably through the approximation of Dirac delta functions and the strategic selection of evaluation points.

Traditional numerical methods like finite elements and finite differences often struggle with the mesh dependence in approximating solutions to Partial Differential Equations (PDEs). However, advancements in scientific machine learning, particularly through Physics-Informed Neural Networks (PINNs), offer a promising alternative. This paper examines the efficacy of these methods in a space-time domain, focusing on groundwater flow equations, leveraging both theoretical insights and applied methodologies from recent literature. Additionally, this study explores how to manage the singularities inherent in PDEs through the regularization of Dirac delta functions.

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On unisolvence of unsymmetric random Kansa collocation

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In some recent papers the authors and some collaborators have investigated almost sure invertibility of unsymmetric random Kansa collocation matrices by some classes of RBF, for the Poisson equation with Dirichlet boundary conditions, i.e.

$$\begin{cases} \Delta u(P) = f(P) \ , \ P \in \Omega \ , \\ u(P) = g(P) \ , \ P \in \partial\Omega \ , \end{cases}$$

over a suitable bounded domain Ω.

The family of RBF includes Thin-Plate splines, Gaussians, MultiQuadrics, Generalized Inverse MultiQuadrics and Matérn. Sketches of the proofs will be shown, depending on the properties of the RBF.

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A new kernel for uniform approximation in RKHS

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We focus on the problem of the uniform approximation of a function in reproducing kernel Hilbert spaces [1]. Recently, such spaces are having a growing interest in literature, since they are widely used in learning theory.

If the trial set of points is well distributed on the underlying manifold, hyperinterpolation associated with a positive quadrature rule can be considered [2, 3]. In the cases that such an approximation is not satisfactory (e.g., the Lebesgue constants grow algebraically) we propose to improve the approximation by using the same data but a new kernel function depending on an additional parameter. The properties of the new approximant will be shown both from the theoretical and experimental point of view. More details can be found in [4].

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Recent Progress in Weighted L_p-Approximation of Functions by Kantorovich-type Operators

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Here we summarize recent results of the authors on weighted L_p-approximation of functions by Kantorovich-type modifications of certain classical linear operators.

It is well-known that the original Baskakov operator, Meyer-König and Zeller operator, Szász-Mirakjan operator, are not suitble for approximation of functions in L_p spaces. By implementing their Kantorovich-type variants we succeeded to obtain direct and strong converse results for the weighted approximation of functions in L_p-norm.

In general, the estimates involve related K-functionals.

Our main results on the subject are as follows.

- For the Meyer-König and Zeller-Kantorovich operator:
  Direct inequality for the weights $w(x) = (1-x)^\alpha$, $\alpha \in \mathbb{R}$ (IG, PP, RU; 2018).
- For the Baskakov-Kantorovich operator:
  Direct inequality for the weights $w(x) = (1+x)^\alpha$, $\alpha \in \mathbb{R}$ (PP; 2020);
  Strong converse inequality for the weights $w(x) = (1+x)^\alpha$, $\alpha < 0$ (IG, RU; 2020).
- For the Szász-Mirakjan-Kantorovich operator:
  Direct inequality for the weights $w(x) = (1+x)^\alpha$, $\alpha \in \mathbb{R}$ (IG, PP; 2021);
  Strong converse inequality for the weights $w(x) = (1+x)^\alpha$, $\alpha < 0$ (IG, PP, RU; 2024).

Furthermore, for the Baskakov-Kantorovich operator and for the Szász-Mirakjan-Kantorovich operator we obtain a complete characterization for the rate of the weighted approximation error by K-functionals.

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Meshfree Domain Decomposition Methods with Fundamental Solutions for Elliptic Boundary Value Problems

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The Method of Fundamental Solutions (MFS) is a Trefftz type meshfree and integration-free numerical scheme for the approximate solution of linear elliptic partial differential equations (PDEs). In its original formulation, see [1], the MFS is restricted to homogeneous PDEs with known fundamental solution. Nevertheless, this method has attracted significant attention from the scientific community due to its simple formulation and computational implementation and, most importantly, due to its remarkable accuracy when applied to boundary value problems (BVPs) posed in smooth settings, e.g. [2].

In the last decades, a large number of variants of the MFS have been developed and successfully applied to direct and inverse problems in the fields of acoustics, elasticity, electromagnetism, fluid mechanics, option prising, etc., e.g. [3]. In [4,5], we addressed the numerical simulation of acoustic and elastic wave propagation problems and extended the MFS to the non-homogeneous Helmholtz and Cauchy-Navier PDEs, respectively. In particular, by considering basis functions (fundamental solutions) that vary not only in terms of the location of their source points (singularities) but also with respect to their frequency, we formulated a domain version of the MFS, called MFS-D.

The main difficulty in the application of the MFS-D is that it requires the solution of a large and fully-populated ill-conditioned linear system, where the PDE and the boundary conditions of the problem are collocated simultaneously. The computational cost of this method becomes prohibitive when large scale problems, posed in domains with complex geometries, are considered. MFS-D is also not applicable to PDEs with singular, e.g. discontinuous, source terms, due to the analyticity of its basis functions in the domain of interest. In view of these problems, most of which are also shared by the classical MFS, it becomes important to investigate the use of domain decomposition techniques in the context of the MFS.

In the first part of this talk, we present some recent results, see [6,7], where we couple the MFS with a non-overlapping domain decomposition method (DDM). In particular, for the modified Helmholtz PDE, we consider an iterative approach, with Robin-Robin type transmission conditions on the interior boundaries, known as Lions non-overlapping DDM, e.g. [8]. The solution of the BVP is calculated in two steps. First, a particular solution of the PDE is approximated by superposition of acoustic plane waves, for a set of test frequencies and directions of propagation. In the second step, we use the classical MFS with Lions DDM to solve the associated homogeneous BVP. Convergence of the iterative scheme is proven theoretically and exemplified numerically for domains with non-trivial geometry. The accuracy of the method is illustrated for PDEs with discontinuous source terms.

In the second part of the talk, we focus on the overlapping domain decomposition approach. In particular, we combine the MFS with the Schwarz alternating DDM, see [9]. The direct application of this hybrid method shows unsatisfactory accuracy due to the presence of singularities in the boundary conditions of the sub-problems. We overcome this issue by augmenting the MFS

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approximation basis with a set of singular particular solutions of the PDE, that correctly fit the local behaviour of the solution of the BVP. The accuracy and convergence of the resulting method are illustrated for BVPs for the Laplace PDE, posed in domains with geometric singularities.

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Approximate solutions to linear fractional integro-differential equations

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We consider a class of fractional weakly singular integro-differential equations

$$(D^{\alpha_2}_{Cap}y)(t) + d_1(t)\,(D^{\alpha_1}_{Cap}y)(t) + d_0(t)y(t) + \int_0^t (t-s)^{-\kappa_0} K_0(t,s)y(s)ds$$

$$+ \int_0^t (t-s)^{-\kappa_1} K_1(t,s)(D^{\theta}_{Cap}y)(s)ds = f(t), \quad 0 \le t \le b, \tag{1}$$

subject to boundary conditions

$$a_i\, y^{(i)}(0) + b_i\, y^{(i)}(b) = \gamma_i, \quad a_i, b_i, \gamma_i \in \mathbb{R}, \quad i = 0, \dots, n-1. \tag{2}$$

Here D^{δ}_{Cap} is the Caputo differential operator of order $\delta > 0$ and $n := \lceil \alpha_2 \rceil$ is the smallest integer greater or equal to the fractional order α_2. We assume that

$$0 < \alpha_1 < \alpha_2 \le n, \quad \theta \in (0, \alpha_2), \quad \kappa_0, \kappa_1 \in [0, 1)$$

and that the given functions d_0, d_1, K_0, K_1 and f are continuous on their respective domains.

On the basis of [1] we study the existence, uniqueness and regularity of the solution y to problem (1)–(2) and show that under suitable conditions this problem can be reformulated as a Volterra integral equation of the second kind with respect to the fractional derivative $D^{\alpha_2}_{Cap}y$. We regularize the solution of this integral equation with the use of a suitable smoothing transformation and construct a numerical solution to the transformed integral equation by applying a piecewise polynomial collocation method on a mildly graded or uniform grid. We show the convergence of the proposed algorithm and present global superconvergence results for a class of specific collocation parameters. Finally, we complement the theoretical results with some numerical examples.

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A Formal Approximation of CNN Filters

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Deep learning approaches, in particular Convolutional Neural Networks (CNNs), are currently a very effective tool in various fields such as prediction, regularization and approximation, classification etc.. Some examples can be found in [1, 2]. However, one of the major criticisms to the aforementioned approaches stems from the lack of a complete comprehension of their behavior, especially from a mathematical point of view. In particular, CNNs are very interesting as their architecture is based on filters that are learnt during the training phase. Only a complete understanding of this important CNN component can help in understanding the whole framework. That's why there are few but insightful approaches in literature that try to solve this problem, such as [3, 4, 5, 6]. This talk will focus on an analysis of the filters learnt by a very simple CNN for a simple task: classification between a rectangular and triangular function under noisy condition. It will be shown that CNN filters have a common approximation in the frequency domain that is independent of their depths and size and is based on Generalized Gabor Filters. Moreover filters at different scales can be linked by a scale parameter. This parametric formulation allows us to describe the whole filters frequency response behavior through a suitable pde. Such an approach paves the way to a better understanding of CNNs from a theoretical point of view and may allow to design swallower CNNs from a practical point of view.

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Minimal cubature rules and interpolation on the square

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For a weight function W on $[-1,1]^2$ that is symmetric with respect to the origin, we consider the cubature rule of degree $2n-1$

$$\int_{-1}^{1}\int_{-1}^{1} f(x,y)W(x,y) = \sum_{k=1}^{N} \lambda_k f(x_k,y_k), \qquad \deg f \le 2n-1.$$

The number of nodes of such a cubature rule satisfies $N \ge N_{\min}(n)$, where

$$N_{\min}(n) := \frac{n(n+1}{2} + \left\lfloor \frac{n}{2} \right\rfloor.$$

The cubature rules with $N_{\min}(n)$ nodes are called minimal. The nodes of such a cubature rule also admit unique interpolation in an appropriate subspace of polynomials of degree $\le n$. The known weight functions that admit minimal cubature rules of degree $2n-1$ include the product Chebyshev weight [1] and, more generally [2,3],

$$W(x,y) = \frac{|x-y|^{2\alpha+1}|x+y|^{2\beta+1}}{\sqrt{1-x^2}\sqrt{1-y^2}}, \qquad \alpha,\beta > -1.$$

In this talk, we discuss several new families of weight functions that also admit minimal cubature rules.

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